

# INCREMENTAL MAINTENANCE OF ONTOLOGIES BASED ON BIPARTITE GRAPH MATCHING

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**Abstract:** Today's *Information Society* demands complete access to available information, which is often heterogeneous and distributed. A key challenge in building the *Semantic Web* is integrating heterogeneous data sources. This paper presents an incremental algorithm for maintaining integration in evolving ontologies. For example, an increased number of smaller, task oriented ontologies, are emerging across the Bioinformatics domain to represent domain knowledge; integrating these heterogeneous ontologies is crucial for applications utilizing multiple ontologies. Most ontologies share a core of common knowledge allowing them to communicate, but no single ontology contains complete domain knowledge. Recent papers examined integrating ontologies using bipartite graph matching techniques. However, they do not address the issue of incrementally maintaining the matching in evolving ontologies. In this paper we present an incremental algorithm, *OntoMaintain*, which incrementally calculates the perfect matching among evolving ontologies and simultaneously updates the labels of the concepts of ontologies. We show that our algorithm has a complexity of  $O(n^2)$  compared to complexity  $O(n^3)$  of traditional matching algorithms. Experimental results prove that our algorithm maintains the correctness of a 'brute force method' while significantly reducing the time needed to find a perfect matching in evolving ontologies.

## 1 INTRODUCTION

Information today is dispersed in a variety of disparate sources like public or proprietary databases, books, journals, scientific publications and national archives. A significant amount of this data exists in heterogeneous format. Unfortunately analysts and scientists are not able to identify and exploit this information easily because of the variety of semantics, interfaces and data formats used by the underlying data sources (Reichhardt, 1999).

Semantic heterogeneity, also referred to as conceptual heterogeneity and logical mismatch, underlines the differences in modelling the same domain of interest. Fundamental to resolving semantic and organizational differences is the task of matching 'semantically heterogeneous data'.

Ontologies are an important paradigm for managing the exponential growth of valuable data generated by new technologies (Gruber, 1993). Ontologies model the structure of data in terms of classes and their attributes; they also represent the semantics of data in the form of axioms, such as

inheritance relationships. These semantics explain the relationship between structure and data in different ontologies, to create a consistent integrated ontology. However, existing integration approaches (Mowbray and Zahavi, 1995), (Paolucci, et. al., 2002), (Pinto and Martins, 2000) result in a static ontology, which is hard to evolve as the core sub-ontologies evolve and expand.

A primary task in designing a data integration method is, establishing a mapping scheme between the data sources; mappings represent semantics of relationships and are used to create an integrated ontology (Doan, et. al., 2002).

Our mapping scheme represents concepts of ontology as vertices of a bipartite graph (Melnik, et. al., 2002); it then invokes a bipartite graph matching algorithm (Bellur and Kulkarni, 2007) to find a perfect matching between these vertices. Furthermore, we use the semantics of ontologies, to create a coherent and consistent integrated ontology.

In this paper we propose an incremental algorithm, **OntoMaintain**, based on the concept of weighted bipartite graph matching. Our algorithm incrementally computes the new perfect matching

between ontologies and updates the labels of vertices as a new pair of vertices are added to the ontology.

Experimental results prove that our algorithm maintains the accuracy of the “brute force” method and significantly reduces the computation time in finding the perfect matching of the extended ontology  $O'$ . Our algorithm results in a time complexity of  $O(n^2)$  compared to complexity  $O(n^3)$  of running the Hungarian algorithm (Kuhn, 1955).

## 2 RELATED WORK

In order to accommodate both semantic and schematic differences between heterogeneous data sources, it is crucial to integrate ontologies across various knowledge domains such as anatomy, drug and disease ontology in *biomedical domains*. A good example is the Gene Ontology (GO). GO provides a consistent description of genes in different biological databases. Each term in GO has a unique numerical identifier and a name; terms are further assigned to an ontology like *molecular function*, *cellular component* or *biological process*. Past publications like OASIS (Song et. al., 2006) have cited GO in their proposed matching system.

