

APPLICATION OF SCALE ANALYSIS ON LEVEL SETS FOR COOPERATIVE IMAGE SEGMENTATION

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Abstract: Image Segmentation has been used by many approaches and techniques in artificial vision but none of them has been proved to be applied completely successfully for any image or object type. We propose in this paper a segmentation approach based on level sets which incorporate low scale cooperative analysis of both image and curve. The image at a low resolution level provides information on coarse variation of grey level intensity. For the same perspective, the curve at a low resolution scale provides a coarser curvature value. The purpose of image scale cooperative approach is to avoid stopping the curve evolution at local minima of images. This method is tested on a sample of a 2D abdomen image, and can be applied on other image types. The results obtained are satisfying and show good precision of the method.

1 INTRODUCTION

Image segmentation is widely used in artificial vision. Its importance is estimated also because of its complexity and the accuracy of results it should provide. Explicit deformable models or active contours were used in image processing and mostly used in medical imaging. Explicit active contours or snakes appeared in the paper of (Kass & al., 1988) and (Caselles & al. 1993, 1997) gave satisfying results especially in medical imaging but suffer from limitations like the difficulty to track a shape of unspecified topology. Implicit deformable models proposed by (Osher and Sethian, 1988), and by (Malladi & al., 1995) offer a good segmentation tool on shapes of unspecified topology, and consequently apply in the case of 2D medical images and can be easily extended to 3D image volumes.

This paper treats the segmentation problem by level sets or implicit deformable models. The model is based on the addition of new constraints to the speed evolution function of level curves. These constraints are:

- Local Variation of grey level intensity of a point P in the contour. In the case of a difference of grey level mean values between pixels at the inside of the contour and pixels at the outside of the contour in a local neighbourhood of P, the function evolves at P,

else the constraint is null and the evolution stops.

- Utilization of a low level scale image and calculation of mean grey level intensity variation with the same manner as the first constraint in order to eliminate local minima.
- Utilization of low level scale curves computed from the current curves in order to smooth discrete curvatures and eliminate concavity and convexity zones present in local minima.

In section 2 of this paper we give an outline of implicit deformable models and level curves evolution principle. Next, we describe in section 3 in detail the proposed segmentation method with Level Sets that incorporate Image and Curve analysis at low Resolution level. In section 4, segmentation results are shown on a 2D abdomen image. At the end, we finish by a conclusion and future perspectives of this work.

2 LEVEL SETS

The principle of Level Sets method is to move and warp temporally any kind of closed curve or surface implicitly represented (Adalsteinsson and Sethian, 1995).

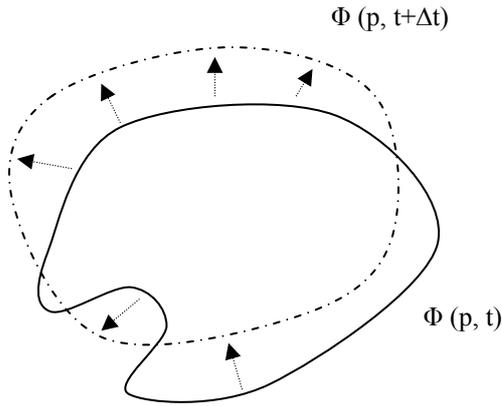


Figure 1: Evolution of a closed curve C represented by a function Φ between time intervals t and $t+\Delta t$.

2.1 Detail of Level Sets Method

C is the level curve of the object in evolution. $\Phi(X, t) < 0$ inside the curve, and $\Phi(X, t) > 0$ outside the curve. $\Phi(X, t)$ is null on the curve C .

The closed contour C –also called front or interface –evolves according to the equation:

$$\frac{\partial \phi}{\partial t} = F \cdot N \quad (1)$$

F : propagation speed defined in each point of the curve.

The level set principle is to consider the moving curve or interface as the set of null values of a function Φ .

