

EFFICIENT SOURCE CODING IN A THRESHOLDING-BASED ECG COMPRESSOR USING THE DISCRETE WAVELET TRANSFORM

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Abstract: The aim of electrocardiogram (ECG) compression is to achieve as much compression as possible while the significant information for diagnosis purposes is preserved in the reconstructed signal. The source coding stage allows us to modify the compression ratio without quality degradation through a lossless encoder. In this work, the performance of this stage is analyzed in a compression scheme that has already presented good results among those from the state of the art. The compressor is based on discrete wavelet transform, thresholding and two-role encoder. The study consists of fixing all the stages except the source coding one in order to obtain an upper compression ratio bound. The assessment is based on the entropy of the independent symbols and the minimum expected length of the codewords. The results reveal a gap to improve the compression ratio, so from the previous entropy study an alternative compression method is proposed. For this purpose the symbols probabilities are analyzed through the normalized histogram. Thus, a Huffman encoder instead of the two-role one is applied in the new compressor to attain the maximum compression ratio. In this way a significant improvement is obtained without decreasing the original retrieved quality.

1 INTRODUCTION

The electrocardiogram (ECG) provides essential information to cardiologists for diagnosing. Therefore, ECG processing has been a topic of great interest and is most commonly used in applications such as monitoring. Since 1961, when Holter (Holter, 1961) introduced new techniques to monitor continuously the electrical activity of ambulatory patients, ECG recording techniques have gone through constant evolution. The current ECG recording systems allow to gather long-term signals with a duration of several hours in a simple, inexpensive and non-invasive way. Then the amount of ECG data increases considerably and compression becomes a necessity in order to provide storage and transmission solutions.

With this in mind, many compression schemes have been proposed during the past decades. A short summary of them can be found in (Blanco Velasco et al., 2004b). Among them, the lossy compression techniques achieve great compression results at the expense of some distortion in the original signal. The point is that the lost information must be the irrelevant

from the diagnosis view (e.g. noise, lowest power frequencies, etc.), which in terms of the ECG means that the morphology of the reconstructed signal does not change significantly. Recently, several thresholding-based compression algorithms using signal decomposition techniques have been developed (Abo-Zahhad and Rajoub, 2002; Benzid et al., 2003; Chen et al., 2006; Blanco Velasco et al., 2004b; Blanco Velasco et al., 2004a; Benzid et al., 2007; Blanco Velasco et al., 2007), because they yield attractive performance and low computational cost. The typical block diagram of this kind of compressors is shown in figure 1. Although some of them utilize modern techniques for the decomposition of the original signal such as nearly-perfect reconstruction cosine modulated filter banks (N-PR CMFB) (Blanco Velasco et al., 2004b; Blanco Velasco et al., 2004a) or wavelet packets (WP) (Blanco Velasco et al., 2007) with good results, most of them are based on the discrete wavelet transform (DWT), which play an interesting role in the ECG data compression applications owing to their easy implementation and efficiency (Abo-Zahhad and Rajoub, 2002; Benzid et al., 2003; Chen et al., 2006;

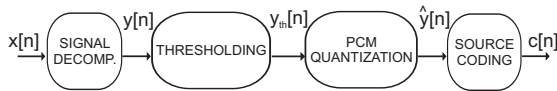


Figure 1: Block diagram of a typical thresholding-based compressor using signal decomposition.

Benzid et al., 2007).

In this paper, the study is focused on the thresholding-based compressors using DWT. The source coding stage of this kind of compression schemes is analyzed in order to obtain the maximum compression ratio (CR_{max}). For this purpose the rest of the blocks presented in figure 1 are fixed utilizing the compression scheme proposed in (Benzid et al., 2007). Then the statistical analysis of the independent symbols used in the source encoder is performed and an upper bound of the compression ratio is derived. Provided that some gap exists for source coding, we propose a new strategy to increase the compression ratio. Therefore a new compression scheme that yields better compression ratio (CR) is proposed.

