# USING MESSAGE PASSING FOR DEVELOPING COARSE-GRAINED APPLICATIONS IN OPENMP

Bielecki Wlodzimierz and Palkowski Marek

Faculty of Computer Science, Technical University of Szczecin, Zolnierska 49 st., 71-210 Szczecin, Poland

- Keywords: Coarse-grained parallelism, send-receive mechanism, synchronization, agglomeration, locks.
- Abstract: A technique for extracting coarse-grained parallelism in loops is presented. It is based on splitting a set of dependence relations into two sets. The first one is to be used for generating code scanning slices while the second one permits us to insert send and receive functions to synchronize the slices execution. Codes of send and receive functions based on both OpenMP and POSIX locks functions are presented. A way of proper inserting and executing send and receive functions is demonstrated. Using agglomeration and free-scheduling are discussed for the purpose of improving program performance. Results of experiments are presented.

### **1** INTRODUCTION

Extracting coarse-grained parallelism available in loops is of great importance to develop shared memory parallel programs. To develop such programs, the **OpenMP** standard can be used (OpenMP, 2007). OpenMP is an Application Program Interface (API), jointly defined by a group of major computer hardware and software vendors. The API supports C/C++ and Fortran on multiple architectures, including UNIX & Windows NT. It permits for specifying parallel regions, work sharing, both barrier and mutual exclusion synchronization. However OpenMP does not support directly message passing (send and receive functions) among threads.

In this paper, we show the need for massage passing while we develop an OpenMP coarsegrained parallel program. Then we present how such a mechanism can be implemented on the basis of both OpenMP and POSIX locks. Agglomeration and free-scheduling are discuss to improve program performance. Results of experiments are presented.

In this paper, we deal with *affine loop nests* (Darte 2000). For extracting coarse-grained parallelism, we use an exact representation of loop-carried dependences and consequently an exact dependence analysis which detects a dependence if and only if it actually exists. To implement our algorithms, we use the dependence analysis proposed by Pugh and Wonnacott (Pugh 1998).

## **2 MOTIVATING EXAMPLE**

Let us consider the following loop.

۴	۴	1	ŕ	ſ	ſ
•	•	<b>\</b>	•	<b>\</b>	•
-	þ	-	-	<b>\</b>	•
•	•	-	-	-	-
•	•	•	•	•	•
6	6	6	6	-	

Figure 1: Dependences in the motivating example.

Using the Petit tool (Kelly 1997), we can extract the following dependence relations for this loop:

 $\begin{array}{l} R1 = \{[i,j] \rightarrow [i,j+1] : 1 <= i <= n \&\& 1 <= j < n\}, \\ R2 = \{[i,1] \rightarrow [i+1,1] : 1 <= i < n \&\& j=1\}. \end{array}$ 

Figure 1 shows the loop iteration space and dependences for n=m=6. To produce a coarsegrained parallel program, we may proceed as follows. Using relation R1, we can generate code scanning synchronization-free slices. For this

Wlodzimierz B. and Marek P. (2008). USING MESSAGE PASSING FOR DEVELOPING COARSE-GRAINED APPLICATIONS IN OPENMP. In Proceedings of the Third International Conference on Software and Data Technologies - PL/DPS/KE, pages 145-152 DOI: 10.5220/0001873801450152 Copyright © SciTePress purpose, we can apply any well-known technique, for example those presented in (Beletska 2007, Bielecki 2008). The following loop, scanning synchronization-free slices being described by relation R1(see Figure 2), is generated by the Omega Calculator *codegen* function (Kelly 1995).

Figure 2: Synchronization-free slices described by R1.

To made this code to be semantically the same as the working loop, we may insert in it send and receive functions responsible for preserving dependences represented by R2. To properly insert and execute send and receive functions, the values of the iteration index vector have to be checked. Let us suppose that a pair of associated send and receive functions are executed in such a way that the receive function will wait until the execution of the correspondent send function is completed. Then for the motivating example, we may apply the following way of inserting and executing send and receive functions.

