OBSERVER-BASED STATE FEEDBACK REMOTE CONTROL WITH BOUNDED TIME-VARYING DELAYS

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Abstract: This paper investigates the problem of remote stabilization via communication networks with uncertain, "non-small", time-varying, non-symmetric transmission delays affecting both the control input and the measured output. More precisely, this paper focuses on a closed-loop Master-Slave setup with a TCP network as communication media, and an observer-based state-feedback control approach to deal with the stabilization objective. First, we establish some asymptotic stability criteria regarding to a Lyapunov–Krasovskii functional derived from a descriptor model transformation, in case of "non-small" delays (that are time-varying delays with non-zero lower bounds). Then, some stability conditions are given in terms of Linear Matrix Inequalities which are used, afterwards, to design the observer and controller gains. Finally, the proposed stabilizing approach is illustrated through numerical and simulation results, related to the remote control of a "ball and beam" system.

1 INTRODUCTION

Over the past few years, the widespread development of low-cost wired and wireless data networks has lead to an increasing interest for Networked Control Systems (NCSs) (for instance, see (Yang, 2006; Tang and Yu, 2007; Hespanha et al., 2007) and references therein). Indeed, such networks seem to be suitable for large scale control systems with sensors, actuators and controllers that communicate over a shared medium. However, most of common network physical configurations and communication protocols[1] lead to transmission delays and even data losses. Then, from a control viewpoint, it is well-known that such undesirable features affect the overall NCS behavior, leading possibly to poor performance and/or instabilities (e.g. (Niculescu, 2001; Ge et al., 2007)). This justifies the increasing investigations on control strategies to insure both closed-loop stability and good performance for time-delayed systems (see (Tipsuwan and Chow, 2003; Richard, 2004) and references therein). Following this, the present paper then deals with the stabilization of a Networked Control System with consideration of TCP (Transmission Control Protocol) networking protocol for bi-directional communications between a Master system (computing the control) and a Slave system (to be controlled). In particular, we investigate the design of an observer-based (static) state-feedback controller (located in the Master system) so as to insure the asymptotic stability of the closed-loop NCS whatever the presence of time-varying, non-symmetric delays in the control and feedback loops. In this purpose, first, we establish some stability conditions by means of a Lyapunov–Krasovskii functional derived from a descriptor model transformation (Fridman and Shaked, 2002). These conditions are given in terms of Linear Matrix Inequalities which are used afterwards to design both controller and observer gains, by means of LMI optimization. This design approach is then illustrated through an example related to the remote control of a "ball and beam" system.

This paper is organized as follows. Section 2 describes the Networked Control System under consideration. Section 3 defines the observer-based control law, while section 4 focuses on the design of both state-feedback controller and observer gains. Section 5 presents a "ball and beam" system as remote controlled plant for illustrating the proposed control strategy. Then, some numerical and simulations results related to the observer-based control of this system are presented. Finally, some concluding remarks are given in section 6.

[1] Such as User Datagram Protocol (UDP), Transfer Control Protocol (TCP), Medium Access Control protocols, etc.

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2 SYSTEM DESCRIPTION

Regarding to Figure 1, the Networked Control System under consideration consists in a Master-Slave setup, with a TCP network as communication media linking these two systems.

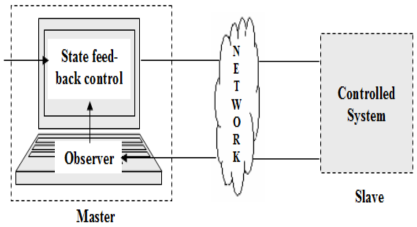

Figure 1: The Networked Control System (Master-Slave configuration).

- The exchanged data correspond respectively to the control input (sent by the Master to the Slave), and a measured output of the remote system (sent by the Slave to the Master). Due to the networking protocol and communication lines properties, we consider some time-delays τ_1 and τ_2, respectively related to the Master-to-Slave and Slave-to-Master transmissions. Moreover these delays are assumed to be time-varying, uncertain (with known lower and upper bounds), and non-symmetric (that is $\tau_1 \neq \tau_2$).