2 THRESHOLDING-BASED COMPRESSORS

This type of compression schemes takes advantage of a signal transformation into a domain where the contribution of some coefficients to the reconstructed signal morphology is less important than others. A common technique would be to cancel the less significant wavelet coefficients, producing large runs of zeros. These bursts of zeros favor the compression through source coding implementation. Thus the original sample values are discarded and the quality of the retrieved signal is degraded. To perform this procedure, a threshold value must be determined. This parameter fixes the edge between the deleted samples and the significant ones. The threshold depends on quality and compression requirements whatever the aim is. Then the threshold can be fixed beforehand (Abo-Zahhad and Rajoub, 2002; Benzid et al., 2003; Chen et al., 2006) or be adjusted according to the desired results (Blanco Velasco et al., 2004b; Blanco Velasco et al., 2004a; Benzid et al., 2007; Blanco Velasco et al., 2007).

The significant coefficients are quantized through pulse code modulation (PCM). Finally, a lossless encoder is applied to the full sequence that includes the successions of zeros and the quantized significant coefficients altogether. This coding procedure consists of two main steps:

- Determining the independent symbols to repre-

sent the original signal that will be later encoded.

- Selecting the codewords that will encode each one of those symbols.

In this study, tests with this kind of compressors are carried out using the MIT-BIH Arrhythmia Database. As measurement criteria to evaluate the quality of the retrieved signal two parameters are used. One is the percentage root-mean-square difference (PRD), which is defined as:

$$PRD = \sqrt{\frac{\sum_{n=1}^N (x[n] - \hat{x}[n])^2}{\sum_{n=1}^N (x[n])^2}} \times 100, \quad (1)$$

where $x[n]$ is the original ECG signal and $\hat{x}[n]$ is the reconstructed one. Since the PRD is strongly dependent on the signal mean value, high mean values can mask the real quality performance assessment. To avoid this, we also use the modified criterion as follows:

$$PRD1 = \sqrt{\frac{\sum_{n=1}^N (x[n] - \hat{x}[n])^2}{\sum_{n=1}^N (x[n] - \bar{x}[n])^2}} \times 100, \quad (2)$$

where $\bar{x}[n]$ is the signal mean value. Furthermore, it is established in (Zigel et al., 2000), that if the $PRD1$ value is between 0 and 9, the quality of the reconstructed signal is either 'good' or 'very good', whereas if the value is greater than 9, its quality group cannot be determined.

Moreover the CR value is calculated as follows:

$$CR = \frac{b_x}{b_c}, \quad (3)$$

where b_x is the amount of bits used to represent the original ECG signal $x[n]$ and b_c is the total number of bits obtained after the source encoder block to represent $c[n]$.

3 COMPRESSION METHODS

The first compression scheme used in this work is based on that of proposed in (Benzid et al., 2007). It utilizes discrete wavelet transform (DWT) and thresholding. The threshold is obtained through an iterative approach that finishes when the PRD reaches the PRD target (PRD_{target}) as is proposed in (Blanco Velasco et al., 2004a). The compressor consists of the following main steps:

1. Choose the PRD_{target} and apply the DWT, up to the fifth level and with the mother wave “bior4.4”.
2. Threshold the wavelet coefficients with an iterative algorithm in order to match the PRD_{target} within a tolerance of 1%.
3. Quantize in PCM the nonzero wavelet coefficients. The quantization resolution B is chosen in an adaptive way. The value of B is increased until the PRD matches the PRD_{target} , with a tolerance of 10%.
4. Lossless source coding of the thresholded and quantized coefficients by means of the two-role encode (TRE) technique, which consists of the following:
 - (a) A baseline of 2^B is added to the significant coefficients. Then each of them is coded with $B + 1$ bits.
 - (b) The runs of zeros are also coded also with $B + 1$ bits. The minimum encodable length is 1 and the maximum is $2^B - 1$, this way these codewords are different from those used in the previous point. Thus longer successions of zeros must be represented with more than one codeword.

