

VARIATIONAL REGION GROWING

Rose Jean-Loïc, Revol-Muller Chantal, Odet Christophe
CREATIS-LRMN, CNRS UMR 5220, Inserm U 630, 69621 Villeurbanne, France

Reichert Christian
Institut Camille Jordan UMR 5208, 69621 Villeurbanne, France

Keywords: Image segmentation, Region growing, Region-based criterion, Variational approach, Shape prior.

Abstract: Region growing is one of the most popular image segmentation methods. The concept of region growing is easily understandable but sometimes criticized for its lack of theoretical background. In order to overcome this weakness, we propose to describe region growing in a new framework which is the variational approach. A variational approach is commonly used in image segmentation methods such as active contours or level sets, but is quite original in the context of region growing. We call this method *Variational Region Growing*. First, we define a region-based criterion. A discrete derivation is applied to this criterion in order to get an evolution rule for the evolving region. The aim of this equation is to guide the evolving region towards a minimum of the criterion. Then, we formalize the iterative process of region growing in the proposed framework. Furthermore, we highlight the relevance of VRG for integrating shape prior. We apply VRG to synthetic and 3D-biomedical images. Results illustrate the improvements of VRG compared to classical methods.

1 INTRODUCTION

Image segmentation is a fundamental topic in image processing. The purpose of segmentation is to extract regions of interest. Since its introduction by (Zucker, 1976), region growing has become a popular method for 3D segmentation. In this approach, a homogeneous region is supposed to correspond to a semantic object. Starting from a seed, manually or automatically located, the iterative process of region growing extracts a region of interest by merging all the neighboring pixels. The merging of a pixel with the evolving region is governed by an aggregation criterion that must be satisfied. At each step, a set of candidate pixels neighboring the evolving region, or already belonging to it, are tested. Candidate pixels that meet the aggregation criterion are added to the evolving region, thus resulting in a new region.

In classical region growing methods, aggregation criterion can be categorized into two groups. In the first group, the criterion governs the growth of a single region. The criterion measures either a similarity between a candidate pixel and another pixel (Sekiguchi et al., 1994) or the homogeneity of the resulting segmented region (Revol-Muller et al., 2002). Such a

criterion requires the use of an arbitrary threshold to fix the minimum value of homogeneity. This method is attractive due to its simplicity, but the choice of the threshold needs further knowledge about the grey-level distribution to avoid trial and error adjustment. In the second group, the criterion governs a competitive growth of several regions. This kind of region growing called *seeded region growing* was introduced by (Adams and Bischof, 1994). At each iteration, the pixel the most similar to a region is looked up in the set of all candidate pixels and merged. This method is thus free of tuning parameters.

Region growing method is appreciated for its simplicity of use and its good segmentation results in various applications. The aggregation criterion usually relies upon low level features of the image such as grey levels of the pixels and the norm of intensity gradient. However, region growing method presents several drawbacks. First, region growing method lacks theoretical framework, whether it be for the description of the iterative process or the definition of aggregation criterion. Moreover, homogeneous regions are not always related to meaningful objects. So, an aggregation criterion only based on grey level measurements is not sufficient to lead to an accurate segmen-

tation. Region growing can not thus distinguish connected structures having similar intensities or statistics. In order to face this problem, added information must be taken into account during the growth such as geometrical constraints or shape prior.

In this paper, we propose to formalize the iterative process of region growing, and to set out a theoretical framework for the definition of the aggregation criterion. Our work is based on variational approach, so it was called *Variational Region Growing*(VRG). The main idea is to minimize a region-based energy by means of a discrete derivation. The major relevance of this framework is its ability to define various aggregation criteria. In (Rose et al., 2007), a new aggregation criterion integrating shape prior was presented. The shape prior was given by a binary volumic model of the target object. Here, we are going to incorporate that criterion into the variational region growing framework. This implies to define a new energy according to the former aggregation criterion. Then, this energy is minimized in order to get the evolution rule of the region growing. The tests were applied to synthetic and 3D-biomedical images. Qualitative and quantitative results show the ability of variational region growing to achieve good segmentation even in case of highly corrupted images, without depending on the pose parameters of the reference model.

