

# Robust Pattern Recognition with Nonlinear Filters

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**Abstract.** Nonlinear composite filters for robust and illumination-invariant pattern recognition are proposed. The filters are based on logical and rank order operations. The performance of the proposed filters is compared with that of various linear composite filters in terms of discrimination capability. Computer simulation results are provided to illustrate the robustness of the proposed filters when a target is embedded into cluttered background with unknown illumination and corrupted by additive and impulsive noise.

## 1 Introduction

Correlation-based filters have been an area of extensive research over past decades [1-4]. A usual way to design filters is by optimizing some performance criteria. Various performance measures for correlation filters have been proposed and summarized [1]. For example, the classical matched spatial filter (MSF) [2] is optimal if an input image is corrupted by additive Gaussian noise. However, many real images are corrupted by non-Gaussian noise. Besides, the MSF is not able to discriminate effectively an object of one class and that belonging to other classes. Composite filters based on synthetic discriminant functions (SDF) [3] can be used for multiclass pattern recognition. SDF filters utilize a set of training images to synthesize a template that yields prespecified correlation outputs in response to training images. A drawback of SDF filters is appearance of false peaks on the correlation plane. A partial solution of this problem is to control the whole correlation plane by minimizing the average correlation energy (MACE) [4]. MACE filters suppress sidelobes while produce sharp correlation peaks at the target location. However, the filters are not tolerant to input noise.

Traditionally correlation-based filters use a linear correlation operation. Minimization of the mean absolute error (MAE) leads to a nonlinear correlation, which is computed as a sum of minima. The MAE criterion is often used to solve optimization problems in rank-order image filtering. This criterion is more robust when the noise has even slight deviations from the Gaussian distribution, and produces a sharper peak at the origin.<sup>5</sup> Recently, local adaptive correlations based on rank order operations were proposed to improve recognition in scenes with non-Gaussian noise [6,7]. However, their performance is poor in scenes with highly illuminated background.

In this paper we propose illumination-invariant nonlinear composite filters derived from the MAE criterion. With the help of computer simulations the performance of

the proposed filters is compared with that of linear composite filters. The paper is organized as follows: Section 2 provides a review of composite linear filters. Section 3 introduces the proposed filters. In section 4 we provide computer simulation results. Section 5 summarizes our conclusions.

## 2 Linear Composite Filters

Composite filters are usually used for distortion-invariant pattern recognition. In this case a set of training images that are sufficiently descriptive and representative of expected distortions can be employed to improve the recognition.

### 2.1 SDF filter

Conventional SDF filters are a linear combination of MSFs for different patterns. The coefficients of the linear combination are chosen to satisfy a set of constraints on the filter output requiring a prespecified value for each pattern used.

Suppose there are  $N$  training images from a true class, each image contains  $d$  pixels. We convert the 2D arrays of the images into the 1D column vector by lexicographical ordering. These vectors are the columns of a matrix  $\mathbf{R}$  of size  $d \times N$ . The column vector  $\mathbf{u}$  contains  $N$  elements, which are the desired values of the output correlation peaks corresponding to each training image. If the matrix  $(\mathbf{R}^+\mathbf{R})$  is nonsingular, the conventional SDF filter can be expressed as follows [3]:

$$\mathbf{h}_{SDF} = \mathbf{R}(\mathbf{R}^+\mathbf{R})^{-1}\mathbf{u} , \quad (1)$$

here superscript  $+$  means conjugate transpose. The main shortcoming of the linear SDF filters is appearance of sidelobes owing to the lack of control over the whole correlation plane.

### 2.2 MACE filter

In order to suppress false correlation peaks, the MACE filter minimizes the average correlation energy of the correlation outputs for a set of training images, satisfying at the same time the correlation peak constraints at the origin. Suppose that there are  $N$  training images, each image with  $d$  pixels. First, the 2D Fourier transform is performed on each training image and converted into 1D column vector. Then, a matrix  $\mathbf{X}$  with  $N$  columns and  $d$  rows is constructed. The columns of  $\mathbf{X}$  are given by the vector version of each transformed image. The frequency response of the MACE filter can be expressed as [4]

$$\mathbf{h}_{MACE} = \mathbf{D}^{-1}\mathbf{X}(\mathbf{X}^+\mathbf{D}^{-1}\mathbf{X})^{-1}\mathbf{u} , \quad (2)$$

where the column vector  $\mathbf{u}$  contains desired correlation peak values of the training images and the  $d \times d$  diagonal matrix  $\mathbf{D}$  contains the average power spectrum of the training images.