The other compression scheme differs from the first one in the 5th stage. In this case, the lossless source coding is carried out with a Huffman encoder (Skretting, 1999) although using the same symbols as obtained with the TRE method. Thus shorter codewords are assigned to less probable symbols and the expected length of the codeword is reduced since the symbol probabilities are different. In the Huffman encoder, the logarithmic encoding and the iterative sequence splitting options have been disabled in order to preserve the original independent symbol set. Therefore a fair comparison analysis between the compression and quality results of both methods can be done.

4 UPPER COMPRESSION RATIO BOUND ACHIEVEMENT

The CR_{max} that can be achieved with a lossless source encoder is studied in order to obtain an upper compression bound. To do so, the entropy of the independent symbols used as input in the source coding stage is analyzed.

The source entropy defines a boundary to the expected length of any instantaneous code (Cover and Thomas, 2006):

Teorema 1. *The expected length L of any instantaneous D -ary code for a random variable X is greater*

than or equal to the entropy $H_D(X)$, with equality when $D^{-l_i} = p_i$, where p_i is the symbol i probability and l_i its corresponding codeword length.

$$L \geq H_D(X). \quad (4)$$

The procedure to derive CR_{max} in the compressor schemes described in section 3 consists of the following steps:

1. The independent symbol set used to encode is chosen. The independent symbols are defined as those that present a biunique relationship with the final codewords. In our case the full symbol set includes two different subsets:
 - The runs of zeros with length less or equal than $2^B - 1$.
 - The significant coefficients, which are quantized with B bits.

Thus the independent symbol set consists of $2^{B+1} - 1$ different symbols, as the value 0 is useless because there are not successions of zeros with this length.

2. The probability mass $p(x)$ of the random variable X is calculated. Then the entropy is derived, considering the binary alphabet $D = \{0, 1\}$:

$$H(X) = - \sum_{x \in \mathcal{H}} p(x) \log p(x). \quad (5)$$

3. The CR is obtained as a result of two independent compression processes: PCM quantization stage and source coding. For PCM quantization, the compression ratio CR_1 only depends on the resolution B used. The lower B , the greater CR_1 . On the other hand, the compression ratio of the source coding CR_2 can be improved without modifying the quality of the retrieved signal. According to theorem 1, the entropy establishes a bound to the expected length L of the codeword and consequently to CR_2 as follows:

$$CR_2 = \frac{B}{L} \leq \frac{B}{H}. \quad (6)$$

Therefore, the relation $\frac{B}{H}$ can be considered as the upper bound of CR_2 (CR_{2max}).

4. To obtain CR_{max} , the CR_1 value is also needed. It follows this equation:

$$CR_1 = \frac{B_x}{B} = \frac{11}{B}, \quad (7)$$

where B_x is the original sample resolution, whose value is 11 for this database, and B refers to the quantization resolution.

5. Finally the CR_{max} value can be calculated applying the following equation:

$$CR_{max} = CR_1 \cdot CR_{2max}. \quad (8)$$

5 EXPERIMENTAL STUDY

Tests are carried out using signals from the MIT-BIH Arrhythmia Database. Files in the database contain two leads sampled at 360 Hz with 11 bits per sample of resolution. The signal length taken is 182 seconds, that is 65520 samples per signal. This value is very close to the two to a power 2^{16} (65536), which allows us to speed the DWT computing up. Moreover the 1024-baseline added to each lead for storage purposes is removed before processing.

A statistical study with the signal 117 is firstly done. The independent symbols used as input in the source encoder are obtained and the normalized histogram is calculated. Moreover the reconstructed waveform is derived and the error signal is calculated as the difference between the original and the retrieved samples. The PRD_{target} as well as the quality and compression results are presented in table 1. Figure 2 shows the histogram. Two graphs are obtained because the whole set of symbols is split into two groups as explained in section 4. Note in the histogram of figure 2 that symbols in the run of zeros have null probability while others are clearly much more likely. Symbols with null probability which does not need any codeword, so it is not necessary to encode them. Also shorter as possible codewords must be assign to the more frequent symbols.