If the iteration index vector, I, belongs to the set SET\_RECV = (Domain(R1) union Range(R1)) intersection Range(R2), comprising all the dependence sources and destinations described by R1 that are simultaneously dependence destinations described by R2, then a receive function must be inserted and executed prior the first loop statement that executes at iteration I. Its arguments are coordinates of the vector  $J = R2^{-1}(I)$ .

The correspondent send function must be inserted and executed after the last loop statement executing at iteration  $J=R2^{-1}(I)$ . Its arguments are also coordinates of J. That is, we may form a set SET\_SEND as  $R2^{-1}(SET\_RECV)$ . It comprises all

the iterations, after which execution, send functions must be executed.

To preserve the presented rules, we may insert send and receive functions as the body of *if* condition statements.

Using the Omega Calculator, we get the following sets for the working example

SET\_RECV={ $[i,1]:2 \le i \le n$ }, SET\_SEND={ $[i,1]:1 \le i \le n-1$ }.

Below we present the loop being formed taking into account the above presented rules. Figure 3 illustrates for which iterations send and receive functions should be executed and what are their arguments.

Figure 3: Parallel threads with synchronization.

# 3 ALGORITHM OF GENERATING COARSE-GRAINED LOOPS WITH SEND AND RECEIVE FUNCTIONS

Below we present an algorithm permitting for proper inserting and executing send and receive functions.

Algorithm. Inserting send and receive functions.

**Input.** 1) set S1 including dependence relations originating at least two synchronization-free slices; 2) set S2 comprising dependence relations to be used for the synchronization of the execution of slices produced by set S1(set S1 and set S2 together comprise all dependence relations extracted for the source loop); 3) relations Rel1 and Rel2 representing the unions of all the relations being comprised in set S1 and set S2, respectively; for sets S1 and S2, the following limitation is satisfied: Domain (Rel2) union Range (Rel2)  $\in$  Domain (Rel1) union Range (Rel1).

Output. Coarse-grained parallel code.

#### Method.

1. Using relations in set S1, generate code scanning synchronization-free slices applying any well-known technique, for example (Beletska 2007, Bielecki 2008) and extract all sets,  $I_j$ , representing the iteration space of statement  $st_j$ , j=1,2,...,q; q is the number of the loop body statements.

2. Calculate set SET\_RECV<sub>i</sub> = (Domain(Rel1) union Range(Rel1)) intersection Range(R2<sub>i</sub>) for each relation R2<sub>i</sub> in S2, i=1,2,...,n; n is the number of relations in S2.

3. Calculate set SET\_SEND = Rel2<sup>-1</sup>(Domain(Rel1)) union Range(Rel1)) intersection Range(Rel2).

4. Before each statement, st<sub>j</sub>, of the code generated in step 1 and for each relation R2<sub>i</sub>, if (SET\_RECV<sub>i</sub> intersection I<sub>i</sub>)  $\neq$  FALSE insert the following code

 $\begin{array}{ll} \text{if}(I \in \text{SET}_{\text{RECV}_i}) \\ & \text{recv}(\text{R2}_i^{-T}(I)); \ /*I \text{ is the} \\ \text{iteration vector being defined by the} \\ \text{loops surrounding st}_i */ \end{array}$ 

5. After each statement  $st_j$  of the code generated in step 1, if (SET\_SEND intersection  $I_j$ )  $\neq$  FALSE insert the following code:

```
if(I ∈ SET_SEND)
    send(I); /*I is the iteration
vector being defined by the loops
surrounding st<sub>j</sub> */
```

Let us generate now code for the motivating example according to the presented algorithm.

#### Input.

 $S1 = \{R1\}, Rell = \{[i,j] \rightarrow [i,j+1] : 1 \le i \le n \&\& \\ 1 \le j \le n\},$   $S2 = \{R2\}, Rel2 = \{[i,1] \rightarrow [i+1,1] : 1 \le i \le n \\ \&\& j=1\}.$  Step 1.if (n >= 2) {
for (t1 = 1; t1 <= n; t1++) }

 $\begin{array}{l} I_1 = \{[t1,1]: t1 >= 1 \&\& t1 <=n \}, \\ I_2 = \{[t1,t2: t1 >= 1 \&\& t1 <=n \&\& t2 >= 2 \&\& t2 <=n \} \end{array}$ 

Steps 2,3. SET\_RECV<sub>1</sub>={[i,1]:2<=i <= n}, SET\_SEND={[i,1] : 1 <= i <= n-1}.