### 3 Nonlinear Composite Filters

We wish to design a nonlinear composite filter, which is invariant to illumination, robust to noise, cluttered background, and false objects. The proposed filtering is a locally adaptive processing of the signal in a moving window. The moving window is a spatial neighborhood containing pixels surrounding the central window pixel geometrically. The neighborhood is referred to as the  $W$ -neighborhood. The shape of the  $W$ -neighborhood is similar to the region of support of the target. The size of the neighborhood is referred to as  $|W|$ , and it is approximately taken as the size of the target. In the case of nonstationary noise or cluttered background (space-varying data), it is assumed that the  $W$ -neighborhood is sufficiently small and the signal and noise can be considered stationary over the window area.

#### 3.1 Illumination-Invariant Correlation

Let  $\{T(k,l)\}$  and  $\{S(k,l)\}$  be a target image and a test scene respectively, both with  $Q$  levels of quantization. Here  $(k,l)$  are the pixel coordinates. The local nonlinear correlation derived from the MAE criterion between a normalized input scene and a shifted version of the target at coordinates  $(k,l)$  can be defined as

$$C(k,l) = \sum_{m,n \in W} \text{MIN} [a(k,l)S(m+k,n+l) + b(k,l), T(m,n)] , \quad (3)$$

where the sum is taken over the  $W$ -neighborhood. The coefficients  $a(k,l)$  and  $b(k,l)$  take into account unknown illumination and a bias of the target respectively. The normalization coefficients can be computed by minimizing the mean squared error between the window signal and the target as:

$$a(k,l) = \frac{\sum_{m,n \in W} T(m,n) \cdot S(m+k,n+l) - |W| \cdot \bar{T} \cdot \bar{S}(k,l)}{\sum_{m,n \in W} (S(m+k,n+l))^2 - |W| \cdot (\bar{S}(k,l))^2} , \quad (4)$$

$$b(k,l) = \bar{T} - a(k,l) \cdot \bar{S}(k,l) , \quad (5)$$

here  $\bar{T}$  and  $\bar{S}(k,l)$  are the average of the target and local window signal over the  $W$ -neighborhood at the  $(k,l)$ 'th window position, respectively.

#### 3.2 Nonlinear Composite Correlation Filters

According to the threshold decomposition concept [8], a gray-scale image  $X(k,l)$  can be represented as a sum of binary slices:

$$X(k,l) = \sum_{q=1}^{Q-1} X^q(k,l) , \quad (6)$$

where  $\{X^q(k,l), q=1, \dots, Q-1\}$  are binary slices obtained by decomposition of the image with a threshold  $q$  as follows:

$$X^q(k,l) = \begin{cases} 1, & \text{if } X(k,l) \geq q \\ 0, & \text{otherwise} \end{cases}. \quad (7)$$

Now, assume that there are  $N$  objects from the true class  $\{T_i(k,l), i=1 \dots N\}$  and  $M$  objects to be rejected  $\{P_i(k,l), i=1 \dots M\}$  (the false class). We construct  $N$  reference images as logical combinations of the training images:

$$\hat{T}_i(k,l) = \sum_{q=1}^{Q-1} T_i^q(k,l) \cap \left[ \bigcup_{j=1}^M P_j^q(k,l) \right], \quad i=1 \dots N, \quad (8)$$

where  $\{T_i^q(k,l), q=1, \dots, Q-1, i=1, \dots, N\}$  and  $\{P_i^q(k,l), q=1, \dots, Q-1, i=1, \dots, M\}$  are binary slices obtained by threshold decomposition from corresponding training images of true and false classes respectively.  $\cup$  and  $\cap$  represent the logical union and intersection, respectively. Finally, the nonlinear composite correlation is computed by

$$\hat{C}(k,l) = \text{MAX} \left\{ \left\{ \frac{u}{t_i} C_i(k,l), i=1 \dots N \right\} \right\}. \quad (9)$$

where  $\hat{C}(k,l)$  is the composite correlation at the coordinates  $(k,l)$ ,  $C_i(k,l)$  is the  $i$ 'th correlation (see equation 3) between the input scene and the  $i$ 'th reference image (computed with equation 8),  $\text{MAX}(X_i)$  is the maximum value among all the  $X_i$ ,  $u$  is the desired value at the correlation output, and  $t_i = \sum_{k,l \in W} \sum_{q=1}^{Q-1} T_i^q(k,l) \cap \left[ \bigcup_{j=1}^M P_j^q(k,l) \right]$ .