We represent Φ by a 2 dimension matrix of real numbers $\Phi(x,y)$. (x,y) are pixel coordinates on the image. Values of $\Phi(x,y)$ that coincide with the position of the curve C are initialized to zero. Values of $\Phi(x,y)$ outside of the curve are positive and equal to the euclidian distance to the curve, and the values inside of the curve are negative.

The propagation front C is defined as:

$$C = \{ (x) | \phi(x, t) = 0 \} \quad (2)$$

The Set $\{ (x) | \phi(x, t = 0) = 0 \}$ defines the initial contour.

Φ evolves according to the equation:

$$\frac{\partial \phi}{\partial t} + \vec{N} \cdot \nabla \phi = 0 \quad (3)$$

N : normal unit vector to the curve,
 $\vec{N} = -\nabla \phi / |\nabla \phi|$

F (curve evolution speed): it depends on external properties, such that physical image properties like gray level intensity, and of intrinsic properties

concerning the curve itself like the discrete curvature.

Generally, the most used speed propagation formula is function of image gradient g and curvature of curve κ :

$$F = \alpha \cdot g |\nabla I| (c + \varepsilon \cdot \kappa) \quad (4)$$

This function is used for comparison in section 4 of experimental results. c : constant, generally equal to 1. ε : term $0 < \varepsilon < 1$.

$\alpha = \pm 1$. For $\alpha = -1$, the curve expands or increases. For $\alpha = +1$, the curve shrinks.

$g |\nabla I|$: term that computes the stopping criterion by image gradient. It allows to minimize the distance –variation- between the external contour and real image borders, so that the contour of the object coincides with the gradient of the image.

The typical formula of g (image gradient) is ($p=1$ or 2):

$$g(|\nabla I(x, y)|) = \frac{1}{1 + |\nabla G_\sigma(x, y) * I(x, y)|^p} \quad (5)$$

κ : curvature that represents the viscosity term of the speed evolution function F and improves smoothing of the curve. The formula below shows the relation between normal to the curve φ and curvature κ :

$$\kappa = \text{div} \left(\frac{\nabla \phi}{|\nabla \phi|} \right) = \frac{\phi_{xx} \phi_y^2 - 2\phi_x \phi_y \phi_{xy} + \phi_{yy} \phi_x^2}{\sqrt{(\phi_x^2 + \phi_y^2)^3}} \quad (6)$$

The function F is proportional to the curvature and inversely proportional to the grey level intensity. It means in general that if $F(p) \approx 0$, the curve is stable at the point p , on the other hand if $\text{abs}(F(p)) > 0$, the contour is instable and a curve deformation at the point p is necessary.

The general evolution principle of «Level Sets» or level curves (Chopp, 1993) is to calculate F on all image positions and to evolve each time the curve or the front at the point having the maximal value of F . A permanent update of the value F on each new position is computed. Since calculation on all pixel positions is time computing expensive, the narrow band principle developed by (Sethian, 1996, 1999) and (Adalsteinsson & Sethian, 1995) and introduced initially by (Chopp, 1993) reduces strongly time computing and limits computing of F at pixels situated on a narrow band of width d pixels at the inside or the outside of the evolving front. We fixed the value of d equal to 1 in our approach.

The Fast Marching Method (FMM) is applied on all level sets if the curve is applied on level sets

where the curve is always moving in the same directions (UpWind for the expansion and DownWind for shrinkage). The evolution to the negative direction can be realized by inverting the sign of the speed function.

2.2 Image Scale Analysis with Level Sets

The Multiscale approach has been recently used with Level Sets and Active Contour models in several research works.

(Lin & al., 2003) apply multi-scale level set framework to echocardiographic ultrasound image sequences by using pyramid level resolutions. They specify that the intensity distribution of an ultrasound image at a very coarse scale can be approximately modelled by Gaussian. And they combine region homogeneity and edge features in a level set approach to extract boundaries automatically at this coarse scale. At finer scale levels, these coarse boundaries are used to both initialize boundary detection and serve as an external constraint to guide contour evolution.

