MATHPORT
Web Application to Support Enhancement in Elementary Mathematics Pedagogy

Barbara Koroušić Seljak, Gregor Papa
Computer Systems Department, Jožef Stefan Institute, Jamova c. 39, Ljubljana, Slovenia

Barbara Japelj Pavešić
Educational Research Institute, Gerbičeva 62, Ljubljana, Slovenia

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Abstract: The paper presents a Web application MatPort that is aimed to support enhancement in elementary mathematics pedagogy. In Slovenia, there is a lack of e-Learning courses on elementary school mathematics that would provide teachers and students with verified mathematics problems in Slovene language. The MatPort offers a databank of verified mathematics problems that may be solved on paper or at a computer, or combined into a paper and pencil form of tests. In this paper, we describe the structure and the functionality of the application, providing an insight into the MatPort decision system that supports automatic search of math problems so that students may reach a certain level of knowledge in a thoughtful way. The decision support system is based upon the genetic algorithm. The application has been evaluated by a group of teachers and students, who have been actively involved into the MatPort development.

1 INTRODUCTION

One of modern ways of education delivery is *e-Learning*, which is a broad term used to describe learning done at a computer, usually connected to the World Wide Web. It is widely accepted that e-Learning can be as rich and as valuable as the classroom experience or even more so. With its unique features e-Learning is an experience that leads to comprehension and mastery of new skills and knowledge, just like its traditional counterpart.

Instructional design for e-Learning, which is the systematic process of translating general principles of learning and instruction into plans for instructional materials and learning, has been perfected and refined over many years using established teaching methods, with many benefits to students.

1.1 Learning Styles

There are three predominant learning styles (Donovan, 2000):

- Visual;
- Auditory;
- Tactile/kinesthetic.

Broken down further, people learn by reading and seeing (visual style), listening and speaking (auditory style) and doing (tactile/kinesthetic style). Reading, listening and seeing are *passive types* of learning, while speaking and doing are *active types* of learning. How much we tend to remember is a function of the type of learning we prefer and our level of involvement in the learning. People often learn through a combination of the ways described above. To a lesser degree, environment is a factor too.

Given a good learning environment, most people tend to understand and remember (i.e., learn) best when practicing the real thing. Being actively engaged in hard and challenging activities strongly supports them to construct knowledge for themselves. Next, a combination of observing people we respect, doing and speaking about what we learn produces a high retention rate, followed by speaking alone.

In e-Learning, it is important to provide a variety of activities supported by human intervention. A lot

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of passive learning may be done through reading text, listening to audio clips, and seeing graphics, while the active mode should be stimulated through practising, e.g., solving mathemathics problems.

1.2 Unique e-Learning Features

There are several unique features of e-Learning:

- e-Learning is self-paced and gives students a chance to speed up or slow down as necessary;
- e-Learning is self-directed, allowing students to choose content and tools appropriate to their differing interests, needs, and skill levels;
- Designed around the learner;
- Geographical barriers are eliminated, opening up broader education options;
- Enhances computer and internet skills.

1.3 Mathematics e-Learning Support in Slovenia

Učiteljska.net (http://uciteljska.net/) and *e-um* (http://www.e-um.si/) are good examples of Slovene Web applications aimed to support e-Learning of elementary school mathematics. While the first one is intended to support exchange of teacher experience and information, the second one provides interactive online courses for students.

However, there is a lack of online courses in Slovene language on solving verified mathematics problems for children under 15 years of age. For this reason, we have developed a new Web application for e-Learning of elementary school mathematics in an active learning mode, called *MatPort* (http://sinica.ijs.si/matport/).

The rest of the paper is organized as follows: Section 2 describes the content and the technology aspects of the Web application. Section 3 outlines the concept of the decision support system that may be used as an automatic search facility. The experimental work is presented in Section 4. Conclusions and directions for further work are given in Section 5.

2 WEB APPLICATION MATPORT

The Web application MatPort is based upon the national curriculum for elementary school mathematics and upon verified mathematical problems.

2.1 Web Content Aspect

For the pilot stage of the project, we decided to use a collection of verified sets of mathematics problems in Slovene language, published in the form of flash cards, which have been used for many years in our schools. These problem solving items already passed through many steps of evaluation, therefore, they are valuable. Some of the real life items needed to be updated for the present time. By this stage, we have focused on the grades 6 to 9 or the age group 12 to 15 years old students.

With the help of experienced teachers we classified the items into subgroups with respect to the knowledge required for problem solving by content and three difficulty levels.