2 VARIATIONAL REGION GROWING

Region growing can be described as an iterative process making a region evolve. The originality of VRG is to recover an object of interest by means of a discrete function that switches according to the minimization of an energy also called criterion. This criterion is designed so that its minimum corresponds to the sought solution. In the literature, many region-based energies were introduced into the variational framework. The purpose of VRG is to make a region evolve towards a meaningful partition of the image, using the minimization of such a region-based criterion.

2.1 Discrete Region Function

In our region growing formalism, the evolving region at the iteration n is represented by a discrete binary function $\phi_{\mathbf{x}}$ given by :

$$\begin{cases} \phi_{\mathbf{x}}^n = 0, & \text{for } \mathbf{x} \in \Omega_{in} \\ \phi_{\mathbf{x}}^n = 1, & \text{for } \mathbf{x} \in \Omega_{out} \end{cases} \quad (1)$$

with Ω the image domain in \mathbb{R}^d . Ω_{in} is a region in Ω . Ω_{out} is defined as $\Omega_{out} = \Omega \setminus \Omega_{in}$ and \mathbf{x} an element of Ω .

2.2 Discrete Derivation of the Region-based Criterion

In this section, we study the discrete derivation of a region-based energy in order to get an aggregation rule. This rule will be applied to candidate pixels and will drive the evolving region near the solution. Of many proposed region-based criteria, a general region-based energy $J(\Omega_{in})$ was defined by (Jehan-Besson, 2003) as:

$$J(\Omega_{in}) = \int_{\Omega_{in}} k(\mathbf{x}, \Omega_{in}) d\mathbf{x} \quad (2)$$

where k is the region descriptor of Ω_{in} . For an iterative process, it is assumed that $J(\Omega_{in}^{n+1})$ the energy of the evolving region at iteration $n+1$ is the sum of $J(\Omega_{in}^n)$ the energy of the region at previous iteration and $\Delta J(\Omega_{in}^{n+1})$ the variation of energy due to aggregation of one pixel. This is expressed in the following equation:

$$J(\Omega_{in}^{n+1}) = J(\Omega_{in}^n) + \Delta J(\Omega_{in}^{n+1}) \quad (3)$$

Our objective is to determine the variation of energy $\Delta J(\Omega_{in}^{n+1})$, also called evolution equation of the region. We distinguish two cases whether the region descriptor depends or not on the evolving region. For both cases, we propose a discrete expression of the energy criterion, and we describe how to get the evolution equation of the region. This equation will help the region evolve towards a minimum of energy. Depending on the sign of $\Delta J(\Omega_{in}^{n+1})$ value, the considered pixel will be aggregated or rejected.

2.2.1 Region-independent Energy Derivation

(Jehan-Besson, 2003) gives a general definition of a region-based energy computed from a “region-independent” descriptor:

$$J(\Omega_{in}) = \int_{\Omega_{in}} k(\mathbf{x}) d\mathbf{x} \quad (4)$$

Since in region growing methods, the evolving region is represented by a set of pixels, we decide to translate the previous definition into a discrete expression. We thus propose a new discrete energy computed from ϕ^n previously defined in (1):

$$J(\phi^n) = \sum_{\mathbf{x} \in \Omega} k_{\mathbf{x}} \cdot \phi_{\mathbf{x}}^n \quad (5)$$

Starting from this energy, we determine the variation of $J(\phi^n)$ induced by the state switch of a candidate pixel. We define the state switch of a pixel \mathbf{v} by the following equation:

$$\phi_{\mathbf{v}}^{n+1} = 1 - \phi_{\mathbf{v}}^n, \quad (6)$$

thus,

$$\phi_{\mathbf{x}}^{n+1} = \phi_{\mathbf{x}}^n \text{ if } \mathbf{x} \neq \mathbf{v}. \quad (7)$$

