MULTI-PERSPECTIVE PANORAMAS OF URBAN SCENES WITHOUT SAMPLING ERRORS

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Keywords: Image Based Rendering, Multi-perspective panorama, City Visualization.

Abstract: In this paper we introduce a framework for producing multi-perspective panoramas of urban streets from a dense collection of photographs. The estimated depth information are used to remove sampling errors caused by depth parallax of non-planar scenes. Then, different projections are automatically combined to create the multi-perspective panorama with minimal aspect ratio distortions, which is achieved by a two-phase optimization: firstly, the global optimal configuration of projections is computed and then a local adjustment is applied to eliminate visual artifacts caused by undesirable perspectives.

1 INTRODUCTION

Rendering a street usually needs to combine different input photographs, as the field of view of a single photograph is limited to a portion of the street. Traditional image mosaicing techniques (Szeliski and Shum, 1997; Shum and Szeliski, 2000) assume input images are captured at a single viewpoint. In this case, the input images can be registered based on certain alignment models, e.g., the homography. However, it is usually impossible to place the viewpoint far enough to encompass the entire street. To acquire more scenes, we need to change the viewpoint of the camera. Generating panoramas from images captured at different viewpoints is much more challenging as an uniform alignment model for nonplanar scenes does not exist. In this paper, we present a framework for constructing panoramas from image sequences captured from a moving camera.

Recently, many approaches have been proposed to combine images captured at different viewpoints into a panoramic mosaic. These approaches can be grouped into the following three categories:

View Interpolation. These approaches warp pixels from input images to a reference viewpoint using the pre-computed 3D scene structure (Chen and Williams, 1993; Kumar et al., 1995). There are two main problems with these approaches: to establish an accurate correspondence between images for stereo is still a hard vision problem, and there will likely be

holes in the result image due to sampling issues of the forward mapping and the occlusion problem.

Optimal Seam. These approaches (Davis, 1998; Agarwala et al., 2006) formulate the composition into a labeling problem, i.e., pixel values are chosen to be one of the input images. To avoid discontinuity, the partition of different labeling is searched to minimize certain cost metrics such as pixel value difference. However, for scenes with large depth variations, it is often impossible to find such an optimal partition that can create seamless mosaics.

Strip Mosaic. The basic idea of the strip mosaic is to cut a thin strip from a dense collection of images and put them together to form a panorama. In the push-broom model (Zhu et al., 2001; Zheng, 2003), the result image exhibits parallel in one direction and perspective in the other, while the crossedslits (Zomet et al., 2003) model is perspective in one direction and is perspective from a different viewpoint in the other direction. The aspect ratio distortion is inherent due to the different projections along the two directions. Moreover, since the pinhole camera is used to capture input images, the result exhibits sampling errors due to the depth parallax. By combining different projection models, multiperspective panoramas can be synthesized, e.g., (Roman et al., 2004; Wexler and Simakov, 2005; Roman and Lensch, 2006).

Our approach is based on the strip mosaic, as it has many advantages. Strip mosaic are more efficient

than view interpolation, and thus can be easily scaled to long image sequences. Furthermore, unlike the optimal seam approach, even for scenes with complex depth, strip mosaic can produce satisfactory results by removing the sampling error and minimizing the aspect ratio distortion. In general, we have made two contributions:

- 1. We propose an approach for eliminating the sampling error based on the 3D scene structure. The principle behind our approach is similar to view interpolation, but we only perform the "interpolation" along one direction, and thus avoid the forementioned problems with the classic view interpolation techniques.
- 2. We present a two-phase optimization framework to create the multi-perspective panorama. Firstly, the optimal configuration of projections is searched to minimize the aspect ratio distortion. Then, local adjustment is applied to eliminate artifacts caused by undesirable perspectives.

The rest of this paper is organized as: Section 2 introduces the use of strip mosaic for rendering streets and the sampling error. Section 3 presents our approach for eliminating sampling errors. Section 4 presents the framework for generating the optimal multi-perspective panorama. Section 5 presents the result and Section 6 concludes this paper.