A level set approach for multiscale vessel segmentation is proposed by (Yu & al., 2005). They incorporate the prior knowledge about the vessel shape into the energy function as a region information term. Multiscale mechanism is mainly used in vessel enhancement filtering.

(Paragios and Deriche, 2000) propose a multiscale technique combined with level sets and geodesic active contours. Specifically, a Gaussian pyramid of images is built upon the full resolution image and similar geodesic contour problems are defined across the different levels. The multiresolution structure is then utilized according to a coarse-to-fine strategy, an extrapolation of the current contour from a level with low resolution to levels with finer contour configuration takes place. They apply their method to a pyramid with 2 or 3 levels of resolution. The multiscale approach especially permits moving objects to be tracked with considerable speedup.

In the next section, we propose our method that computes and analyses both image and level set curve at lower scale level.

3 COOPERATIVE IMAGE SCALE ANALYSIS WITH LEVEL SETS

We have adopted a new segmentation approach with level sets by the integration of a Multi-scale

approach. The classical formula of F (4) has been modified by a new one in order to improve avoiding local minimum. In our work, we have used Multiscale for image intensity and curve computation at lower resolution level. The lower scale image and curve enable respectively to calculate local image intensity variation and discrete curvature value at a coarse level.

3.1 Local Gray Level Constraint

This new constraint is added to the speed function F .

We consider a local rectangular zone F with size $(m \times m)$ at the neighbourhood of the point P . Generally, the evolution of a contour at a given point P can affect the evolution of the close pixels with P in the same direction.

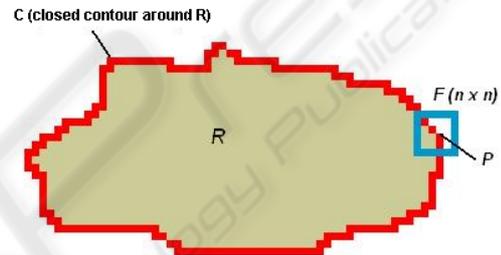


Figure 2: Local Window $(n \times n)$ for computing the local gray level variation in the image.

The neighbourhood zone F is centered at the point P_{ij} with radius n . The radius value used here is 2.

$$F = \{x | i - n \leq x \leq i + n, j - n \leq y \leq j + n\} \quad (7)$$

The actual interface or Contour C delimits the region R (Fig. 3).

$$E_1 = \{x | x \in R; x \in F\}, E_2 = F - E_1 \quad (8)$$

$$l_1 = \frac{\sum_{i(x) \in E_1} i(x)}{\text{Card}(E_1)}, l_2 = \frac{\sum_{i(x) \in E_2} i(x)}{\text{Card}(E_2)} \quad (9)$$

l_1 (resp. l_2): mean grey level pixels of the set E_1 (resp. E_2).

l_1 : mean grey level of pixels inside the window F and belonging inside the region R delimited by the contour C . $i(x)$: image intensity of a pixel in the window F .

If $|l_1 - l_2| \approx 0 \Rightarrow$ there is no local grey level variation at the point P , then the level curve C must evolve at P .

If $|l_1 - l_2| > 0 \Rightarrow$ there is local gray level variation at the point P.

The local mean gray level formula is:

$$MG_{loc} = \frac{v_1}{(1 + |l_1 - l_2|^k)} \quad (10)$$

v_1 is a weighting coefficient, $k = 1$.

3.2 Lower Scale Image and Curve computation

3.2.1 Lower Scale Image Computation

To compute a lower scale resolution image, we apply Bartlett filter to the image, by reducing with a scale value of four (4).