Finally, we designed and developed the MatPort Web application with the following modules for:

1. Entering math items, their solutions and teaching instructions into a database for teachers and administrators;
2. Solving math items;
3. Preparing a paper and pencil form of test;
4. Providing other information relevant for teaching the elementary school mathematics.

The first and the third module are aimed for teachers, the second one for students and the last one for teachers and parents.

2.1.1 Entering Math Items

A user registered as a teacher may enter his/her items into the MatPort database (Fig. 1). Each item is described with a problem definition, solution(s) in terms of values and units, content area, difficulty level, source or author, and teaching instructions. The user may copy an already validated math item and adapt it to its needs.

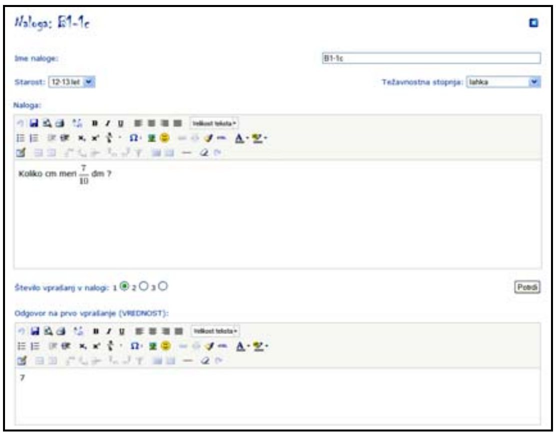

Figure 1: Entering Math Items.

The math items saved in the database are accessible only to the user. However, (s)he can outbid any of his/her items to other MatPort users. Before publishing this item as a public one, experts verify it and suggest modifications if required.

Teachers are informed of their activities through a simple statistics on math items collected in the MatPort database.

2.1.2 Solving Math Items

A user registered as a teacher or a student may solve any public or his/her own math item from the MatPort databank. The application provides *manual* and *automatic search* of math items in content areas defined by the national curriculum for elementary school mathematics. The automatic search is provided by a decision support system that is described in detail in Section 3.

Ones an item is selected, the user can print it and solve the problem in a classical paper and pencil way, or directly provide solution(s) to the MatPort. The application evaluates the answer(s) and denotes the item as correctly or incorrectly solved problem by this user.

Users are given information on the progress in solving math items through graphycal symbols and stimulative words (Fig. 2). More specifically, if the user correctly solves more than 45%, 60%, 75% or 90% of dealt items, respectively, (s)he is rewarded with one, two, three, four or five stars, respectively.

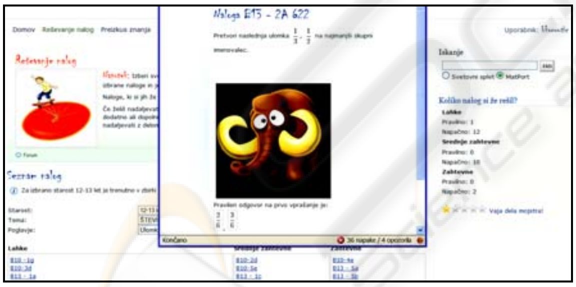

Figure 2: Solving Math Items – information.

If required, users may get teaching instructions for math items (Fig. 3). They can also use a forum integrated into the application to communicate with other users or experts.

2.1.3 Preparing Tests

The MatPort supports design of paper and pencil form tests to examine the students' knowledge in a classroom. A user may select a set of items, modify them if needed, define the test's header and print the test. The application also offers a printout of solutions to the selected math items.

Figure 3: Solving Math Items - teaching instructions.

2.1.4 Providing Information

We support teachers and parents with information on elementary school mathematics, such as (Fig. 4):

- Results on the *IEA (International Association for the Evaluation of Educational Achievement) TIMSS Advanced Assessment Frameworks*, which – carried out every four years at the fourth and eighth grades provides data about trends in mathematics and science achievement over time;
- The national curriculum for elementary school mathematics;
- Practical advices to parents on helping students learning mathematics at home;
- Education debate on motivation methods and tools for engaging students in the process of active learning mathematics.

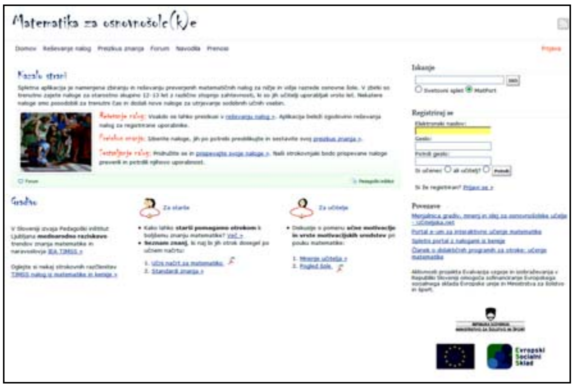

Figure 4: MatPort web application.