Like equation (5), the energy at iteration $n+1$ can be expressed as a function of ϕ^{n+1} :

$$J(\phi^{n+1}) = \sum_{\mathbf{x}} k_{\mathbf{x}} \cdot \phi_{\mathbf{x}}^{n+1} \quad (8)$$

Using equations (6) and (7), we can also write:

$$J(\phi^{n+1}) = k_{\mathbf{v}} \cdot \phi_{\mathbf{v}}^{n+1} + \sum_{\mathbf{x} \neq \mathbf{v}} k_{\mathbf{x}} \cdot \phi_{\mathbf{x}}^n \quad (9)$$

$$J(\phi^{n+1}) = k_{\mathbf{v}} \cdot (1 - \phi_{\mathbf{v}}^n) - k_{\mathbf{v}} \cdot \phi_{\mathbf{v}}^n + k_{\mathbf{v}} \cdot \phi_{\mathbf{v}}^n + \underbrace{\sum_{\mathbf{x} \neq \mathbf{v}} k_{\mathbf{x}} \cdot \phi_{\mathbf{x}}^n}_{J(\phi^n)} \quad (10)$$

$\Delta J(\phi^{n+1})$ is obtained by identification of equations (10) and (3):

$$\Delta J(\phi^{n+1}) = (1 - 2\phi_{\mathbf{v}}^n) \cdot k_{\mathbf{v}} \quad (11)$$

Note that this variation of energy is defined whatever the region-independent descriptor used.

2.2.2 Region-dependent Energy Derivation

This section deals with the minimization of a discrete region-based energy when the descriptor is ‘‘region-dependent’’. We propose the following general expression for the energy:

$$J(\phi^n) = \sum_{\mathbf{x} \in \Omega} k_{\mathbf{x}}(\phi_{\mathbf{x}}^n) \cdot \phi_{\mathbf{x}}^n \quad (12)$$

where $k_{\mathbf{x}}(\phi_{\mathbf{x}}^n) = k(\mathbf{x}, \phi_{\mathbf{x}}^n)$ is a region-dependent descriptor. Such a descriptor changes as the segmentation progresses, so we need to define the variation of the descriptor between two iterations:

$$k_{\mathbf{x}}(\phi_{\mathbf{x}}^{n+1}) = k_{\mathbf{x}}(\phi_{\mathbf{x}}^n) + \Delta k_{\mathbf{x}}(\phi_{\mathbf{x}}^{n+1}) \quad (13)$$

Moreover,

$$J(\phi^{n+1}) = \sum_{\mathbf{x}} k_{\mathbf{x}}(\phi_{\mathbf{x}}^{n+1}) \cdot \phi_{\mathbf{x}}^{n+1} \quad (14)$$

The energy variation $\Delta J(\phi^{n+1})$ is obtained as in the above subsection.

$$\Delta J(\phi^{n+1}) = (1 - 2 \cdot \phi_{\mathbf{v}}^n) \cdot k_{\mathbf{v}} + \Delta K_{\mathbf{x}} \quad (15)$$

with,

$$\Delta K_{\mathbf{x}} = (1 - \phi_{\mathbf{v}}^n) \cdot \Delta k_{\mathbf{v}}(\phi_{\mathbf{v}}^{n+1}) + \sum_{\mathbf{x} \neq \mathbf{v}} \Delta k_{\mathbf{x}}(\phi_{\mathbf{x}}^{n+1}) \cdot \phi_{\mathbf{x}}^n \quad (16)$$

This last term $\Delta K_{\mathbf{x}}$ coming from the descriptor variation is most often negligible.