Moreover the first 4096 samples of the original, retrieved and error signal are shown in figure 3. The cardiologists actually diagnose through visual analysis of the ECG, so both waveforms must look as similar as possible. In this case, taking into account the great upper compression bound obtained, an optimal visual quality of the reconstructed signal is attained. Furthermore the error is equally distributed over time, so cannot mask significantly any part of the signal by accumulation.

Table 1: Upper compression bound for signal 117.

| Signal | PRD_{target} | B | CR_{max} | PRD | $PRD1$ |
|--------|----------------|-----|------------|-------|--------|
| 117 | 2 | 7 | 17.24 | 2.12 | 8.01 |

The first compression study is also carried out with the signal 117. Several values of B are taken in order to check the efficiency of the adaptive quantization technique. The results are shown in table 2. As can be seen, the criterion to select B holds for $B = 7$, which is that of chosen by the adaptive quantization procedure in table 1. This strategy permits to reduce the value of B , and as a result increases the CR , in a dynamic way according to the nature of each ECG signal. So the adaptive quantization algorithm is re-

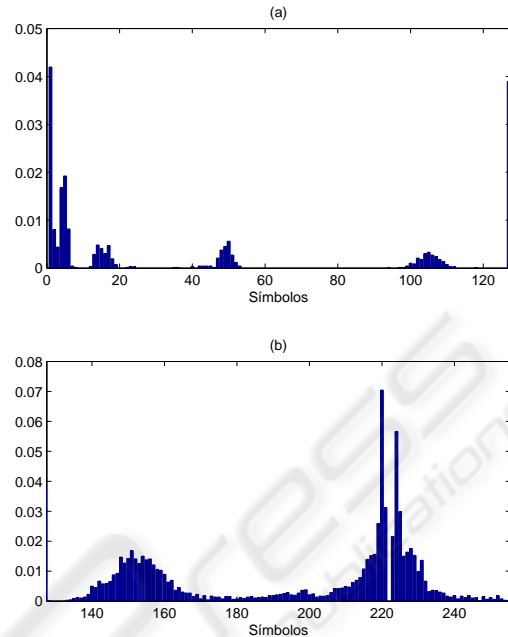


Figure 2: Normalized histograms of signal 117 for $PRD_{target} = 2$. (a) Runs of zeros (b) Significant coefficients.

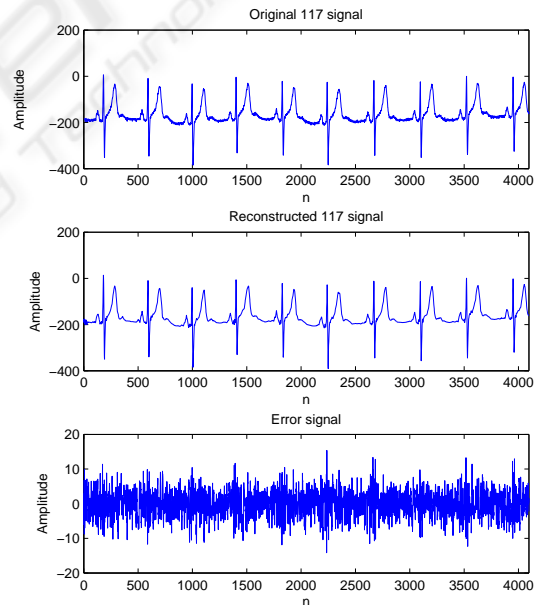


Figure 3: Compression waveform of signal 117 for $PRD_{target} = 2$.

vealed as efficient and valuable to be included in the quantization stage of the proposed method. Also the compression ratios are greater and closer to the upper bound for all the tested B values.