2.2 Web Technology Aspect

The MatPort was designed and developed using state-of-the-art technologies. We applied a *UML* (Unified Modelling Language) based model-driven methodology (http://www.uml.org/) to cover the life-cycle of the Web application development.

2.2.1 Design

To design the application sympathetically with the way students, teachers and parents actually use the Web, not how we think they should, we directly involved experienced teachers into the project.

The MatPort **database** was designed as a rational database, consisting of tables that store data on:

- User profiles;
- Math items;
- Knowledge required for solving items;
- Relations between the content areas;
- History of items' solving.

The database structure is given in Fig. 5.

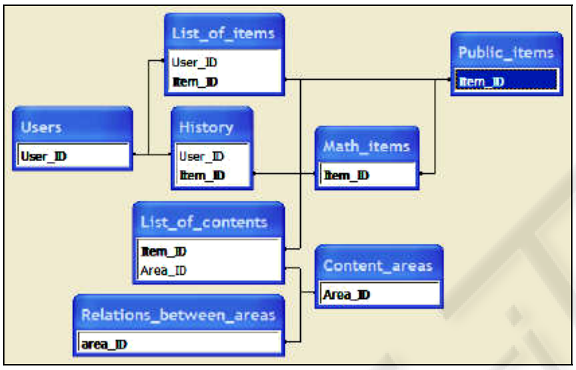

Figure 5: Rough draft of the MatPort database structure.

The application consists of several dynamic and static **modules**, such as:

- Module for providing math items;
- Math item solver module;
- Test generator module;
- Informer module;
- Forum;
- Download center.

The informer module has a static content that is managed by a content management system, while the others may change their content in a dynamic way. The Web application structure is shown in Fig. 6.

2.2.2 Development

As the project's budget was low and we needed to minimize the cost, we decided to make good use of open-source and freely available software. We applied:

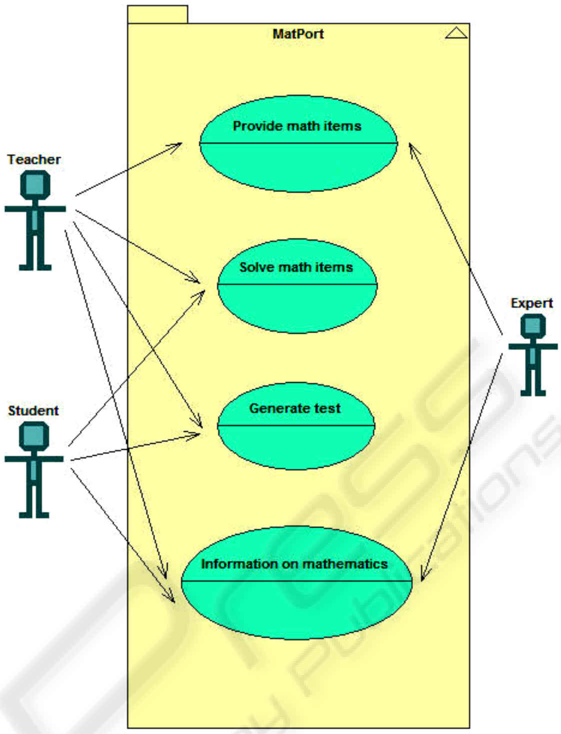

Figure 6: Rough draft of the MatPort use case.

- *Apache* (http://www.apache.org/) as a **Web server** that replies to Web clients' requests via HTTP (HyperText Transfer Protocol);
- *MySQL* (http://www.mysql.com/) as a **database management system** that is based on the relational model;
- *PHP5* (HyperText Preprocessor) (http://www.php.net/) as a server-side technology. This is also a **scripting language** that has evolved to include a command line interface capability and can be used in standalone graphical applications. PHP5 offers standardized means for specifying the variety of property scopes typically offered by object-oriented languages (Hayder, 2007);
- *JavaScript* (Darie *et al*, 2006) as a client-side technology. This is another **scripting language**, whose code is written in plain text and can be embedded into HTML (HyperText Markup Language) pages to empower them.