2.3 Implementation Issues

In this section, we focus on the VRG algorithm implementation. In a first step, we describe how to choose the set of candidate pixels required by VRG algorithm. Then, we detail the rules of aggregation applied to these pixels. These rules depend on the state of the pixel and the sign of the energy variation. To sum up, we give a synoptic view of the VRG algorithm.

2.3.1 Candidate Pixels

The candidate pixels are the set of pixels tested at each iteration by the predicate of aggregation. This set is noted C^n , and depends on the segmented region Ω_{in} . The pixels of C^n can either be non segmented ones i.e. belonging to Ω_{out} or already segmented ones i.e. belonging to Ω_{in} , but in all cases in the vicinity of the contour of Ω_{in} .

We define C^n as:

$$C^n(\Omega_{in}, \varepsilon) = \{\mathbf{m} \in \Omega_{out}, \mathbf{n} \in \Omega_{in} | d(\mathbf{m}, \mathbf{n}) \leq \varepsilon\} \quad (17)$$

where $\varepsilon > 0$ is the maximum Euclidean distance between two pixels at each side of the contour of Ω_{in} .

2.3.2 Variational Region Growing algorithm

In section 2.2, we studied the derivation of the region-based criterion leading to the evolution equation. This section considers the state switch of a candidate pixel according to the variation of energy given in equation (11). The main difference between our approach and other region growing methods lies in the strategy of the aggregation. In VRG, our iterative process tries to achieve a minimization of energy, whereas the other methods merge pixels while the homogeneity criterion is satisfied. This subsection provides the aggregation rules that ensure VRG to reduce the energy criterion at each iteration and converge to a minimum.

At iteration n , a pixel \mathbf{v} undergoes a state switch if it leads to a lower energy. Table 1 displays a summary of the possible state switches according to the variation $\Delta J(\phi_{\mathbf{v}}^{n+1})$ and the initial state of \mathbf{v} :

The aggregation criterion $\Delta J(\phi_{\mathbf{v}}^{n+1})$ governs the state switch of \mathbf{v} . If $\Delta J(\phi_{\mathbf{v}}^{n+1}) < 0$, then \mathbf{v} is added to

Table 1: State switch rules.

ϕ_v^n	Initial state: 0		Initial state: 1	
$(1 - 2 \cdot \phi_v^n)$	-1		+1	
k_v	< 0	> 0	< 0	> 0
$\Delta J(\phi_v^{n+1})$	> 0	< 0	< 0	> 0
ϕ_v^{n+1}	0	1	0	1

($\phi_v^{n+1} = 1$) or rejected from the evolving region ($\phi_v^{n+1} = 0$).

Figure 1 sums up the different steps of VRG algorithm.

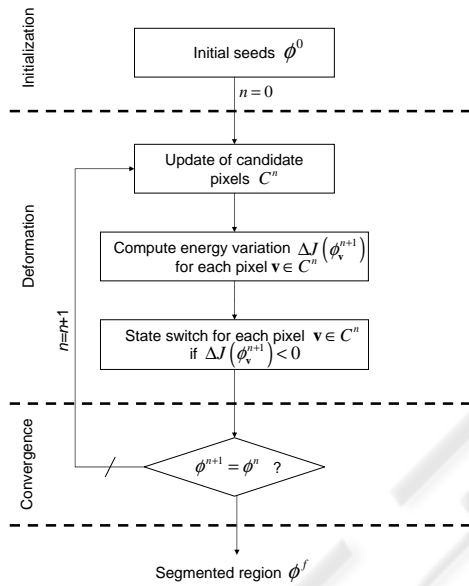


Figure 1: Variational Region Growing algorithm.

We have laid out a general framework that may fit many segmentation applications by only changing the region descriptors. In the following sections, we give guidance on how to integrate different energy criteria in VRG. We will see, in particular, that it is quite easy to integrate shape prior in VRG or to combine several energy criteria.