2 STRIP MOSAIC AND THE SAMPLING ERROR

In our system, street scenes are captured by a precalibrated video camera mounted on a vehicle, which is moving down a street with a slow and smooth speed to capture it looking sideways. Strips are cut from the captured image sequence and pasted into the result image. From the plan view of the capturing setup, each strip represents a sampled ray used to render an image from a novel horizontal projection center, which is actually a vertical slit in the 3D view. Figure 1 illustrates projection models relevant in our application, which are four special cases of the general linear camera summarized in (Yu and McMillan, 2004).

Because scenes within each strip are rendered from a particular pinhole perspective, given a certain strip width, there is a depth at which scenes show no distortion. For a further depth, some portions of the surface might be duplicately rendered, i.e., over-sampled, while for a closer depth, some portions of the surface can not be fully covered, i.e., under-sampled. In the literature, this kind of artifact is named the sampling error (Zheng, 2003). Figure 2(a)

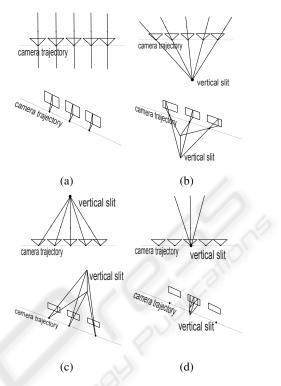


Figure 1: Projection Models. (a) The push-broom model, where the horizontal projection center is placed at infinity. (b) The crossed-slits model, where the horizontal projection center is placed off the camera trajectory. (c) The inverse perspective, where the horizontal projection center is put behind the camera trajectory. (d) The pinhole model, where the horizontal projection center is just placed at a camera's optical center.

illustrates the sampling error and Figure 2(b) gives a real example.

3 MOSAICING WITHOUT SAMPLING ERRORS

3.1 Single Direction Interpolation

In our system, the mosaicing result is rendered on a picture surface, which is defined by a 3D plane π_f . We assume the camera trajectory lies on a plane π_c . If scenes are exactly located on the picture surface, a point of the result image (p',q') can be mapped to a point (p,q) of an input frame by a projective transformation, i.e., the homography:

$$\begin{bmatrix} p \\ q \\ 1 \end{bmatrix} = H_i \begin{bmatrix} p' \\ q' \\ 1 \end{bmatrix} = P_i G \begin{bmatrix} p' \\ q' \\ 1 \end{bmatrix}$$
(1)

where $P_i = KR_i[I | -C_i]$ is the camera matrix of the *i*th

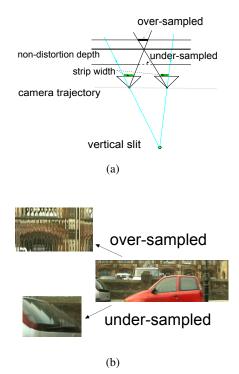


Figure 2: The Sampling Error (a) The sampling error is caused by the depth parallax. (b) A real example of the

sampling error.

frame. The camera parameters are extracted from the video sequence by the structure-from-motion (SFM) algorithm (Hartley and Zisserman, 2004). *G* is a 4×3 matrix that establishes the mappings between a 2D point of the result image and a 3D point on the picture surface.

We assume the horizontal projection center C_v lies on the camera plane and the vertical slit vl is the line that passes through C_v and perpendicular to the camera plane. We project the camera center C_i onto the result image c'_i along the line connecting C_i and C_v see Figure 3. A given point of the result image is rendered with the frame corresponding to the closest camera center projection c'_i .

For scenes with complex depth structures, a pixel from the input frame should be warped onto the result image based on the actual 3D coordinate, which is estimated by an approach resembling that in (Goesele et al., 2006). We search along the back-projected ray of a pixel and for each depth h, we project the corresponding 3D coordinate onto a neighboring frame and compute the normalized cross-correlation (NCC). The 3D coordinate is that with the highest NCC score. To enforce multi-view consistency, we compute the average value of h in a set of neighboring frames and use the robust estimation (RANSAC) to remove out-

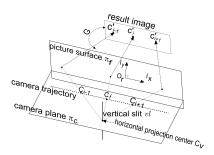


Figure 3: The mosaic is rendered on the picture surface. Camera centers are projected onto the picture surface and then mapped to the final result image.