For  $I_2$ , we have SET\_RECV<sub>2</sub> intersection  $I_2 =$  FALSE, SET\_SEND<sub>2</sub> intersection  $I_2 =$  FALSE, therefore send and receive functions are not inserted for statement st<sub>2</sub>.

# 4 IMPLEMENTATION OF SEND AND RECEIVE FUNCTIONS

In order to implement send and receive functions, we use lock functions. A lock function takes a lock variable as an argument. Firstly we present send and receive functions based on OpenMP lock functions, then we demonstrate how they can be implemented on the basis of POSIX lock functions.

### 4.1 Send and Receive Functions based on OpenMP Lock Functions

For each vector I, belonging to set S\_SEND or set S\_RECV, a structure, *s\_synch*, is created. The structure consists of a binary released lock *mutex* and a boolean variable *executed* initialized with the false value. Below we present the code of the send

The arguments of the send function are coordinates of vector I, n is the number of coordinates of I. Firstly the function gets the reference to a correspondent structure. The function *omp\_set\_lock* acquires the correspondent lock and waiting until it becomes available, if necessary. The next statement sets the variable *executed* to true. The function *omp\_unset\_lock* releases the lock *mutex*, resuming a waiting thread.

Below is the code of the receive function.

```
void recv(int t1, int t2,...,int tn)
{
struct s_comm *set_c =
          &SET_RECV[t1][t2]...[tn];
  while(1)
  {
     omp set lock(&set c->mutex);
     if(!(set c->executed))
         omp_unset_lock(
                 &set c->mutex);
     else {
         omp_unset lock(
                   &set c->mutex);
         break;
     }
  }
}
```

The arguments of the receive function are coordinates of vector I. Firstly the function gets the reference to a correspondent structure. In order to check the variable *executed*, the lock has to be set by means of the function *omp\_set\_lock*. A thread executes the while loop as long as the value of the variable *executed* is false (at each iteration, it releases the lock and next locks the lock again).

When the value of the variable *executed* becomes true(a correspondent send function was executed), the execution of the receive function is terminated allowing a waiting thread to be resumed.

#### 4.2 Send and Receive Functions based on POSIX Lock Functions

The disadvantage of the OpenMP lock functions is that waiting is active, causing wasting processor cycles. The send and receive functions, implemented with POSIX Threads (Posix 2004), do not posses such a disadvantage.

A condition variable in POSIX allows threads to wait until some event is occurred without wasting CPU cycles. Several threads may wait on the same condition variable until some other thread signals this condition variable (sending a notification). Then, one of the threads waiting on this condition variable wakes up. It is possible also to wake up all threads waiting on a condition variable by using a broadcast method on this variable.

It is worth to note that a condition variable does not provide locking. Thus, a lock is used along with a condition variable, to provide necessary locking when accessing this condition variable.

The code of the send function is as follows.

Firstly, the function *pthread mutex lock* acquires the lock on the *mutex* variable. If the *mutex* variable is already locked by another thread, the call of this function will block the calling thread until the mutex variable is unlocked. Then the variable executed is set to true, signaling that the iteration defined by vector I=(t1,t2,...,tn) has been executed. POSIX function *pthread\_cond\_brodcast* The signals all waiting threads on the condition variable cond and causes their awaking up. Finally, the function *pthread mutex unlock* unlocks the *mutex* variable. Below the code of the receive function is presented.

Firstly the lock is locked. Then the variable *executed* is checked. If its value is false, the thread, calling the receive function, is put to sleep. The function *pthread\_cond\_wait* blocks the calling thread until the specified condition, *cond*, is signaled. This function is called while the mutex variable is locked, and it will automatically release the mutex variable while it waits. After the signal is received and the tread is awakened, the mutex variable will be automatically locked for use by the tread. Finally the thread releases the lock calling the function *pthread\_mutex\_unlock*. If the variable *executed* was set to *true*, the thread only releases the *mutex* variable and continues computing.