Remark 1. *The consideration of TCP networking protocol insure that all transmitted data are received in the emission order. Thus, when considering a first data packet emitted at time t_1 undergoing a delay τ_1, and a second data packet emitted at time t_2 undergoing a delay τ_2, the correct scheduling of data implies that (see (Witrant et al., 2003)):*

$$t_1 + \tau_1 < t_2 + \tau_2 \quad \Leftrightarrow \quad -1 < \frac{\tau_2 - \tau_1}{t_2 - t_1} \simeq \frac{d\tau}{dt} \tag{1}$$

Therefore, the Master-to-Slave and Slave-to-Master delays $\tau_i(t)$ (with $i = 1,2$) can be expressed as differentiable functions, and such that:

$$\begin{aligned} &\forall t \geq 0, \quad \tau_i(t) = h_i + \eta_i(t), \\ with \quad &0 \leq \eta_i(t) \leq \mu_i, \quad \dot{\eta}_i(t) \leq d_i < 1 \end{aligned} \tag{2}$$

where the $\tau_i(t)$ (with $i = 1,2$) are considered as time-varying bounded delays with non-zero lower bounds $h_i > 0$ (sometimes referred to as "non-small delays"). $\eta_i(t)$ is a differentiable function which characterizes a (bounded) time-varying perturbation with bounded time-derivative $\dot{\eta}_i(t) < 1$ (so that $\tau_i(t)$ are commonly referred to as slowly-varying delays – e.g. (Shustin and Fridman, 2007)), and μ_i and d_i are strictly positive, constant upper-bounds (see (Fridman, 2004)).

Moreover, we can define $\tau_i^* = h_i + \mu_i$ as an upper-bound for $\tau_i(t)$, leading finally to $h_i \leq \tau_i(t) \leq \tau_i^*$.

Remark 2. *Such an assumption on non-zero lower bounds h_i of delays is realistic. Indeed, zero or close to zero delays (corresponding to instantaneous or quasi-instantaneous transmissions) are usually not met in most of real networks (due to, at least, propagation phenomena).*

- The controlled system (within the Slave part), is supposed to be linear, controllable and observable, with a known state-space representation $(A;B;C)$. By taking into account the time-delay τ_1 (intrinsic to the Master-to-Slave transmission), this Slave system is then given by:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t - \tau_1(t)) \\ y(t) &= Cx(t) \end{aligned} \tag{3}$$

  where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the delayed control input with an input time-delay $\tau_1(t) > 0$ that we assume to be a differentiable function satisfying to relation (2). $y(t) \in \mathbb{R}^p$ is the system output, and A, B and C are constants matrices of appropriate dimensions.

- The Master system includes an observer which aims at providing an estimation $\hat{x}(t)$ of the full state-vector $x(t)$ of the Slave system, from the output $y(t)$ it receives after a delay $\tau_2(t)$ (assuming this delay also satisfies to relation (2)). From this estimation $\hat{x}(t)$, the Master then computes the control and forwards it to the Slave.

3 THE OBSERVER-BASED STATE-FEEDBACK CONTROL

3.1 The Full-state Observer

As already mentioned, this paper considers, from the Master system viewpoint, a full-state reconstruction of the Slave state-vector $x(t)$ from the transmitted, delayed, scalar output $y(t - \tau_2)$ coming from the Slave system. As this last system is assumed to be linear, we propose here to perform this full-state estimation, by means of a Luenberger-type observer (Luenberger, 1971).

With respect to the NCS setup, the observer can then be defined by:

$$\begin{aligned}\dot{\hat{x}}(t) &= A\hat{x}(t)+Bu(t-\tau_1(t)) \\ &\quad -L\left[y(t-\tau_2(t))-\hat{y}(t-\tau_2(t))\right] \\ \hat{y}(t) &= C\hat{x}(t) \end{aligned} \tag{4}$$

where L is the observer gain which has to be designed so as to ensure a sufficiently fast convergence of $\hat{x}(t)$ towards the true system state $x(t)$, regardless of time-varying delay $\tau_2(t)$.