The mask (3 x 3) of the Bartlett filter below is applied to derive a lower scale image by a value of 2 after a Gaussian smoothing of the image. In the case of computing an image with a lower scale value of 4, we apply to the image a composition operation of Bartlett filter 2 times successively.

$$\begin{aligned} h_{bartlett}^{3 \times 3} &\stackrel{def}{=} \frac{1}{16} \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} \\ &= \frac{1}{4} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \cdot \frac{1}{4} \begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \end{aligned}$$

The figure (3) shows the original image and the image obtained by division with a scale factor of 4, by a successive reduction for 2 times with a scale factor of 2. Section 3.3 will give more details about this constraint.



Figure 3: Original Image and image I_s with a scale factor of $1/4$.

3.2.2 Lower Scale Curve Approximation

The computation of a curve at a lower scale and corresponding to the original curve is generated approximately.

The original closed curve represents a Narrow Band of a zero level set of width 1 that delimits the deforming object in segmentation. Pixels P (x, y) of the closed curve are represented by a list L of points.

The computation of a curve at a low scale is realized by dividing each pixel position (x, y) of L by the same scale value applied to the image. We obtain a new list L_s of points (x_s, y_s) , redundant pixel values of (x_s, y_s) are automatically eliminated.

Figure 4 shows the region R_s delimited by the low scale Curve Φ_s after image reduction by a scale factor of 4. Section 3.4 will give details about the application of this constraint.

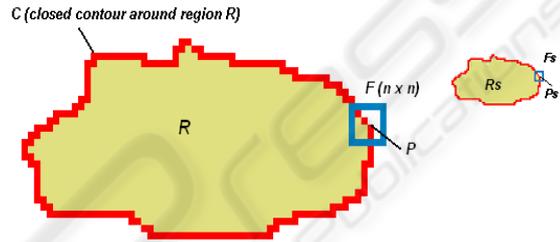


Figure 4: The region R_s is derived from region R after scale factor applying in the image.

3.3 Local Gray Level Variation Constraint in Lower Scale Image

The neighbourhood zone F_s of the scaled image is centered at the point P_{ij}^s with radius n. The radius value used here is 2.

$$F_s = \{x | i - n \leq x \leq i + n, j - n \leq y \leq j + n\} \quad (11)$$

$$S_1 = \{x | x \in R_s; x \in F_s\}, S_2 = F_s - S_1 \quad (12)$$

Sl_1 : mean grey level of pixels of the local zone F_s and belonging inside of the region R_s delimited by the low scale Curve Φ_s after image reduction by a scale factor s.

$$sl_1 = \frac{\sum_{x \in S_1} x}{Card(S_1)}, sl_2 = \frac{\sum_{x \in S_2} x}{Card(S_2)} \quad (13)$$

Sl_1 (resp. Sl_2): mean grey level pixels of the set E_1 (resp. E_2).

If $|sl_1 - sl_2| \approx 0 \Rightarrow$ the curve must evolve, there is no intensity variation in the image.

If $|sl_1 - sl_2| > 0 \Rightarrow$ there is a local coarse intensity variation.

$$MG_{-Scale}_{loc} = \frac{v_2}{\left(1 + |sl_1 - sl_2|^k\right)} \quad (14)$$

v_2 is a weighting coefficient, $k = 1$.

Example: figure 5 presents 2 images with an initial contour (yellow) in the left image and a final contour (full red) in the right image, and where the final contour cannot segment and add the local grey level variation inside the contour by using simply the classical level set evolution function

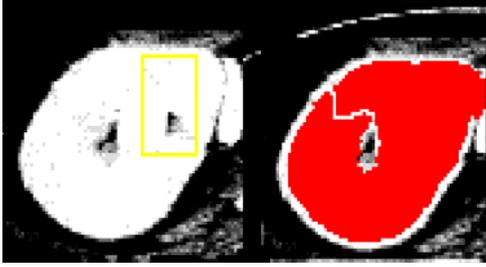


Figure 5: Image Segmentation Result by classical Level Sets method.