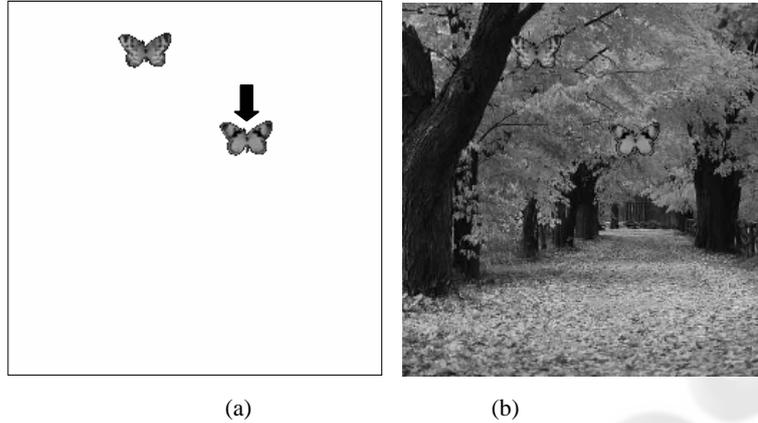
One can show that the composite correlation yields the value  $u$  at output correlation for objects belonging to the true class, while the output correlation peaks for the false objects are zeros.

#### 4 Computer Simulations

In this section computer simulation results obtained with the proposed filters are presented. The performance of nonlinear filters is compared with that of SDF and MACE filters in terms of discrimination capability (DC). The DC is formally defined as the ability of a filter to distinguish a target among other different objects [9], and can be expressed as:

$$DC = 1 - \frac{|C^B(0,0)|^2}{|C^O(0,0)|^2}, \quad (10)$$

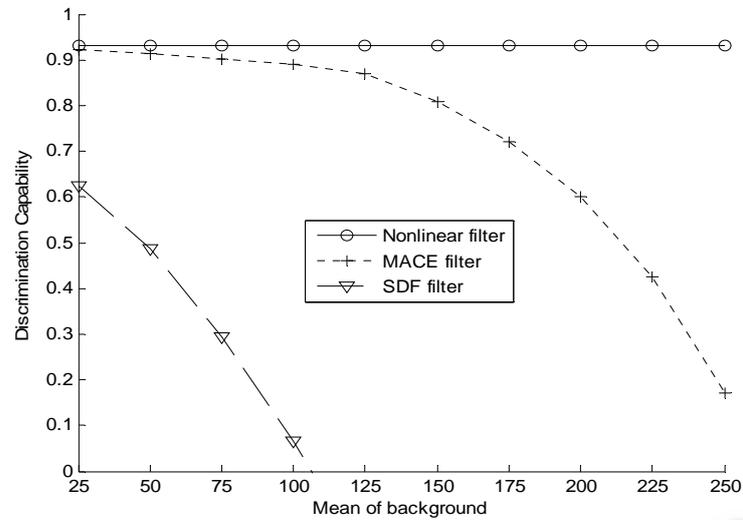
where  $C^B(0,0)$  is the maximum in the correlation plane over the background area to be rejected and  $C^O(0,0)$  is the maximum in the correlation plane over the area of the object to be recognized. The area of the object to be recognized is determined by the region of support of the target. The background area is complementary to the area of the object to be recognized.



**Fig. 1.** (a) Objects used in experiments (target is marked with the arrow), (b) test scene with objects embedded into a cluttered background.

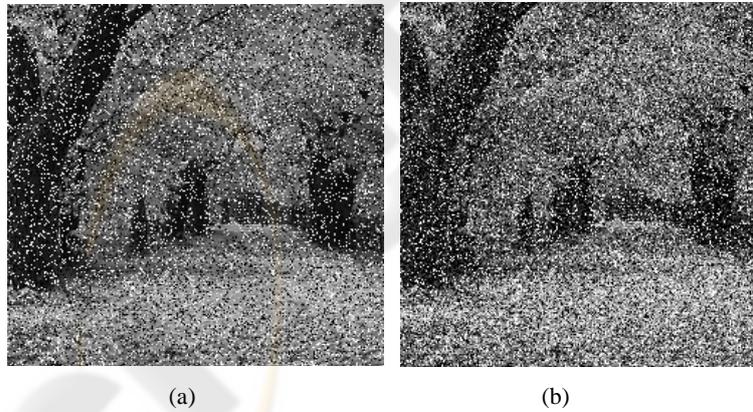
Figure 1(a) shows the objects used in computer simulations. The target is marked with an arrow. The size of the moving window is about  $19 \times 35$  pixels. The signal range of images is  $[0-255]$ . The objects of the true class are the target and its version rotated by 5 degrees. Figure 1(b) illustrates the objects embedded into a background. The size of scenes is  $256 \times 256$ . The mean of target is 92.3 and its standard deviation is 47.9. The mean of background is 93.8 with standard deviation of 48.7. We designed a filter with 2 objects from the true class and 1 object from the false class. 30 statistical trials in different positions of the objects were conducted and averaged. The DC values for the SDF, the MACE and the nonlinear filter are 0.13, 0.90, and 0.93, respectively.