# 5 SLICES AGGLOMERATION AND ORDER OF ITERATIONS SCANNING

To reduce the volume of synchronization among threads, it is worth to agglomerate several slices into one macro-slice to be executed by an OpenMP thread. This permits for eliminating synchronization among slices within a macro-slice. For this purporse, we can combine several slices into one macro-slice. Let us suppose that applying agglomeration we produced N macro-slices. Then we add an additional parallel loop to scan those N macro-slices. The remaining inner loops with new lower and upper bounds *lb*, *ub*(their values depend on N) are executed serialy and scan iterations of a particular macro-slice.

To avoid inner synchronization in a macro-slice, additional constraints in *if* statements must be added (step 4, 5 in the algorithm presented in Section 3). We have to check for each vector I whether iterations  $R2_i^{-1}(I)$  and R2(I) belong to the iterations of a macro-slice. If this is not true, synchronization is required.

Program performance will depend not only on the volume of synchronization but also on how we scan iterations of macro-slices. Figure 4 represents two different ways to scan iterations of macro-slices. On the left-hand side, iterations are scanned in lexicographic order, for example, for the first macroslice the order of the iteration execution is (1,1), (1,2),..., (2,1), (2,2),... On the right-hand side, iterations are executed according to the freescheduling policy (Darte 2000) that supposes that in each step we execute those iterations whose input data is already completed, i.e., in each step the iterations between a pair of skewing neighborhood

lines are executed (the order of their execution can be arbitrary because they all have completed data). There is barrier synchronization in the end of each step. For example, for the first macro-slice the following order of the iterations execution is valid: step1: (1,1), step2: (2,1), (1,2), step3: (3,1), (2,2), (1,3) and so on. It is obvious that applying the freescheduling policy improves the program performance because it reduces the waiting time. For example, applying lexicographic scanning, the second macro-slice can start its execution after the execution of 13 iterations(see Figure 4) while applying the free-scheduling policy, we can start the execution of the second macro-slice after the execution of 4 iterations only.

In paper (Bielecki 2008), we proposed how to generate code scanning iterations according to the free-scheduling policy. For scanning macro-slices, we should add a tuple to a set holding iterations to be executed at the next step (Bielecki 2008), only if an iteration  $R2_i(I)$  belongs to the iterations of a macro-slice.

Below, we present the pseudo-code that scans iterations of macro-slices for our working example according to the free-schedule policy. This code is generated on the basis of the approach presented by us in paper (Bielecki 2008), and the described above way to insert send and receive functions. The additional constraints permitting for avoiding inner synchronization are bolded in the pseudocode below.

```
par for w = 1 to N {
     calculate
                  upper
                            and lower
  //bounds, ub and lb as follows
 // pack = ((o ub-1)+1)/N,
 //lb = pack *(w-1)+o_lb;
 // ub = (w==N) ? n : pack *w;
// where o_lb, o_ub are the lower
//and upper bounds of the outermost
//loop generated in step 1 of the
//algorithm presented in Section 3
  S' = \emptyset;
  for t=lb to ub {
      I = [t, 1]
      Add I to S';
  }
  while (S' != \emptyset) {
    S tmp= Ø;
    foreach(vector I=[i,j] in S') {
       if(j==1 && 2 <= i && i <= n &&
  !(i-1>=1b && i-1 <=
                              ub))
                                    /*
            message
 receive
                        from
                                another
 macro-slice */
        recv(i-1,1);
       s1(I);
       if(j==1 && 1 <= i && i < n &&
  ( !(i+1>=1b && i+1 <= ub) ) )
```



Figure 4: Illustrating slices agglomeration and the order of iterations scanning.