Remark 3. *Delay $\tau_2(t)$ is supposed to be time-varying and uncertain. Nevertheless, we assume the knowledge of an upper-bound $\tau_2^* = h_2+\mu_2 \geq \tau_2(t)$.*

3.2 The Control Law

Regarding to the literature, many control strategies have been proposed to deal with the stabilization problem of NCS with delays. In our case, as the Luenberger-type observer is supposed to provide a full-state reconstruction $\hat{x}(t)$ of the Slave state-vector $x(t)$, we propose to investigate the use of a simple state-feedback control $u(t)$ of the following form:

$$u(t) = K\hat{x}(t) \tag{5}$$

where K is the control gain to design so as to guarantee the closed-loop stability of the controlled system (the Slave), regardless of the control input time-varying delay $\tau_1(t)$.

4 DESIGN OF CONTROLLER AND OBSERVER GAINS

With respect to (4) and (5), this section is devoted to the design of both controller and observer gains that guaranty the closed-loop stabilization of the NCS (despite of both input and output time-varying delays $\tau_1(t)$ and $\tau_2(t)$). In this aim, let us establish some asymptotic stability criteria, by applying a Lyapunov-Krasovskii methodology based on a descriptor model transformation (see (Fridman and Shaked, 2002)).

4.1 Control Design

First, let us focus on the design of an ideal controller $u(t) = Kx(t)$ by considering a perfect observer (such that $\hat{x}(t) = x(t)$), before to deal, in a later subsection, with the influence of the observation error on the whole system stability.

Thus, first, let us recall that the controlled system is represented by a linear system with bounded, time-varying, input delay (see Remark 1), whose dynamics can be expressed as:

$$\begin{aligned}\dot{x}(t) &= Ax(t)+BKx(t-\tau_1(t)) \\ x(\theta) &= \varphi(\theta), \quad \theta\in[-\tau_1^*,0]\end{aligned} \tag{6}$$

where $\tau_1^* = h_1+\mu_1$ is an upper-bound for time-delay $\tau_1(t)$. Then, following a similar approach as in (Fridman, 2004), let us express a result that gives some asymptotic stability conditions for system (6), in terms of Linear Matrix Inequalities, for a given K.

Theorem 1. *Given a gain matrix K, system (6) is asymptotically stable if there exists $n\times n$ matrices $0 < P_1, P_2, P_3, S_1, S_{a_1}, Y_{1i}, Y_{a_{1i}}, Z_{1k}, Z_{a_{1k}}$, and R_1, R_{a_1} satisfying the LMI conditions for $i = 1,2$ and $k = 1,2,3$:*

$$\Gamma = \begin{bmatrix} \Psi_1 & P^T\begin{bmatrix}0\\BK\end{bmatrix}-Y_{a_1}^T & Y_{a_1}^T - Y_1^T \\ * & -(1-d_1)S_{a_1} & 0 \\ * & * & -S_1 \end{bmatrix} < 0 \tag{7}$$

$$\textit{and,}\quad \begin{bmatrix} R_1 & Y_1 \\ * & Z_1\end{bmatrix}\geq 0, \begin{bmatrix} R_{a_1} & Y_{a_1} \\ * & Z_{a_1}\end{bmatrix}\geq 0 \tag{8}$$

with,

$$Y_1 = [Y_{11}\ Y_{12}] \qquad Y_{a_1} = [Y_{a_{11}}\ Y_{a_{12}}]$$

$$Z_1 = \begin{bmatrix} Z_{11} & Z_{12} \\ * & Z_{13}\end{bmatrix} \quad Z_{a_1} = \begin{bmatrix} Z_{a_{11}} & Z_{a_{12}} \\ * & Z_{a_{13}}\end{bmatrix}$$

where $*$ denotes the symmetric, and Ψ_1 is given by:

$$\begin{aligned}\Psi_{11} &= P_2^TA+A^TP_2+S_1+h_1Z_{11}+Y_{11}+Y_{11}^T+S_{a_1} \\ &\quad +\mu_1Z_{a_{11}} \\ \Psi_{12} &= A^TP_3+P_1^T-P_2^T+h_1Z_{12}+Y_{12}+\mu_1Z_{a_{12}} \\ \Psi_{13} &= -(P_3+P_3^T)+h_1(Z_{13}+R_1)+\mu_1R_{a_1}+\mu_1Z_{a_{13}}\end{aligned}$$