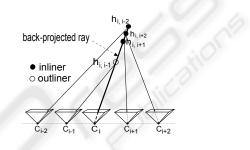


Figure 4: 3D Scene coordinate reconstruction.

liers. Figure 4 illustrates the depth estimation approach.

We define a vertical center line CL_i that passes c'_i on the result image. A vertical boundary line $BL_{\{i \leftrightarrow i+1\}}$ is drawn between any consecutive camera center projections. The center line CL_i is then mapped to CL_i on the source frame I_i . For each individual pixel (p,q), suppose its corresponding 3D coordinate is X_d , its mapping onto the picture surface is the intersection of 3 planes: the picture surface π_f , the plane π_v that contains X_d and the vertical slit vl and the plane π_h that contains X_d and the tangent line of the camera trajectory at C_i on the camera plane, see Figure 5. Once the intersection is recovered, it is mapped to the result image by G^+ , the *pseudo-inverse* of G. For a given input frame I_i , we only examine pixels within a region around \widehat{CL}_i . For each row of I_i , we take the pixel on \widehat{CL}_i as the starting point and search

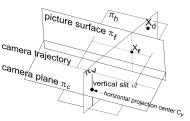


Figure 5: A pixel from the input frame is warped to the picture surface based on its corresponding 3D coordinate.

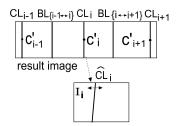


Figure 6: The center lines and boundary lines on the result image. The center line is mapped to the corresponding frame. The pixel warping is carried out within a region around the center line mapping.



Figure 7: The rendered image without sampling errors.

into both sides, once the warped point onto the result image is beyond the boundary line $BL_{\{i \leftrightarrow i+1\}}$ or $BL_{\{i-1 \leftrightarrow i\}}$, we proceed to the next row, see Figure 6.

However, this approach is sensitive to incorrect depth estimations. In practice, we assume the X-axis of the camera is coincident with the tangent line of the camera trajectory. Therefore, the value of q' can be directly computed using the homograph H_i . On the other hand, the value of p' depends on the actual 3D coordinate of (p,q). Suppose the picture surface π_f intersects π_v at a 3D line, and X_s and X_t are two points on that 3D line, then its mapping onto the result image is defined as:

$$\begin{bmatrix} ((G^{+})^{2\mathsf{T}}X_{s})((G^{+})^{3\mathsf{T}}X_{t}) - ((G^{+})^{2\mathsf{T}}X_{t})((G^{+})^{3\mathsf{T}}X_{s}) \\ ((G^{+})^{3\mathsf{T}}X_{s})((G^{+})^{1\mathsf{T}}X_{t}) - ((G^{+})^{3\mathsf{T}}X_{t})((G^{+})^{1\mathsf{T}}X_{s}) \\ ((G^{+})^{1\mathsf{T}}X_{s})((G^{+})^{2\mathsf{T}}X_{t}) - ((G^{+})^{1\mathsf{T}}X_{t})((G^{+})^{2\mathsf{T}}X_{s}) \end{bmatrix} \begin{bmatrix} p' \\ q' \\ 1 \end{bmatrix} = 0$$
(2)

where $(G^+)^{k\top}$ denotes the k^{th} row of the matrix G^+ . By solving this equation, the value of p' can be derived. Because with one direction the pixel warping adopts the original projective transformation, while the other is based on the real 3D coordinate, we name our rendering strategy a "single direction interpolation" as opposed to the full perspective interpolation. Figure 7 shows a rendered result.

In principle, the picture surface should lie along the dominant plane of street scenes, such as the building facet. One can fit the plane equation of the picture surface to the 3D points discovered by the SFM algorithm. However the fitting result is often a slanted plane, which would cause an non-uniform scaling of



Figure 8: The result image is rendered on a slanted picture surface.

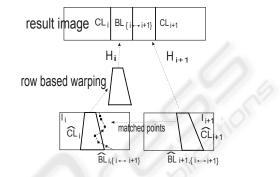


Figure 9: The fast algorithm using depth-variant strips.

scenes, see Figure 8. Therefore, we choose the picture surface to be perpendicular to the camera plane and parallel with the camera trajectory, i.e., frontoparallel. Based on this constraint, we use the least square fit to find its plane equation.