Loop 1	Loop 2	Loop3
for i=1 to n do for j=1 to n do a(i,j) = a(i,j-1) + a(i-1,1)/1.1 endfor endfor	for i=1 to n do a(i, 1) = a(1, i) for j=1 to n do a(i,j) = a(i,j+1)/1.001 endfor endfor	for i=1 to n do for j=1 to n do if(i==2 and j==1) then a(i,j) = a(i-1,j) endif a(i,j) = a(i,j-1)/1.1 + a(i-2,j+1)/1.1 endfor endfor

```
Table 1: Loops for experiments.
```

```
/*send message to another macro-
slice */
```

send(i,1);

```
if(j==1 && 1<=i && i<n &&
lb<=i+1 && i+1>=ub) /*if
R2(I) &= domain R2
and I &= Iterations of macro-slice
*/
        add J=[i+1,j] to S_tmp;
/* J=[ip,jp]= R2(I) */
}
S' = S tmp; }
```

# 6 **EXPERIMENTS**

To evaluate the performance of programs being formed by the proposed approach, we have tested three loops (see Table 1). OpenMP parallel coarsegrained codes were produced by us on the basis of presented approach. The POSIX library (POSIX threads C+ access library2.2.1-2) was included in OpenMP sources to permit us to use send and receive functions based on POSIX lock functions. We have carried experiments on the machine Intel Xeon 1.6 Ghz, 8 processors (2 x 4-core CPU, cache 4 MB), 2 GB RAM, Ubuntu Linux.

The results are presented in Table 2, where time is presented in seconds, S and E denote speed-up and efficiency, respectively. The second column in Table 2 indicates whether agglomeration was used (+) or not(-), F means that free scheduling was used, N is the number of macro-slices.

For Loop 1 and Loop 2, we examined how agglomeration impacts speed-up and efficiency. Loop 3 originates only two slices, hence agglomeration was not considered.

Experiments with Loop 1 and Loop 2 demonstrate that i) increasing the volume of calculations executing by a thread(increasing n) increases S and E; ii) the positive impact of agglomeration on S and E grows with increasing the number of processors; iii) free scheduling is important for the number of processors greater than 2. However it is not obvious what is the value of N that optimizes program performance. As N grows, the waiting time to start the macro-slice execution decreases while the volume of synchronization increases.

	Aggl.	CPU	1		2			4			8			
Loop		n	seq. for	synch	time	S	Е	time	S	Е	time	S	Е	
	-	1500	0,0441	0,0462	0,0296	1,490	0,745	0,0231	1,281	0,320	0,0288	1,028	0,128	
Loop1		2000	0,0782	0,0814	0,0475	1,646	0,823	0,0346	2,263	0,566	0,0390	2,005	0,251	
		2500	0,1222	0,1271	0,0713	1,714	0,857	0,0461	2,651	0,663	0,0680	1,797	0,225	
	+/F	1500	0,0441	0,0470	0,0299	1,475	0,737	0,0213	2,070	0,518	0,0272	1,621	0,203	
Loop1	N = 200	2000	0,0782	0,0861	0,0479	1,633	0,816	0,0350	2,234	0,559	0,0312	2,506	0,313	
		2500	0,1222	0,1300	0,0721	1,695	0,847	0,0487	2,509	0,627	0,0461	2,651	0,331	
	+	1500	0,0446	0,0452	0,0308	1,449	0,724	0,0251	1,777	0,444	0,0247	1,806	0,226	
Loop2	N = 16	2000	0,0793	0,0801	0,0516	1,537	0,768	0,0314	2,525	0,631	0,0318	2,494	0,312	
		2500	0,1238	0,1249	0,0720	1,719	0,860	0,0458	2,703	0,676	0,0382	3,241	0,405	
	-	1500	0,0578	0,0580	0,0391	1,478	0,739	-	-	-	-			
Loop 3		2000	0,0843	0,0844	0,0527	1,600	0,800	-	-	-	-		0-	
		2500	0,1148	0,1153	0,0648	1,772	0,886	-	-	- //	0-/	No.	-	

Table 2: Results of experiments (POSIX functions).

radie 5. Influence of the number of macro-snees on program performance (1 051X functions	Table	3:	Influence of	of the	number	of r	nacro-s	slices	on	program	performance	$(\mathbf{P}$	OSIX	[ ft	inction	s)
--	-------	----	--------------	--------	--------	------	---------	--------	----	---------	-------------	---------------	------	------	---------	----