Another study is performed with the dataset com-

Table 2: Quality and compression results as a function of B for the 117 signal.

| $PRD_{target} = 2$ | PRD | CR_{max} | CR TRE method | CR prop. method |
|--------------------|-------|------------|-----------------|-------------------|
| $B = 6$ | 2.44 | 18.68 | 14.68 | 18.40 |
| $B = 7$ | 2.12 | 17.24 | 13.65 | 16.96 |
| $B = 8$ | 2.02 | 15.73 | 12.38 | 15.36 |
| $B = 9$ | 2.00 | 14.33 | 11.25 | 13.81 |
| $B = 10$ | 1.99 | 13.11 | 10.28 | 12.36 |

posed of the records 100, 101, 102, 103, 107, 109, 111, 115, 117, 118 and 119. In this case different PRD_{target} values are taken, which can be seen in table 3. In the table 3 we show the averaged CR , PRD and $PRD1$ of the whole dataset for different PRD_{target} . The obtained $PRD1$ values, except the last one, are all under 9, which stands for good quality of the reconstructed signal. The CR as a function of PRD is shown in figure 4. As can be seen in the graphic, our methods yields better compression ratio in the full range.

Table 3: Compression and quality results for the signal set.

| PRD_{target} | | 2 | 2.5 | 3 | 3.5 | 4 | 4.5 | 5 |
|-----------------|------------|------|-------|-------|-------|-------|-------|-------|
| PRD | | 2.13 | 2.63 | 3.18 | 3.67 | 4.20 | 4.78 | 5.25 |
| $PRD1$ | | 3.77 | 4.64 | 5.64 | 6.51 | 7.41 | 8.42 | 9.27 |
| Comp. limit | CR_{max} | 9.12 | 10.98 | 13.08 | 14.61 | 15.99 | 17.52 | 18.51 |
| TRE method | CR | 6.35 | 8.02 | 9.71 | 10.99 | 12.14 | 13.37 | 14.20 |
| Proposed method | CR | 8.95 | 10.75 | 12.82 | 14.31 | 15.68 | 17.19 | 18.17 |

As was said above, several symbols with null probability can be noticed in the runs of zeros histogram, meanwhile other symbols are clearly more frequent than any other. In the TRE method, same length codewords are used for every symbol, but it does not take advantage of the different symbols probabilities. As a result, figure 4 shows that the compression ratios derived with this technique are significantly lower than the maximum ones. On the other hand, the proposed compression scheme takes advantage of the different probabilities through a Huffman encoder. Thus shorter codewords are assigned to more probable symbols so the expected length of the generated code decreases and consequently the compression performance improves. In this way we obtain CR values closer to the upper bounds.

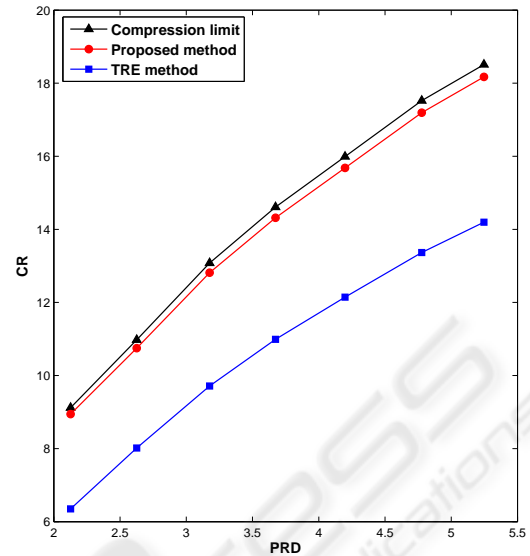


Figure 4: Compression comparative study results.

6 CONCLUSIONS

In this work a thresholding-based compression scheme using DWT is proposed. The results show a better performance than similar compressors in the state of the art. The entropy analysis of the independent symbols used to encode is carried out and the upper compression ratio bound is derived. The CR_{max} calculation is revealed as an effective tool to evaluate the compression bound that a specific scheme can provide. In this work, the TRE method has been analyzed as it has demonstrated good performance among those in its category. The analysis of the upper CR bound tells us that there is still a gap for the improvement. Therefore, a Huffman encoder (Skretting, 1999) is applied as source coder attaining better results for a wide range of PRD values. The obtained CR values keep very close to the upper bound. Therefore, an optimal result has been obtained for this particular scheme.

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