3.2 A Fast Approximation

It is very costly to compute the actual 3D coordinate for every warped pixel, and for large texture-less area, the depth estimation is not reliable. Therefore, we implement a fast approximation. Assuming \widehat{CL}_i and \widehat{CL}_{i+1} are mappings of the center line CL_i and CL_{i+1} , and $\widehat{BL}_{i,\{i \leftrightarrow i+1\}}$ and $\widehat{BL}_{i+1,\{i \leftrightarrow i+1\}}$ are mappings of the boundary line $BL_{\{i \leftrightarrow i+1\}}$ from the result image onto two consecutive frames I_i and I_{i+1} , see Figure 9. We search along the line $\widehat{BL}_{i+1,\{i \leftrightarrow i+1\}}$ and match a set of corresponding points on I_i with high NCC values. By interpolating and extrapolating these matched points, a curved stitching line is defined on I_i . We warp each row based on this stitching line, then the new derived quadrilateral is transformed to the result by H_i . On the other hand, the quadrilateral encompassed by \widehat{CL}_{i+1} and $\widehat{BL}_{i+1,\{i \leftrightarrow i+1\}}$ on I_{i+1} is directly transformed to the result image by H_{i+1} . The illustration is presented in Figure 9.

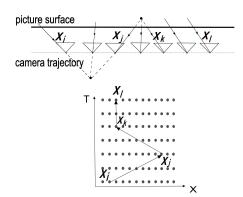


Figure 10: The multi-perspective panorama and the path on the X-T space.

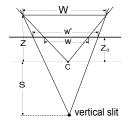


Figure 11: The aspect ratio distortion.

4 MULTI-PERSPECTIVE PANORAMAS

4.1 Global Optimization

This section describes how the projection models listed in Figure 1 are automatically combined to create a multi-perspective panorama. According to the paradigm proposed by Wexler and Simakov (Wexler and Simakov, 2005), the transitions of the strip location for creating a panorama form a path through the X-T space of the stacked volume of frames. To adopt this paradigm, the camera trajectory is restricted to be linear, i.e., straight. The picture surface is chosen to be fronto-parallel. In this setup, given a particular horizontal projection center, the center line CL_i in the result image is mapped to a vertical line \widehat{CL}_i in I_i , so that its X-direction location is fixed across rows. We denote this location as x_i . For illustration see Figure 10.

Figure 11 shows the aspect ratio distortion in this case, defined by:

$$\alpha = \frac{w'}{w} = \frac{W \frac{s+z_0}{s+z}}{W \frac{z_0}{z}} = \frac{z(z_0+s)}{z_0(z+s)}$$
(3)

To search the optimal path, we need a proper cost

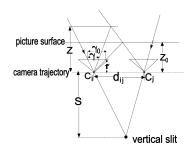


Figure 12: The relationship between the aspect ratio distortion and strip width.

metric for strip transition. The fast approximation algorithm gives us an intuition that the warping rate of a row reflects the aspect ratio distortion in the result. As shown in Figure 12, if scenes are exactly located on the picture surface the strip width γ_0 is: $\frac{1}{2}d_{ij}(\frac{f}{z_0} + \frac{f}{s})$, while for off-plane scenes, the strip width γ is: $\frac{1}{2}d_{ij}(\frac{f}{z} + \frac{f}{s})$. The rate between γ_0 and γ is equal to the aspect ratio distortion:

$$\frac{\gamma_0}{\gamma} = \frac{\frac{1}{2}d_{ij}(\frac{f}{z_0} + \frac{f}{s})}{\frac{1}{2}d_{ij}(\frac{f}{z} + \frac{f}{s})} = \frac{\frac{1}{z_0} + \frac{1}{s}}{\frac{1}{z} + \frac{1}{s}} = \frac{z(z_0 + s)}{z_0(z + s)} = \alpha \quad (4)$$

Based on (4), we define our error metric as:

$$E_{\alpha} = \begin{cases} \frac{\|\frac{\eta_{\gamma}}{2} - 1\|}{\max(\|\frac{\eta_{\gamma}}{\gamma}\|, 1)} & x_i \le x_j \\ \\ \eta^{\|x_i - x_j\|} \frac{\|\frac{\eta_{\gamma}}{2} - 1\|}{\max(\|\frac{\eta_{\gamma}}{2}\|, 1)} & x_i > x_j \end{cases}$$
(5)