3.4 Lower Scale Discrete Curvature Constraint

This constraint is based on the computation of the geometrical shape contour at a lower scale to obtain a coarser contour, then the computation of the discrete curvature value at a point P_s of the lower curve (fig. 6B), in order to eliminate coarser concave or convex shapes that are also smoothed.

The figure 6 shows an example of the contour and its scaling by a factor of 4.

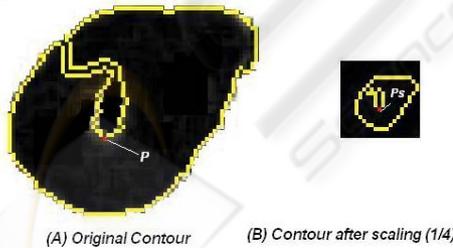


Figure 6: Original Contour (A) and the generated Contour (B) after scaling, the point P_s (red) after scaling corresponds to the point P (red) of the original contour, so the point P_s corresponds to more than one point P (approx. 4 points for a scale value of 4) of the original contour.

κ_s : discrete curvature: the curvature formula is the same as the formula (6) applied for the curvature κ , after applying a transformation –reduction with a scale $1/s$ - to the original contour in order to smooth

contour positions that still present convex or concave parts at a large scale.

$$\kappa_s = \text{div} \left(\frac{\nabla \phi_s}{|\nabla \phi_s|} \right) \quad (15)$$

This method estimates the curvature value at a lower resolution level. Φ_s : curve of the right image (fig 6.B) obtained at a lower resolution level.

3.5 Speed Function F or Level Set Evolution Function

The precedent constraints (sections 3.1 to 3.4) are integrated to the speed evolution function F. The value of F is computed at each point of the curve C.

The formula of F is as follows by integrating formulas (10) and (14) for local gray level constraint and local gray level at lower scale constraint respectively :

$$F = \pm g(\alpha \kappa + \alpha_2 \kappa_s) \quad (16)$$

$$g = MG_{loc} + MG_{-Scale}_{loc} \quad (17)$$

$$g = \frac{v_1}{1 + |m_1 - m_2|^{k_1}} + \frac{v_2}{1 + |sl_1 - sl_2|^{k_2}} \quad (18)$$

g: image intensity variation. $k_1, k_2 = 1$.

κ : discrete curvature for curve smoothing.

κ_s : discrete curvature of the scaled curve (this value is coarser and is computed only for points whose curvature value κ is not high).

v_1, v_2 : weighting coefficients. α, α_2 : weighting coefficients, generally lower than intensity coefficients v_1, v_2 .

The coefficient values are determined empirically and experimentally.

The sign of F indicates the evolution direction of the curve. In the case of this approach, the direction is manually chosen by the user, and by default negative, which means that the initial curve is in expansion or dilation, and hence limits the evolution to Fast Marching where the curve evolves only in one direction, UpWind or DownWind.

If $F < 0$ → the contour expands and the front evolves only at the outside of the curve.

If $F > 0$ → the contour shrinks and the front evolves only at the inside of the curve.

If $F \approx 0$ → the contour is stable.

What position P of the curve to choose for moving the front? Select the pixel or the position having the absolute value of F maximal.

4 EXPERIMENTAL RESULTS

We applied our approach to a 2D abdomen image. Figure 7 shows the original image with the initial contour.

Figure 8 shows the result of the segmentation with the classical level set function by applying the formula (4) of the propagation speed related in section 2. The 2 local minima inside the object are not segmented.

Figure 9 shows a partial result of the segmentation progression by integrating the constraints of our approach described in section 3.

Weighting coefficient values for grey level variation v_1 and v_2 are equal to 2 and 1 respectively.

The value of coefficient α used is 0.20. The value of α_2 is 0.20 too if the curvature value κ is weak ($\kappa < \kappa_{\max}/2$); else $\alpha_2 = 0$ is not used.

In Figure 10, the final segmentation result is presented. The contour does not stop at the 2 local grey level minima inside the object to be segmented.