Proof — Representing (6) in an equivalent descriptor form ((Fridman and Shaked, 2002)) leads to:

$$\begin{aligned}\dot{x}(t) &= z(t) \\ 0 &= -z(t)+Ax(t)+BKx(t-\tau_1(t))\end{aligned} \tag{9}$$

By posing $\bar{x}(t) = col\{x(t),z(t)\}$ and $E = diag\{I_n,0\}$, then (9) can be rewritten as:

$$E\dot{\bar{x}} = \begin{bmatrix}\dot{x}(t)\\0\end{bmatrix} = \begin{bmatrix}z(t)\\-z(t)+\Lambda x(t)\end{bmatrix} - \begin{bmatrix}0\\BK\end{bmatrix}\int_{t-h_1}^{t} z(s)ds - \begin{bmatrix}0\\BK\end{bmatrix}\int_{t-h_1-\eta_1}^{t-h_1} z(s)ds \tag{10}$$

where $\Lambda = A+BK$.

Now, considering a Lyapunov-Krasovskii functional (LKF) of the form:

$$V(t) = V_n(t)+V_a(t); \tag{11}$$

where $V_n(t)$ is a nominal LKF corresponding to the nominal system (10) with $h_1 \neq 0$ and $\eta_1(t) = 0$, and such that (see (Fridman, 2004)):

$$\begin{aligned}V_n(t) &= \bar{x}(t)^TEP\bar{x}(t)+\int_{-h_1}^{0}\int_{t+\theta}^{t} z(\beta)^TR_1z(\beta)d\beta d\theta \\ &\quad +\int_{t-h_1}^{t} x(s)^TS_1x(s)ds\end{aligned} \tag{12}$$

and $V_a(t)$ is an additional term (which corresponds to the perturbed system), which vanishes when the delay perturbation approaches to 0 (that is when $\eta_1(t) \simeq 0$) and such that:

$$V_a(t) = \int_{-\mu_1}^{0}\int_{t+\theta-h_1}^{t} z(s)^T R_{a_1} z(s) ds d\theta + \int_{t-\tau_1(t)}^{t} x(s)^T S_{a_1} x(s) ds \qquad (13)$$

with $P = \begin{bmatrix} P_1 & 0 \\ P_2 & P_3 \end{bmatrix}$, P_1, R_1, R_{a_1}, S_1 and $S_{a_1} > 0$.

Noting that $V_1 = \bar{x}(t)^T EP\bar{x}(t) = x(t)^T P_1 x(t)$, then differentiating this term in t along the trajectories of the perturbed system (9) leads to:

$$\frac{dV_1(t)}{dt} = 2\bar{x}(t)^T P^T \begin{bmatrix} \dot{x}(t) \\ 0 \end{bmatrix}$$

Then, replacing $[\dot{x}(t) \quad 0]^T$ by the right side of (10), the derivative of $V_n(t)$ in t along the trajectories of the perturbed system (9) satisfies the following relation:

$$\begin{aligned} \dot{V}_n(t) &= \bar{x}(t)^T \Psi_0 \bar{x}(t) + \delta_1(t) + \delta_2(t) + h_1 z(t)^T R_1 z(t) \\ &\quad - \int_{t-h_1}^{t} z(s)^T R_1 z(s) ds + x(t)^T S_1 x(t) \\ &\quad - x(t-h_1)^T S_1 x(t-h_1) \end{aligned} \qquad (14)$$

with $\Psi_0 = P^T \begin{bmatrix} 0 & I_n \\ \Lambda & -I_n \end{bmatrix} + \begin{bmatrix} 0 & I_n \\ \Lambda & -I_n \end{bmatrix}^T P$.