3 INTEGRATION OF SHAPE PRIOR IN VRG

The purpose of this section is to show the ability of VRG to take into account shape prior. We start from a previous work (Rose et al., 2007) which was focused on the integration of shape prior in a 3D region growing and we propose to improve this work by translating the former shape prior criterion into a region-based energy.

3.1 Shape Prior-based Criterion

Let us recall some definitions given in (Rose et al., 2007):

$d(\mathbf{x}, \Gamma^{ref})$ is the normalized signed distance from \mathbf{x} to the nearest pixel belonging to the reference contour, negative inside the reference object, positive outside.

$\varphi_{shape}(d(\mathbf{x}, \Gamma^{ref})) \in [0, 1]$ is a function encoding shape prior which takes a value close to 1 (resp. 0) when \mathbf{x} is inside (resp. outside) the reference object:

$$\varphi_{shape}(d(\mathbf{x}, \Gamma^{ref})) = \frac{\frac{\pi}{2} - \tan^{-1}\left((\lambda \cdot d(\mathbf{x}, \Gamma^{ref}))^3\right)}{\pi} \quad (18)$$

where λ is a tuning parameter.

We rescale $\varphi_{shape}(d(\mathbf{x}, \Gamma^{ref}))$ to the range $[-1, 1]$, thus defining $\varphi_{new}(d(\mathbf{x}, \Gamma^{ref}))$:

$$\varphi_{new}(d(\mathbf{x}, \Gamma^{ref})) = 2 \times \varphi_{shape}(d(\mathbf{x}, \Gamma^{ref})) - 1 \quad (19)$$

The new shape prior-based criterion J_P is defined, so that it supports (resp. penalizes) aggregation of pixels when they are inside (resp. outside) the object of reference:

$$J_P(\Omega_{in}) = \sum_{\mathbf{x} \in \Omega_{in}} -\varphi_{new}(d(\mathbf{x}, \Gamma^{ref})) \quad (20)$$

Note that $\varphi_{new}(d(x, \Gamma^{ref}))$ depends on the affine position of the reference object. This problem is easily settled by using a classical affine transformation $\mathbf{x}' = T^{aff} \cdot \mathbf{x}$.

Then, we express the criterion J_P as a function of ϕ^n :

$$J_P(\phi^n) = \sum_{\mathbf{x}} -\varphi_{new}(d(\mathbf{x}', \Gamma^{ref})) \cdot \phi_{\mathbf{x}}^n \quad (21)$$

The shape prior-based criterion J_P penalizes large discrepancies between the evolving region and the reference model.

3.2 Derivation of the Shape Prior-based Criterion

We derive the shape prior-based criterion according to our variational approach. By definition, the descriptor in equation (21) is region-independent. Consequently, the variation of the criterion for a pixel \mathbf{v} is directly determined from equation (11):

$$\Delta J(\phi_v^{n+1}) = (1 - 2\phi_v^n) \cdot (-\varphi_{new}(d(\mathbf{v}', \Gamma^{ref}))) \quad (22)$$

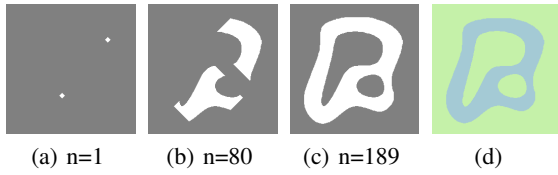


Figure 2: Region evolution using only shape prior constrain ($\lambda = 15$): (a) original image with two initial seeds , (b) intermediate step, (c) final result, and (d) reference shape

3.3 Experimental Result

We propose to illustrate and quantify the ability of the shape prior criterion defined in equation (21) to make the evolving region converge towards the shape reference, while enabling free changes of topology during the growing.