GO represents two forms of relationships: *is a* and *part of* relationships. The *is a* relationship represents a simple class subclass relationship, where *A is a B* means that A is a subclass of B. The *part of* relation is slightly more complex; *C part of D* means that whenever C is present, it is always a part of D, but C does not always have to be present. An example would be nucleus *part of* cell; nuclei are always part of a cell, but not all cells have nuclei.

(Bellur and Kulkarni, 2007) propose an improved version of semantic matchmaking algorithm by (Paolucci, et al., 2002) to dynamically discover and invoke a *web service*. They use the Hungarian algorithm (Kuhn, 1955) to compute a complete matching of a bipartite graph but minimize the *maximum weighted edge* of graph to get an optimal matching; this optimization criterion is different from the Hungarian algorithm.

None of the works cited above discusses the issue of maintaining mappings in evolving ontologies; how to calculate a perfect matching incrementally, every time new vertices are added to the graph?

## 3 ONTOLOGY MAPPING

### 3.1 Similarity Measures

By comparing terms of ontologies, we first calculate the *similarity values* between them, based on certain metrics; each metric establishes a value based on certain characteristics. Similarity values are in the range [0, 1]. Larger the value, more similar the two terms are.

**Linguistic Similarity.** Is based on term names. A string match algorithm is used to match the names.

**Definition Similarity.** Certain terms in the ontology database have definitions attached; a text classification algorithm compares two definitions and calculates a similarity value between two terms.

**Structure Similarity.** Neighbours of a term are parents and children of the node; probability of a term being similar to another is high if neighbours of both terms are matched.

Based on the above three metrics, a similarity chart is computed. If we have  $n$  similarity charts, each of which has similarity values  $Sim_i(a, b)$ ,  $i = 1..n$ , for any pair of elements  $(a, b)$ ; the overall similarity for each pair  $(a, b)$  is calculated as:

$$Sim(a, b) = \frac{\sum_{i=1}^n (Sim_i(a, b) \times w_i)}{n}, \text{ where } \sum_{i=1}^n (w_i) = 1$$

Higher the weight  $w_i$ , more is its importance and preference.

Table 1: Match according to similarity values.

Value Range	Match
0.50 – 1.00	Exact
0.25 – 0.50	Subsumption
0.25 – 0.00	Conditional

### 3.2 The Gene Ontology

The Gene Ontology is a well known web based bio-ontology; an open source of bioinformatics knowledge. We will use two key GO ontologies; **cellular component** and **molecular function** as examples to explain how the Match conditions mentioned in Table 1 are assigned.

The cellular component and molecular function ontologies are each represented as a directed acyclic graph or DAG. Relationships between any two concepts in a DAG are depicted by *semantic edges*.

A gene may have more than one annotation in different branches of GO. For example, the gene nuclear chromosome is annotated as GO: 0000228 (Figure 1a) and GO: 0000784 (Figure 1b); it is located in more than one cellular component, during which it performs more than one molecular function.

**Exact.** Is the most desirable match. The degree *exact* is assigned when a term in one ontology is equivalent to or superclass of a term in another ontology.

**Subsumption.** The degree *subsumption* is assigned when a term of ontology includes a term in another ontology i.e. it could be used as a substitute.

**Conditional.** The degree *conditional* is assigned when a term of ontology doesn't necessarily match a term in the other ontology but may still be used to achieve the desired result.

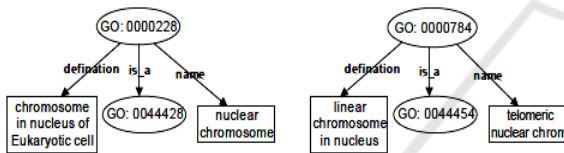


Figure 1a: Ontology  $O_1$ . Figure 1b: Ontology  $O_2$ .

Table 2: Weights according to degree of Match.

Match	Weight
Exact	$w_1$
Subsumption	$w_2$
Conditional	$w_3$

$Exact > Subsume > Conditional$

The criteria in Table 2 are implemented as follows:

**Algorithm 1:** match(value)

- 1: if value  $\geq 0.50$  then
- 2: return Exact
- 3: else if  $0.50 \leq \text{value} \leq 0.25$  then
- 4: return Subsume
- 5: else if value  $< 0.25$  then
- 6: return Conditional
- 7: else
- 8: return Fail
- 9: end if

We match the concepts of ontologies  $O_1$  and  $O_2$  based on the matching criteria proposed in Table 1. The final match is an integrated ontology  $O$  shown in Figure 2 below.