Moreover, it comes that $\delta_1(t)$ and $\delta_2(t)$ are given by:

$$\begin{aligned} \delta_1(t) &= -2\bar{x}(t)^T P^T \begin{bmatrix} 0 \\ BK \end{bmatrix} \int_{t-h_1}^{t} z(s) ds \\ \delta_2(t) &= -2\bar{x}(t)^T P^T \begin{bmatrix} 0 \\ BK \end{bmatrix} \int_{t-h_1-\eta_1}^{t-h_1} z(s) ds \end{aligned}$$

Now, let us bound $\delta_1(t)$ and $\delta_2(t)$ by applying the bounding given in (Moon et al., 2001), where, for any $a \in \mathbb{R}^n$, $b \in \mathbb{R}^{2n}$, $R \in \mathbb{R}^{n\times n}$, $Y \in \mathbb{R}^{n\times 2n}$, $Z \in \mathbb{R}^{2n\times 2n}$, $N \in \mathbb{R}^{2n\times n}$ the following holds:

$$-2b^T Na \leq \begin{bmatrix} a \\ b \end{bmatrix}^T \begin{bmatrix} R & Y - N^T \\ Y^T - N & Z \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$$

with $\begin{bmatrix} R & Y \\ Y^T & Z \end{bmatrix} \geq 0$.

By considering such bounding condition and taking $N = P^T [0 \quad BK]^T$, $a = z(s)$, $b = \bar{x}(t)$, $R = R_1$, $Z = Z_1$, $Y = Y_1$ such that:

$$\begin{bmatrix} R_1 & Y_1 \\ * & Z_1 \end{bmatrix} \geq 0$$

then, we can find the following bound for δ_1 :

$$\begin{aligned} \delta_1(t) &\leq \int_{t-h_1}^{t} z(s)^T R_1 z(s) ds + h_1 \bar{x}(t)^T Z_1 \bar{x}(t) \\ &\quad + 2(x(t)^T - x(t-h_1)^T) \\ &\quad (Y_1 - \begin{bmatrix} 0 & (BK)^T \end{bmatrix} P)\bar{x}(t) \end{aligned} \qquad (15)$$

Similarly to $\delta_1(t)$, by posing $N = P^T [0 \quad BK]^T$, $a = z(s)$, $b = \bar{x}(t)$, $R = R_{a_1}$, $Z = Z_{a_1}$, $Y = Y_{a_1}$, such that:

$$\begin{bmatrix} R_{a_1} & Y_{a_1} \\ * & Z_{a_1} \end{bmatrix} \geq 0$$

the bound of $\delta_2(t)$ is given by:

$$\begin{aligned} \delta_2(t) &\leq \int_{t-h_1-\eta_1}^{t-h_1} z(s)^T R_{a_1} z(s) ds + \mu_1 \bar{x}(t)^T Z_{a_1} \bar{x}(t) \\ &\quad + 2(x(t-h_1)^T - x(t-h_1-\eta_1)^T) \\ &\quad (Y_{a_1} - \begin{bmatrix} 0 & (BK)^T \end{bmatrix} P)\bar{x}(t) \end{aligned} \qquad (16)$$

Then, the time-derivative of $V_a(t)$ is given by:

$$\begin{aligned} \dot{V}_a(t) &= \mu_1 z(t)^T R_{a_1} z(t) - \int_{t-h_1-\eta_1}^{t-h_1} z(s)^T R_{a1} z(s) ds \\ &\quad + x(t)^T S_{a_1} x(t) \\ &\quad - (1-d_1) x(t-h_1-\eta_1)^T S_{a_1} x(t-h_1-\eta_1) \end{aligned}$$

Substituting (15) and (16) into (14), we find that the derivative of $V(t)$ along the trajectories of the perturbed system satisfies the following inequality:

$$\dot{V}(t) \leq \zeta(t)^T \Gamma \zeta(t) \qquad (17)$$

where $\zeta(t) = col\{\bar{x}(t), x(t-h_1-\eta_1), x(t-h_1)\}$, and Γ is a negative matrix given by (7). Thus $\dot{V}(t)$ is negative definite if conditions (7) and (8) are satisfied, while $V(t) \geq 0$. Therefore, system (6) is asymptotically stable, and the proof is achieved. ■