A backward edge $(x_i > x_j)$, corresponds to an inverse perspective, see Figure 1(c). We penalize this with a higher cost $\eta^{||x_i-x_j||}$, and $\eta \approx 1.2$. Based on this error metric, the cost function associated with a strip transition is defined as:

$$E = \frac{1}{n_p} \left(\sum_p E_\alpha \right) + \beta \frac{\|x_j - x_i\|}{d_{ij}} \tag{6}$$

We only consider warping rates of rows with those matched points rather than the entire strip. p denotes such a matched point and n_p denotes the number of matched points involved. The second term of (6) is used to suppress strips that are too wide, because in this case discontinuities at strip borders are likely to be visible. Dijkstra's algorithm is used to find the shortest path. After the optimal projection configuration is achieved, we use the fast approximation algorithm to create the sampling-error-free panorama. We first search along the optimal path to locate all the maximal connected forward segments and render the result with these forward segments. Then the remaining backward segments are processed. Figure 13 presents an example, where some portions exhibit

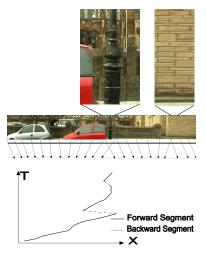


Figure 13: The result of the global optimization and the corresponding optima path.

heavy artifacts caused by the backward segment. In the next section, we describe how this problem can be handled by a local adjustment step.

4.2 Local Adjustment

The idea of the local adjustment is to avoid the use of the inverse perspective (the backward segments), i.e., we only consider those forward segments. For simplicity, we use the term "virtual camera" to denote these forward segments ¹. There are two possible spatial relationships between two adjacent virtual cameras: their rendered areas overlap on the picture surface Figure 14(a), or disjoint Figure 14(b). For the latter, we need to extend the field of view of the two virtual cameras to make them overlap, see Figure 14(c).

To make a seamless composition, we divide the overlapping region of the two adjacent virtual cameras into two parts, each of which is labeled with pixel values from the rendered result of a single virtual camera, see Figure 15. The optimal partition can be cast into a graph cut problem. We define the cost of a cut between any two neighboring pixels p and q as:

$$C(p,q) = C_d(p,q) + \mu C_g(p,q) \tag{7}$$

 $C_d(p,q)$ is the pixel value difference and $C_g(p,q)$ measures the partition cost in the gradient domain. The weight μ is chosen to be 0.01. $C_d(p,q)$ is defined

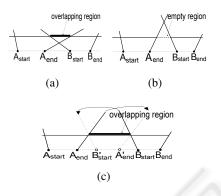


Figure 14: The Spatial relationship between adjacent virtual cameras. (a) Overlapping. (b) Disjoint. (c) The disjoint virtual cameras are expanded based on the bordering projection direction.

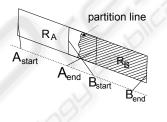


Figure 15: The Optimal Partition.

as:

$$C_d(p,q) = \sum_{channels} (NSSD(R_A, R_B, \omega(p)) + NSSD(R_A, R_B, \omega(q)))$$
(8)

where R_A and R_B denote the rendered images of the two virtual cameras A and B. $NSSD(R_A, R_B, \omega(p))$ is the normalized sum of squared pixel value difference between R_A and R_B computed in a patch around a given pixel ($\omega(p)$).

The gradient partition cost is the sum of two terms measuring the gradient magnitude and similarity:

$$C_{g}(p,q) = M_{R_{A}}(p) + M_{R_{A}}(q) + M_{R_{B}}(p) + M_{R_{B}}(q) + \rho \sum_{l \in \{x,y\}} (\|\nabla_{l}R_{A}(p) - \nabla_{l}R_{B}(p)\| + \|\nabla_{l}R_{A}(q) - \nabla_{l}R_{B}(q)\|)$$
(9)

where $M_{R_A}(p)$ denotes the magnitude of the gradient at a pixel, and $\|\nabla_l \cdot\|$ denotes the gradient along one dimension of the image space, *x* or *y*. We choose the weight $\rho = 0.8$.