Next the mean of background is varied while its standard deviation is fixed. Figure 2 shows the results. Note that the performance of linear filters deteriorates quickly when the background becomes highly illuminated, while the proposed nonlinear filter is illumination-invariant. The SDF filter fails to recognize the target in highly illuminated background.



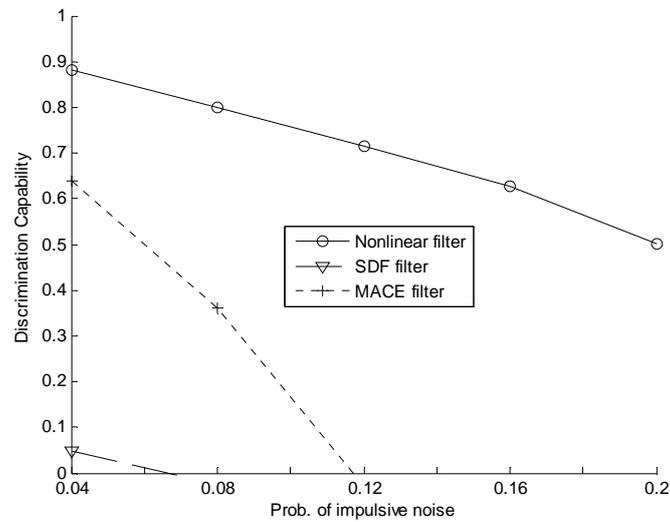
**Fig. 2.** Performance of filters in terms of DC as a function of the mean of background.

Now we show the robustness of filters to different kinds of noise. First, the scene is corrupted by impulsive salt and pepper noise. The probability of impulsive noise is varied from 0.04 to 0.2 with equal probability of occurrence for negative and positive impulses. To guarantee statistically correct results, 30 statistical trials of each experiment for different realizations of random processes were performed. Figure 3(a) is an example of the test scene corrupted by impulsive noise with probability of 0.2.

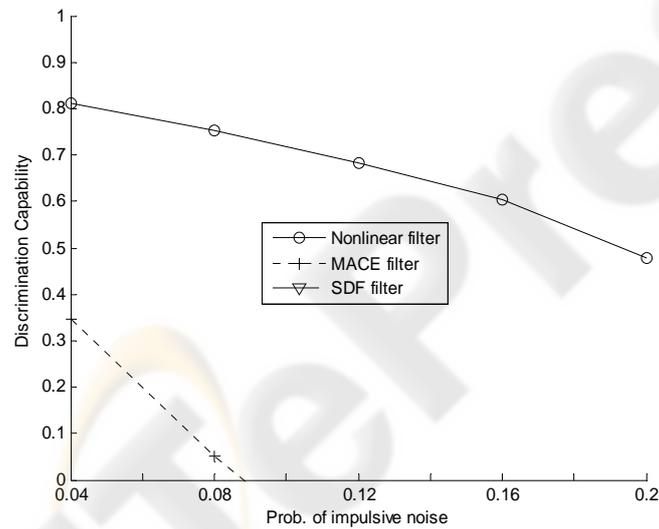


**Fig. 3.** Test input scene corrupted by (a) impulsive noise with probability of 0.2, (b) mixed additive noise with standard deviation of 40 and impulsive noise with probability of 0.2.

Figure 4(a) illustrates the performance of filters as a function of the probability of impulsive noise.



(a)



(b)

**Fig. 4.** Performance of filters in terms of DC as a function of impulsive noise probability for (a) impulsive noise only, (b) mixed additive and impulsive noise.

Note that performance of linear filters degrades rapidly, while the proposed filter is able to recognize targets. Finally the scene is corrupted by mixed additive Gaussian and impulsive noise. The standard deviation of additive noise is 40 and the probability of impulsive noise is varied from 0.04 to 0.2.

Figure 3(b) shows an example of a test scene corrupted with mixed noise, the probability of impulsive noise is 0.2. Figure 4(b) shows the computer simulation results. It

can be seen that the linear filters rapidly fail to recognize the objects, while the nonlinear filter is able to correctly detect objects in extremely noisy scenes.

## 5 Conclusions

In this paper, composite nonlinear filters were proposed. Their recognition performance and noise robustness were compared to those of composite linear filters in terms of discrimination capability. Extensive computer simulations illustrated an improvement in pattern recognition of multiple objects when the proposed filters are used.

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