Note that conditions (7) and (8) are satisfied for a given state-feedback gain K. However, in our case (that is a stabilization problem involving a control law $u(t) = Kx(t)$), K is an unknown control gain to be designed so as to insure the closed-loop stability of system (9). In such a case, the LMI condition (7) contains a bilinear term coming from the product of the LMI variable with K, leading (7) to be a Bilinear Matrix Inequality. Therefore, to give rise to a LMI condition for computing of gain K, we can apply the transformation given in (Suplin et al., 2004). In this aim, let us define: $P_3 = \varepsilon P_2$ where $\varepsilon \in \mathbb{R}$ is a tuning scalar parameter. Moreover, let us note that P_2 is nonsingular since the only matrix which can be negative definite in the second block on the diagonal of Ψ_1 is $-\varepsilon(P_2 + P_2^T)$. Therefore we can also define: $\bar{P} = P_2^{-1}$. In addition, for any matrix $V = \{P_1, S_1, S_{a_1}, Y_{1i}, Y_{a_{1i}}, Z_{1k}, Z_{a_{1k}}, R_1, R_{a_1}\}$, for $i = 1, 2$ and

$k = 1,2,3$, let us define an other matrix $\bar{V} = \bar{P}^T V \bar{P}$. Then, by multiplying (7), from the right and the left sides respectively, by $\Delta_4 = diag\{\bar{P},\bar{P},\bar{P},\bar{P}\}$ and its transpose Δ_4^T, and multiplying (8) by $\Delta_3 = diag\{\bar{P},\bar{P},\bar{P}\}$ and its transpose Δ_3^T, from the right and the left sides respectively, and posing $W = K\bar{P}$, the proof of the following theorem is straightforward.

Theorem 2. *Suppose that, for some positive number* ε, *there exists a positive-definite matrix* $\bar{P}_1$, $n \times n$ *matrices* $\bar{P}$, $\bar{S}_1$, $\bar{S}_{a_1}, \bar{Y}_{1i}, \bar{Y}_{a_{1i}}, \bar{Z}_{1k}, \bar{Z}_{a_{1k}}$, $\bar{R}_1, \bar{R}_{a_1}$ *and* $W \in \mathbb{R}^{m\times n}$ *satisfying the LMI conditions for* $i = 1,2$ *and* $k = 1,2,3$*:*

$$\Gamma = \begin{bmatrix} \Psi_2 & \begin{bmatrix} BW \\ \varepsilon BW \end{bmatrix} - \bar{Y}_{a_1}^T & \bar{Y}_{a_1}^T - \bar{Y}_1^T \\ * & -(1-d_1)\bar{S}_{a_1} & 0 \\ * & * & -\bar{S}_1 \end{bmatrix} < 0 \quad (18)$$

$$\textit{and,} \quad \begin{bmatrix} \bar{R}_1 & \bar{Y}_1 \\ * & \bar{Z}_1 \end{bmatrix} \geq 0, \quad \begin{bmatrix} \bar{R}_{a_1} & \bar{Y}_{a_1} \\ * & \bar{Z}_{a_1} \end{bmatrix} \geq 0 \quad (19)$$

where,

$$\bar{Y}_1 = [\bar{Y}_{11}\ \bar{Y}_{12}] \qquad \bar{Y}_{a_1} = [\bar{Y}_{a_{11}}\ \bar{Y}_{a_{12}}]$$

$$\bar{Z}_1 = \begin{bmatrix} \bar{Z}_{11} & \bar{Z}_{12} \\ * & \bar{Z}_{13} \end{bmatrix} \quad \bar{Z}_{a_1} = \begin{bmatrix} \bar{Z}_{a_{11}} & \bar{Z}_{a_{12}} \\ * & \bar{Z}_{a_{13}} \end{bmatrix}$$

and matrix Ψ_2 *is given by:*

$$\begin{aligned} \Psi_{21} &= A\bar{P} + \bar{P}^T A^T + \bar{S}_1 + h_1 \bar{Z}_{11} + \bar{Y}_{11} + \bar{Y}_{11}^T \\ &\quad + \bar{S}_{a_1} + \mu_1 \bar{Z}_{a_{11}} \\ \Psi_{22} &= \varepsilon \bar{P}^T A^T + \bar{P}_1 - \bar{P} + h_1 \bar{Z}_{12} + \bar{Y}_{12} + \mu_1 \bar{Z}_{a_{12}} \\ \Psi_{23} &= -\varepsilon(\bar{P} + \bar{P}^T) + h_1(\bar{Z}_{13} + \bar{R}_1) + \mu_1 \bar{R}_{a_1} \\ &\quad + \mu_1 \bar{Z}_{a_{13}} \end{aligned}$$

Then, the gain,

$$K = W\bar{P}^{-1} \quad (20)$$

asymptotically stabilizes the system (6) for delay $\tau_1(t) \leq \tau_1^*$.