The MatPort is aimed for doing mathematics, which means that mathematical symbols need to be supported. For this reason, we integrated *TinyMCE* (http://tinymce.moxiecode.com/) as a platform independent JavaScript **WYSIWYG editor**. It has the ability to convert HTML TEXTAREA fields or other HTML elements to editor instances. Because

TinyMCE supports only standard HTML math symbols, we extended its library with a module for handling mathematical expressions. Once entered, these are rendered into images using the *LatexRender* scripts (http://www.mayer.dial.pipex.com/tex.htm).

All MatPort printouts have the standard *PDF* (Portable Document Format) format.

The MatPort forum is based upon the *PHPBB*™ (http://www.phpbb.com/) open-source forum solution.

Last but not least, we set formatting MatPort visual options in a centralized document that is referenced from PHP files by using *CSS* (Cascading Style Sheets).

3 DECISION SUPPORT SYSTEM

We are aware of the fact that only providing a dataset of math items is not enough. We need to incorporate an *extrinsic* motivation system to "bribe" the 6th to 9th grade students to practice mathematics. There are rare children who are *intrinsically motivated* to do repetitive, boring tasks.

3.1 How to Motivate Children?

A student who solves a MatPort math item receives information on the progress through graphical symbols and stimulative words (Fig. 2).

In addition, the application provides information on math items that need to be further solved to receive a higher score. These can be supplementary or additional items to help strengthen or increase knowledge, respectively. The information is provided by the MatPort decision support system, when the automatic search facility is used.

3.2 Genetic Algorithm

The MatPort decision support system that motivates students to continue solving exercises is based upon an evolutionary computation method, i.e., the *genetic algorithm* (GA) (Goldberg, 1989; Bäck, 1996).

The GA is based on a heuristic method, which requires little information to search effectively in a large search space. The algorithm employs an initial population of chromosomes, which evolve to the next generation by probabilistic transition rules (randomized genetic operators) such as selection, crossover and mutation. The objective function evaluates the quality (fitness) of solutions coded as chromosomes. This information is used to perform an effective search for better solutions. There is no need of other auxiliary knowledge. The GA tends to take advantage of the fittest solutions by giving them greater weight and by concentrating the search in the regions of the search space with likely improvement. The GA mechanism is presented in Fig. 7.

```
Initialize the population of
chromosomes;
While stop condition not met do:
 ▪ Calculate the fitness for each
    member in the population using the
    fitness function;
 ▪ Select and reproduce individuals
    according to their fitness;
 ▪ Perform genetic operators
    (crossover and mutation) on the
    population.
```

Figure 7: The GA's pseudocode.

The GA is a population-based evolutionary approach that allows searching within a broad set of solutions from the search space simultaneously. Namely, because there are many math items (few hundreds or even more than thousand math items per a grade) and interrelated content areas (more than 100 content areas per a grade), the student may continue solving items in many possible ways that may or may not lead to a higher score. Moreover, math items are dynamically generated by teachers (i.e., the item dataset expands with time) and the student may start solving them anywhere in the dataset. In the GA, there is a risk of converging to a local optimum, but good results of various research work obtained in other optimization problem areas (Papa and Koroušić Seljak, 2005; Koroušić Seljak, 2006; Tušar et al, 2007; Korošec and Šilc, 2008) encouraged us to consider the GA approach as a promising approach to the decision making problem.

The idea is to **find a set of math items within different content areas that, when solved correctly, improve the user's knowledge** and increase his/her score as much as possible. The set of items should consist of math problems from all poorly scored content areas and the areas that precede these areas. Therefore, before start searching, the system identifies all the feasible items, i.e., math problems from the poorly scored content areas. These items form some kind of a pool of relevant items *P* for current score improvement.

3.2.1 Encoding

The suggested list of math items needed to improve the score is encoded into a chromosome, where each gene represents the identification (ID) number of the item in the MatPort database. The chromosome length has been fixed to 15, while this number represents a reasonable number of items to perform, in order to significantly improve the score. Fig. 8 presents the chromosome with such number of genes and the IDs of the items to be performed in the order as encoded in the chromosome.

1	2	3	4	5		14	15
45	124	33	79	186	…	247	31

Figure 8: Chromosome of length 15 that represents a set of items to be solved to increase the current score.

3.2.2 Population Initialization

The initial population consists of n chromosomes. Each chromosome is initialized with randomly chosen items from the pool of relevant math items.

3.2.3 Genetic Operators

In the selection process, the elitism strategy is applied through the substitution of the least-fit chromosomes with the equal number of the best-ranked chromosomes.