In Figure 2, VRG is tested by only using the shape prior-based criterion computed from the reference model given in Figure 2(d), i.e. without taking into account any grey level information. Figure 2(a) depicts the original constant image with two initial seeds. Figure 2(b) displays the VRG segmentation in progress with two disconnected components. Figure 2(c) shows the resulting segmented object with only one connected component. This test highlights two major properties of VRG: (i) how the shape prior-based criterion forces the evolving region to look like the reference model and (ii) how VRG allows free changes of topology during the segmentation.

Figure 3 displays the evolution of the shape prior-based criterion during the above described segmentation of VRG. The decrease of the energy clearly appears during the progression of the segmentation. At iteration $n = 189$, the minimum of the shape prior-based criterion is reached, VRG has converged towards the sought object. After this iteration, random pixels were added to the segmented region, thus leading to an increase of the criterion.

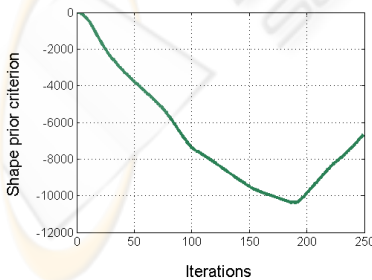


Figure 3: Shape prior criterion minimization is obtained for the iteration $n = 189$.

4 APPLICATION TO BIOMEDICAL IMAGE SEGMENTATION

Biomedical images are often difficult to segment due to artifacts and low signal to noise ratio. Most of region growing methods only based on homogeneity criterion fail to recover object of interest. The purpose of this section is to show that VRG provides a general framework which can be adapted to the needs of the targeted application. We do not present an extensive experiment validating the accuracy of our segmentation, we just want to exemplify a possible use of VRG. VRG can answer the problem raised before if its energy criterion integrates both grey level information and shape prior.

So, we propose a new region-based criterion for VRG, that mixes image data information and shape prior. This new criterion expressed in equation (24) is the sum of the shape prior-based criterion defined in (21) and the well-known image data criterion introduced by (Chan and Vese, 2001) as:

$$J_I(\phi_{\mathbf{x}}) = \sum_{\mathbf{x}} |I_{\mathbf{x}} - \mu_{in}|^2 \cdot \phi_{\mathbf{x}} + \sum_{\mathbf{x}} |I_{\mathbf{x}} - \mu_{out}|^2 \cdot (1 - \phi_{\mathbf{x}}) \quad (23)$$

where μ_{in} (resp. μ_{out}) is the average intensity in the domain Ω_{in} (resp. Ω_{out}). $I_{\mathbf{x}}$ is the intensity value of the pixel \mathbf{x} .

$$J_T(\phi_{\mathbf{x}}) = J_I(\phi_{\mathbf{x}}) + \alpha * J_P(\phi_{\mathbf{x}}) \quad (24)$$

where α is an arbitrary hyper-parameter required to balance the influence of shape prior and image data criteria.

By minimizing $J_T(\phi_{\mathbf{x}})$, we obtain:

$$\Delta J(\phi_{\mathbf{v}}^{n+1}) = (1 - 2\phi_{\mathbf{v}}^n) \cdot \left(|I_{\mathbf{v}} - \mu_{in}|^2 + |I_{\mathbf{v}} - \mu_{out}|^2 - \alpha \cdot \varphi(d(\mathbf{v}', \Gamma^{ref})) \right) \quad (25)$$

where μ_{in} and μ_{out} are updated at each iteration.

VRG was applied to three dimensional micro-CT scans of mice kidney. The framework of the application is the phenotyping of mice kidneys. The 3D reference model was obtained by a previous segmentation of a reference image. The method was tested on a random input volume. Slices of x-plane and y-plane are shown in Figures 4(a), and 4(d).