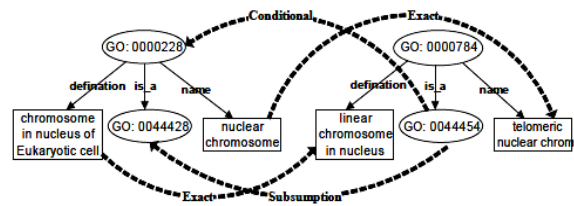


Figure 2: Ontology  $O$ .

## 4 BIPARTITE GRAPH MATCHING

### 4.1 Overview

A *Bipartite Graph* is a graph  $G = (V, E)$  in which the vertex set can be partitioned into two disjoint sets,  $V = V_1 \cup V_2$  such that every edge  $e \in E$  has one vertex in  $V_1$  and the other in  $V_2$ .

A weighted bipartite graph  $G = (X \cup Y, X \times Y)$  having partitions with size  $V$  can be represented by a weight matrix  $W$  of size  $V \times V$ . In the weight matrix, rows correspond to the  $X$  partition and columns correspond to the  $Y$  partition of vertices. Each entry  $w_{ij}$  represents the weight of the edge between the vertices  $x_i$  and  $y_j$ .

Feasible vertex labelling  $l$  is defined on the vertices of both partitions of the bipartite graph as follows:

$$l(x_i) + l(y_j) \geq w_{ij} \quad \forall x_i \in X, \forall y_j \in Y$$

The subgraph corresponding to the feasible vertex labelling  $l$  defined by the edges satisfying the following equality is called the equality subgraph  $G_l$

$$l(x_i) + l(y_j) = w_{ij}$$

We seek the max-weighted perfect matching in bipartite graphs.

## 5 INCREMENTAL ALGORITHM

Our algorithm is based on the Hungarian algorithm, which produces the feasible vertex labelling of the vertices together with the max-weighted matching. OntoMaintain updates the new feasible vertex labelling of the extended graph while producing the max-weighted matching or so called perfect matching.

The Hungarian algorithm assumes the existence of a weighted bipartite graph,  $G = (X \cup Y, X \times Y)$  as described in Section 4.1. The edge weights,  $w_{ij}$  are stored in a matrix  $W$ . An edge  $(x_i, y_j)$  is matched if  $x_i$  is matched to  $y_j$  and unmatched otherwise. We

represent matched edges with solid lines and unmatched edges with dotted lines. Furthermore, let  $S \subseteq X$  and  $T$  be neighbour of  $S$  i.e.  $T = \text{nhbor}(S)$  but  $T \neq Y$ . So  $T$  contains only those vertices from partition  $Y$ , which are matched to a vertex in  $S$ .

The Hungarian algorithm outputs a complete matching  $M^*_V$ .

The incremental algorithm adds a new pair of vertices to the max-weighted matched bipartite graph whose feasible vertex labelling is also given, together with the max-weighted matching. Then, it assigns any feasible labelling to the newly added pair of vertices and, by using this labelling, determines the maximum-weighted matching of the whole extended bipartite graph.

When we add a new pair of vertices to the max-weighted-matched graph, a new weight matrix is generated by adding a new row and a new column to the previous weight matrix as the  $(V+1)$ st row and column corresponding to the edges incident to the new vertices.

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**Algorithm 2:** OntoMaintain.

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*Input:* The extended bipartite graph

$G' = (X \cup Y, X \times Y)$ , with partitions of size  $V+1$  represented by a weight matrix  $W'$  of size  $(V+1) \times (V+1)$

A feasible vertex labeling  $l$  of the first  $V$  vertices such that it corresponds to  $G_l$  containing the max-weighted matching among the first  $V$  vertices

The max-weighted matching  $M^*_V$  among the first  $V$  vertices of the partitions

*Output:* Perfect matching  $M^*_{V+1}$  and the updated labels  $l$  of the vertices of the extended bipartite graph  $G'$ .