The graph cut problem is solved using the maxflow/min-cut algorithm described in (Boykov and Kolmogorov, 2004). Figure 16 presents the improved panorama of Figure 13. In addition, a given portion of the picture surface might be covered by more than two virtual cameras. We adopt a straightforward solution: virtual cameras are processed in a series and for each incoming virtual camera, the optimal parti-

¹It should be noted that the term virtual camera is only used to denote forward segments, in fact, as shown in Figure 13, they are usually composed of several different projections.

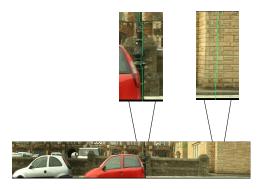


Figure 16: The multi-perspective image after local adjustment. The zoom-in view shows the optimal partition line (seam).

tion is performed on overlapping region between the current virtual camera and areas already rendered on the picture surface. If the incoming virtual camera has no overlapping region with areas so far rendered, the new virtual camera and its immediately previous one are expanded.

5 RESULTS

We have conducted experiments on our framework using image sequences captured by a digital video camcorder (Canon XM1), which captures at 25 frames/second. Compared to existing multiperspective panorama generation techniques, e.g., (Wexler and Simakov, 2005; Roman and Lensch, 2006), the essential improvement of our approach lies in the local adjustment as it makes our system capable of achieving the best trade-off between the seamless result and the maximal preservation of the human-eye perspective. Approaches described in (Wexler and Simakov, 2005; Roman and Lensch, 2006) are equivalent to the global-optimization step in our framework. In this sense, results with and without the local adjustment shown in Figure 13 and 16 present a comparison between these two kinds of approaches.

We have applied our techniques to longer streets. The result in Figure 17(a) visualizes a street that spans around 80 meters, and the street visualized in Figure 17(b) spans around 160 meters.

For the mosaicing result in Figure 7, the camera pose is extracted by Voodoo camera tracker [http://www.digilab.uni-hannover.de/docs/manual.html] with bundle adjustment. For long streets shown in Figure 16, 17(a) and 17(b), we rectify the input sequence to compensate for the camera tilt and we assume a translational motion along the horizontal direction at a constant speed. While, along the

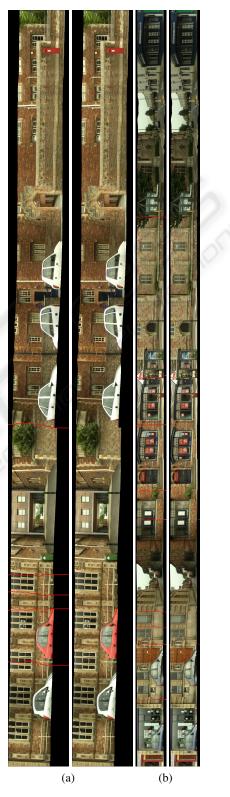


Figure 17: Multi-perspective panoramas. The first row of each image set shows the partition seam and the second without.

vertical direction, the displacement is computed by matching salient features and RANSAC is used to remove outliers.

The optimization framework is tested on a PC with two Xeon CPUs (2.00 GHz and 1.99 GHz) and 1.50GB ram. The global optimization of the result in Figure 17(a) (with 980 482 × 429 input frames) takes around 12 minutes and the result in Figure 17(b) (with 1200 395 × 227 input frames) takes around 8 minutes². The local adjustment of these two results both takes around 4 minutes.

6 CONCLUDING REMARKS

This paper presents a framework for producing multiperspective panoramas of street scenes. Our approach uses an estimation of 3D scene structure to eliminate the sampling error caused by the depth parallax. Then an automatic optimization is performed to create the panorama with minimal aspect ratio distortions. After that, a further local adjustment step is applied to remove artifacts caused by inverse perspectives. In principle, our approach is restricted to straight camera trajectories and approximately fronto-parallel picture surfaces. For non-straight camera trajectories, we assume they are piece-wise linear. However, for trajectories with abrupt direction changes, although our rendering system can handle this situation, the result of our global optimization is not theoretically accurate, as the aspect ratio distortion in this case is not yet clear.

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²To eliminate the redundant computation of matching points, a dense disparity map for each consecutive frame pair is pre-computed before optimization.