We compare the results of VRG with and without shape prior i.e using $J_T(\phi_{\mathbf{x}})$ or only $J_I(\phi_{\mathbf{x}})$ the classical region-based energy defined in equation (23). Figure 4(b) and 4(e) show the resulting segmentation without shape prior. The segmentation fails to segment the kidney due to strong inhomogeneities in the

image. Moreover, the segmentation spreads through the leaking points induced by an artifact. Figure 4(c) and 4(f) illustrate VRG results with shape prior constrain. The parameter λ stepping in $J_P(\phi_x)$ was set to 15. This value was not chosen too high in order to let flexibility with regard to the reference model (for further details, see (Rose et al., 2007)). The hyper-parametre α was fixed to 1000 and achieves a good compromise between $J_P(\phi_x)$ and $J_I(\phi_x)$ since the kidney surface was recovered more accurately and without any leakage.

5 CONCLUSIONS

This work presents a new region growing approach. We define a discrete function which evolves according to the minimization of an energy functional including region-based terms. Our approach is based on a discrete derivation and allows to readily take into account both region-dependent and region-independent descriptors. An evolution equation is determined and enables to govern the state switch of the candidate pixels during the progression of the segmentation.

In order to demonstrate the interest of Variational Region Growing, we have integrated shape prior into the region-based criterion governing the algorithm. The shape prior-based criterion enables to constrain the shape of the evolving region. Our tests have pointed out the convergence of the criterion towards a minimum during the segmentation and also the ability of the criterion to constrain the segmentation.

Our method was tested and applied to small animal imaging in order to highlight the performance of shape prior constrain. The results show the improvement provided by VRG when its region-based criterion takes into account both image data information and shape prior.

ACKNOWLEDGEMENTS

This work was funded by the EUMORPHIA project (QLG2-CT-2002-00930) supported by the European Commission under FP5. It is in the scope of the scientific topics of the PRC-GdR ISIS research group of National Center for Scientific Research CNRS.

REFERENCES

- Adams, R. and Bischof, L. (1994). Seeded region growing. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 16(6):641–647.

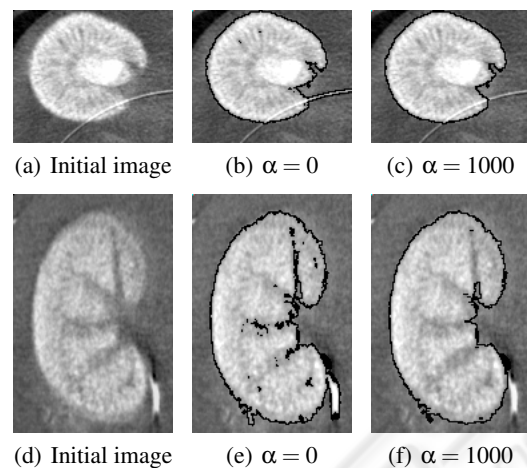


Figure 4: 3D μ -CT image segmentation: (a,d) slices of the input volume, (b,e) segmentation result without shape prior, (c,f) segmentation result using our shape prior constrain.

- Chan, T. and Vese, L. (2001). Active contours without edges. *Image Processing, IEEE Transactions on*, 10(2):266–277.
- Jehan-Besson, S. (2003). *Modèles de contours actifs basés région pour la segmentation d’images et de vidéos*. PhD thesis, Universit de Nice-Sophia Antipolis, France.
- Revol-Muller, C., Peyrin, F., Carrillon, Y., and Odet, C. (2002). Automated 3d region growing algorithm based on an assessment function. *Pattern Recognition Letters*, 23(1-3):137–150.
- Rose, J.-L., Revol-Muller, C., Almajdub, M., Chereul, E., and Odet, C. (2007). Shape prior integrated in an automated 3d region growing method. In *IEEE ICIP*, volume 1, pages I – 53–I – 56.
- Sekiguchi, H., Sano, K., and Yokoyama, T. (1994). Interactive 3-dimensional segmentation method based on region growing method. *Systems and Computers in Japan*, 25(1):88–97.
- Zucker, S. W. (1976). Region growing: Childhood and adolescence. *Computer Graphics and Image Processing*, 5(3):382–399.