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1. Begin with matching  $M^*_V$  derived by Hungarian algorithm.

Assign labels to new vertices  $Y_{V+1}$  and  $X_{V+1}$  as follows:

$$l(Y_{V+1}) = 0$$

$$l(X_{V+1}) = \max_{y_j \in Y} (w_{v+1,j})$$

2. On the equality subgraph  $G_l$ , pick an unmatched vertex in partition  $X$  (called  $U$ ) using matching  $M^*_V$ . Grow a Hungarian tree rooted at this vertex, while doing so; include all vertices encountered in  $X$  (with  $U$ ) into  $S$  and all vertices encountered in  $Y$  into  $T$ .

3. If an augmenting path  $A$  is found, interchange matched and unmatched edges in the augmenting

path. Calculate new matching by increasing size by one.

4. If no augmenting path is found, revise the labeling  $l$ . After label revision, labels of vertices in  $S$  are decreased by the smallest possible amount which adds at least one edge between  $S$  and  $Y-T$ . Concurrently, labels of vertices in  $T$  are increased by the same amount in order to maintain the current matching between  $S$  and  $T$ . Adding such edges will increase the chance of finding an augmenting path. Revise labels as:

$$\alpha_l = \min_{xi \in S, yj \in Y-T} \{ l(x_i) + l(y_j) - w_{ij} \}$$

$$l'(v) = \begin{cases} l(v) + \alpha_l \\ l(v) - \alpha_l \\ l(v) \text{ otherwise} \end{cases}$$

5. Go to Step 2 to search for an augmenting path with the new equality subgraph  $G'_l$  defined by the new labeling

## 5.1 Complexity Analysis

Our Incremental Algorithm, OntoMaintain, has a time complexity of  $O(n^2)$ , where  $n$  denotes the total number of vertices in the bipartite graph. Finding the feasible labelling for the new row and column takes linear time, so it has time complexity of  $O(n)$ . In order to find an augmenting path, provided we don't find one straight away, we have to modify the labels of the vertices. This adds at least one edge to the tree, so in the worst case after  $n$  iterations we will find an augmenting path. Each iteration requires  $O(n)$  operations. The computation of  $\alpha_l$  requires  $O(n^2)$  complexity, adding edges from  $S$  to  $Y-T$  requires  $O(n^2)$  operations. Thus, the total complexity of our algorithm is  $O(n^2)$ .

## 5.2 Correctness of the Algorithm

When we add a new pair of vertices to the maximum-weighted-matched bipartite graph, feasible labelling for the extended bipartite graph can be determined by using the labelling on the already determined max-weighted-matched part of the graph. After adding the new vertices, all the matched edges will be in the equality subgraph.

If there is no edge between the new pair of vertices, then they will be the only unmatched vertices in the graph. Because there is only a single unmatched vertex pair, it is possible to find only a single augmenting path in the equality subgraph. So

when the augmenting path is discovered and the matching inverted, the size of the matching must only be increased by one. This in turn would give us the perfect matching.

If we don't discover an augmenting path, we modify the vertex labels to add another edge to the tree. In the end, an augmenting path starting with the only unmatched vertex in one partition and ending with the only unmatched vertex in the other partition will be discovered. That equality subgraph will contain the perfect matching or the max-weighted matching of the bipartite graph.

### 6 PERFORMANCE ANALYSIS

We implemented our algorithm in Java and tested it on a Windows operating environment under optimal conditions.

#### 6.1 Experimental Results

*Environment:* The code was tested on a Windows System, running Windows Vista Home Premium, Intel Core 2 Duo 2.4 GHz with 2037 MB of RAM.

Table 3: Computation Time.

No. of Nodes	Hungarian Algorithm (ms)	OntoMaintain Algorithm (ms)	From Scratch (ms)
3	0	0	0
10	0	0	0
50	16	0	31
100	187	16	202
200	2980	140	2777
300	14118	312	14102
400	39873	1280	40528
500	91369	2168	97407
600	209602	2824	212206

In Table 3: Column 1. Shows number of nodes, column 2. Shows time taken to run the original algorithm, column 3. Shows time taken to run the incremental algorithm and column 4. Shows time taken to recalculate matching from scratch.