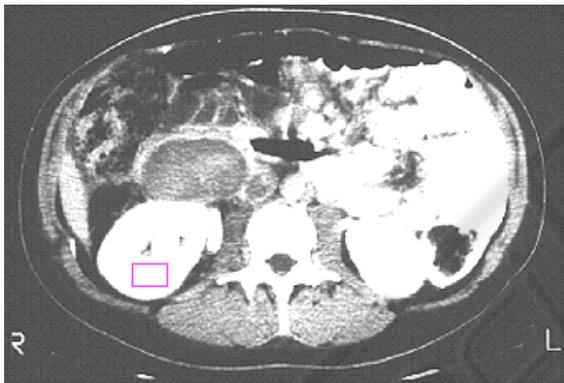


Figure 7: Abdomen Image. Contour Initialisation by a red rectangular zone.

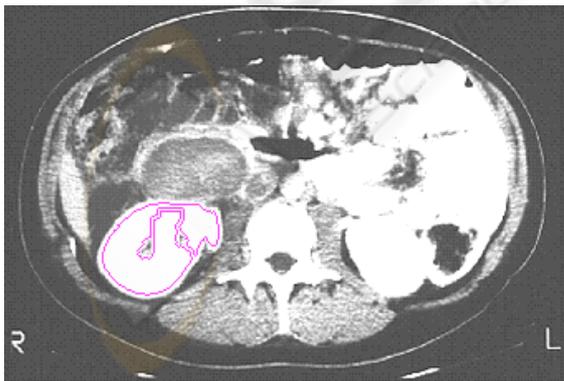


Figure 8: Segmentation by traditional Levels Sets function.

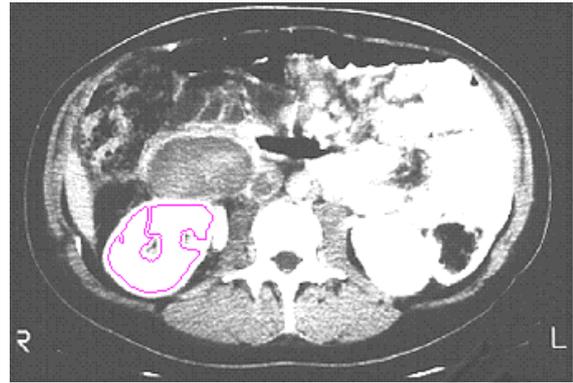


Figure 9: Partial Segmentation Result.

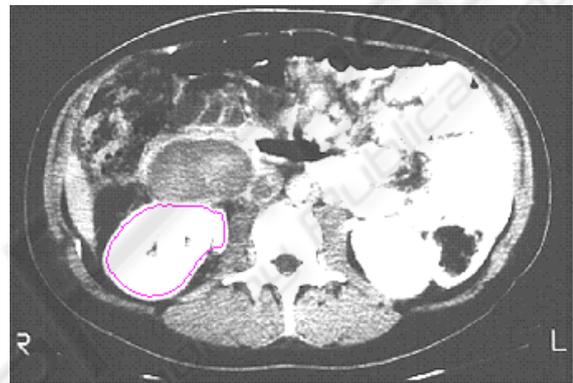


Figure 10: Final Segmentation Result.

5 CONCLUSIONS

In this paper, we proposed a new image segmentation method applied with the level set approach. Zero Level Sets or Level Curves are an efficient tool to segment objects with unspecified topological shape. They are depending essentially on edge gradient for image stopping criterion and on curvature for curve smoothing. However, stopping the curve at a local minimum cannot be resolved only with image gradient and discrete curvature. In our approach, we added three constraints: (i) local image intensity variation criterion, (ii) image intensity variation at a lower scale and (iii) discrete curvatures of the original curve and of the curve obtained at a lower scale in order to avoid stopping the curve at local grey level minima of the images.

We hope to extend our segmentation approach to different image types and to 3D image volumes.

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