CPU	1			2			4		8			
Ν	seq. for	synch	time	S	Е	time S		E	time	S	Е	
32	0,0819	0,0908	0,0507	1,615	0,808	0,0428	1,914	0,478	0,0370	2,214	0,277	
64	0,0819	0,0912	0,0498	1,645	0,823	0,0356	2,301	0,575	0,0346	2,367	0,296	
128	0,0819	0,0878	0,0514	1,593	0,797	0,0324	2,528	0,632	0,0307	2,668	0,333	
256	0,0819	0,0876	0,0538	1,522	0,761	0,0342	2,395	0,599	0,0364	2,250	0,281	
							1	0				

1 CPU, s 2 CPU, s 4 CPU, s 8 CPU, s OpenMP POSIX WS OpenMP POSIX WS OpenMP POSIX WS OpenMP POSIX WS Ν Seq 0,0894 0,0587 0,0516 0,0496 0,0392 0,0381 0,0378 0,0375 0,0819 0,0866 0,0837 0,036 0,0359 512 (18,3%;4%) (4,5%;0,3%) (6,8%;3,5%) (3,7%;0,8%) 0.0853 0,0568 0,0501 0,0499 0.0345 0.0309 0.0306 0,0819 0.0888 0.0852 0.0386 0,036 0,0317 256 (4,2%;3,5%) (13,8%; 0,4%) (12,7%; 1,0%) (21,8%; 13,6%) 0,0576 0,0521 0,0512 0,0819 0,0875 0,0866 0,0856 0,0356 0,0318 0,0316 0,0365 0,0319 0,032 128 (12,5%; 1,8%) (12,7%; 0,6%) (14,1%;0%) (2,2%; 1,2%) 0,0819 0,1001 0,0965 0,0845 0,0585 0,056 0,0509 0,0431 0,0401 0,0371 0,0372 0,0343 0,0311 32 (18,5%; 14,2%) (14,9%; 10%) (16,2%;8,1%) (19,6%;10,3%)

Table 4: POSIX and OpenMP functions.

The size of a macro-slice impacts also program locality – a very important factor defining program performance. Table 3 presents how the number of macro-slices (Loop 1, free-scheduling with agglomeration), N, impacts S and E. For each number of processors, there exists a particular value of N optimizing program performance.

Table 4 presents differences in execution times of programs being written with OpenMP and POSIX lock functions for Loop 1, n=2048. The columns named as WS present program execution times when

send and receive functions are removed from code. In the brackets, the percentage of the program execution time is inserted that characterizes synchronization time on the basis of OpenMP and POSIX functions, respectively. The data in Table 4 demonstrates that POSIX functions permit for better program performance in comparison with OpenMP functions.

## 7 RELATED WORK

Unimodular loop transformations (Banerjee 1990), permitting the outermost loop in a nest of loops to be parallelized, find synchronization-free parallelism. However when there exist dependences between slices, that approach fails to extract coarse-grained parallelism.

The affine transformation framework (polyhedral approach), considered in many papers, for example, in papers (Darte 2000, Feautrier 1994, Lim 1999) unifies a large number of previously proposed loop transformations. It permits for producing affine time and space partitioning and correspondent parallel codes. For the motivating example, there does not exist any affine space partitioning permitting for extracting coarse-grained parallelism; we are able to find only time partitioning to extract fine-grained parallelism.

Our contribution consists in demonstrating how to generate coarse-grained code on the basis of two sets of dependence relations, the first of them permits for extracting synchronization-free slices (our previous results: Beletska 2007, Bielecki 2008) while the second one is used for inserting send and receive functions. The proposed approach extracts coarse-grained parallelism when in a loop there does not exist synchronization-free slices.

# 8 CONCLUSIONS AND FUTURE WORK

We introduced the approach to extract coarsegrained parallelism in loops. Send and receive functions are used to synchronize the slices execution. Our future research direction is to derive techniques permitting for automatic splitting a set of dependence relations into two sets such that the first one is to be used for generating code scanning slices while the second one is to insert send and receive functions.

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