With the one-point crossover scheme, chromosome mates are chosen randomly and with a probability p_c all values after randomly chosen position are swapped, which leads to two new solutions that replace their original sources. Fig. 9 shows a crossover example.

45	124	33	79	186	…	247	31
12	14	233	56	228	…	269	41

⇓ ⇓

45	124	33	56	228	…	269	41
12	14	233	79	186	…	247	31

Figure 9: One-point crossover.

In the mutation process each value of the chromosome mutates with a probability p_m. If the value of the chromosome needs to be changed, than some new value from the pool of relevant math items is chosen. However, since a high mutation rate resulted in a random walk through the GA search space, p_m has to be chosen to be somewhat low. Fig. 10 shows a mutation example.

45	124	33	79	186	…	247	31

⇓ ⇓

45	137	33	56	12	…	269	41

Figure 10: Mutation.

3.2.4 Fitness Evaluation

After the recombination operators modify the solutions, the whole new population of chromosomes is ready to be evaluated. In the evaluation process the set of math items is assumed to be solved correctly and the score improvement is calculated. The calculated score improvement is used as a fitness value of each chromosome. Here all the items are weighted with their difficulty grade; the order of items is relevant when the problems belong to different content areas that derive from each other.

3.2.5 Parameter Settings

In order to ensure optimal solutions in a reasonable response time robust parameter settings need to be found for the population size, number of generations, selection criteria and genetic operator probabilities:

- If the population size and the number of generations are too small, the GA converges too quickly to a local optimal solution and may not find the best solution. On the other hand, a large population and too much iteration require long time to converge to a region of the search space with significant improvement. In our case, we have used the population size n=15 and number of generations n_g=30;
- Applying the elitism strategy, fitter solutions have greater chance to be reproduced. But when the number of worse solutions to be exchanged with better ones (the selection criteria) is too high, the GA is trapped too quickly in a local optimum solution. Our selection rate has been 20%;
- Too low crossover probability preserves solutions to be interchanged and longer time is required to converge. This probability should be large enough to crossover almost all mated solutions. In our case, efficient setting for p_c has been 70%;
- Too high mutation probability may introduce too much diversity and takes longer time to reach an optimal solution. Too low mutation

probability tends to miss some near-optimal solutions. Again, the efficient setting for p_m has been 5%.

3.2.6 Termination

When a certain number of populations are generated and evaluated, the system is assumed to be in a non-converging state. A chromosome with the highest score improvement is chosen as a final result.

On average, the GA finds an optimal selection of math items that need to be further solved by the user to receive a higher score in order of few seconds. As it is implemented as a background process, it does not slow down the application.

4 EXPERIMENTAL WORK

During the current pilot stage of the MatPort application, we have involved a group of elementary school teachers and a group of the 8^{th} grade students to test the application facilities.

We have examined the MatPort effectiveness for delivery of elementary mathematics e-Learning. In the group of teachers, we have been interested in:

1. The number of new math items provided by the teachers;
2. The number of copied and modified already validated math items;
3. The number of written tests designed by the MatPort.

In the group of students, we have been interested in:

1. The time spent in each content area;
2. The number of correctly and incorrectly solved exercises in each knowledge field;
3. The correlation between the MatPort scores and school marks before and after the period of testing the MatPort;
4. Frequency of using the MatPort automatic search (decision support system) facility.

Both groups have been asked to complete a survey that includes an assessment of likeability, intention to use exercises and information, and demographic indicators.

The main aim of the study is to prove a predicted hypothesis that participants keep or even increase their level of knowledge in elementary mathematics by practising math problems provided by the MatPort. In this way, we could conclude that the MatPort actually supports enhancement in elementary mathematics pedagogy.

5 CONCLUSIONS

In this paper, the instructional design of the MatPort Web application, which is aimed for elementary mathematics e-Learning in the active way, were presented. We upgraded this design by incorporating a high-performance evolutionary computation method to support automatic search of relevant math items. In this way, the MatPort may lead its users toward higher scores in a thoughtful way. Finally, we described the method for evaluation of the application's effectiveness.

After the pilot stage of the project, we are planning to expand the dataset of math items to other elementary school grades. We will increase its efficiency through additional motivation tools, such as winner lists or computer games, which will be activated as soon as a student will gain a certain score. Much work needs to be done to find an adequate level of human intervention. In cooperation with teachers, we will try to improve the way of providing teaching instructions and intermediate solutions. Last but not least, we will discuss the problem of cheating.

In addition, we will do some experimental work on the application of the efficient parameter-less evolutionary search method (Papa, 2008) as a substitution for the currently implemented genetic algorithm.

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