#### 6.2 Performance Graph

The performance results of our algorithm are shown in Figure 7 below. As is evident from the graph, the time taken by the Incremental Algorithm to calculate the new matching is significantly less than the time taken to run the complete algorithm from scratch.

Until 50 nodes, the Incremental Algorithm takes about the same time as the Hungarian run from scratch. However with 100 nodes and onwards, the Incremental Algorithm takes dramatically less time to calculate the new perfect matching than the Hungarian Algorithm run from scratch. As the number of nodes in the graph increases, a significant difference in performance time is observed.

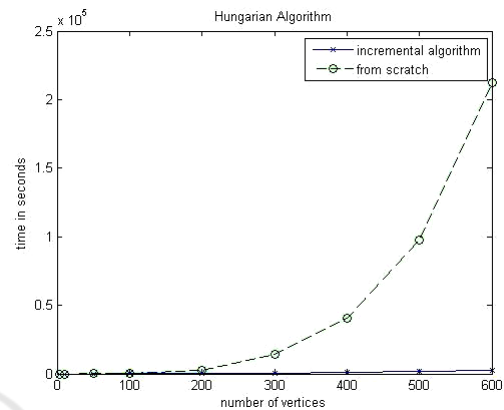


Figure 3: Performance Graph.

#### 6.3 An Illustrative Example

The max-weighted matching  $M^*_V$  among the first  $V$  vertices of the partition and the corresponding weight matrix are displayed in Figure 4a. and Figure 4b.

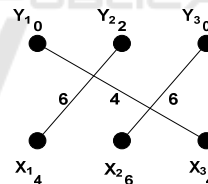


Figure 4a: Max-weighted matching  $M^*_V$ .

	$Y_{1,0}$	$Y_{2,2}$	$Y_{3,0}$
$X_{1,4}$	0	Ⓞ	0
$X_{2,6}$	0	0	Ⓞ
$X_{3,4}$	Ⓞ	0	0

Figure 4b: Weight Matrix.

Now consider the  $(V + 1) \times (V + 1)$  weight matrix  $W'$  in Figure 4c. The last row and column correspond to the newly added vertices. Feasible labels are assigned to  $X_4$  and  $Y_4$  as discussed in Section 4.1

	$Y_1 0$	$Y_2 2$	$Y_3 0$	$Y_4 9$
$X_1 4$	0	6	0	1
$X_2 6$	0	0	6	7
$X_3 4$	4	0	0	9
$X_4 5$	3	5	2	1

Figure 4c: Extended Weight Matrix  $W'$ .

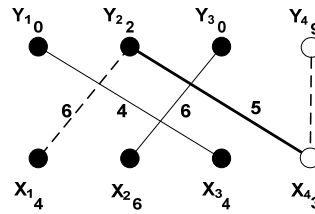


Figure 5c: Final Matching.

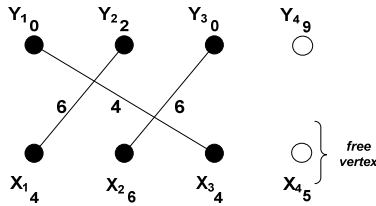


Figure 4d: Unsaturated nodes  $X_4$  and  $Y_4$ .

In Figs. 4c and 4d  $X_4$  and  $Y_4$  are the only unsaturated vertices and there is no augmenting path in the equality graph, so we revise labels. Figure 5a shows the new labelling after labels are revised using Step 4 of **OntoMaintain**. Figure 5b shows the equality subgraph corresponding to the new labelling; the **augmenting path**, in bold, is shown in the Final Matching in Figure 5c.

	$Y_1 0$	$Y_2 2$	$Y_3 0$	$Y_4 9$
$X_1 4$	0	6	0	1
$X_2 6$	0	0	6	7
$X_3 4$	4	0	0	9
$X_4 3$	3	5	2	1

Figure 5a: New Labelling.

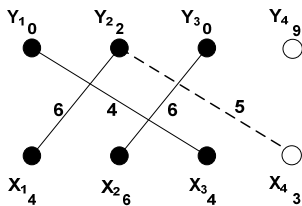


Figure 5b: Equality Subgraph.

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