4.2 Observer Design

Since the pair $(A;C)$ is assumed to be observable, it is possible to determine, in the non-delayed case (that is $\tau_2 = 0$), a gain L such that the Luenberger-type observer leads the estimation error to asymptotically converge towards zero. Now, by taking into account the variable delay $\tau_2(t)$ on the Slave output, then, from (3) and (4), the observation error $e(t) = \hat{x}(t) - x(t)$ is ruled by:

$$\dot{e}(t) = Ae(t) + LCe(t - \tau_2(t)) \quad (21)$$

We then express the following result which insures that the observer state $\hat{x}(t)$ converges sufficiently fast towards the true system state $x(t)$ despite of delay $\tau_2(t)$.

Theorem 3. *Suppose that, for some positive scalar* ε, *there exists* $n \times n$ *matrices* $0 < P_1$, P, S_2, $S_{a_2}, Y_{2i}, Y_{a_{2i}}, Z_{2k}, Z_{a_{2k}}$, R_2, R_{a_2} *and* $X \in \mathbb{R}^{n\times p}$ *satisfying the LMI conditions for* $i = 1,2$ *and* $k = 1,2,3$*:*

$$\Gamma = \begin{bmatrix} \Psi_1 & \begin{bmatrix} XC \\ \varepsilon XC \end{bmatrix} - Y_{a_2}^T & Y_{a_2}^T - Y_2^T \\ * & -(1-d_2)S_{a_2} & 0 \\ * & * & -S_2 \end{bmatrix} < 0 \quad (22)$$

$$\textit{and,} \quad \begin{bmatrix} R_2 & Y_2 \\ * & Z_2 \end{bmatrix} \geq 0, \begin{bmatrix} R_{a_2} & Y_{a_2} \\ * & Z_{a_2} \end{bmatrix} \geq 0 \quad (23)$$

where,

$$Y_2 = [Y_{21}\ Y_{22}], \quad Y_{a_2} = [Y_{a_{21}}\ Y_{a_{22}}]$$

$$Z_2 = \begin{bmatrix} Z_{21} & Z_{22} \\ * & Z_{23} \end{bmatrix}, \quad Z_{a_2} = \begin{bmatrix} Z_{a_{21}} & Z_{a_{22}} \\ * & Z_{a_{23}} \end{bmatrix}$$

with matrix Ψ_1 *is given by:*

$$\begin{aligned} \Psi_{11} &= P^T A + A^T P + S_2 + h_2 Z_{21} + Y_{21} + Y_{21}^T \\ &\quad + S_{a_2} + \mu_2 Z_{a_{21}} \\ \Psi_{12} &= \varepsilon A^T P + P_1^T - P^T + h_2 Z_{22} + Y_{22} + \mu_2 Z_{a_{22}} \\ \Psi_{13} &= -\varepsilon(P + P^T) + h_2(Z_{23} + R_2) + \mu_2 R_{a_2} \\ &\quad + \mu_2 Z_{a_{23}} \end{aligned}$$

Then, the gain

$$L = (P^T)^{-1} X \quad (24)$$

leads the estimation error $e(t) = \hat{x}(t) - x(t)$ *to asymptotically converge towards zero.*

Proof — Representing (21) in an equivalent descriptor form (Fridman and Shaked, 2002):

$$E\dot{\bar{e}}(t) = \begin{bmatrix} z(t) \\ -z(t) + Ae(t) + LCe(t - \tau_2(t)) \end{bmatrix} \quad (25)$$

where $\bar{e}(t) = col\{e(t), z(t)\}$, $E = diag\{I_n, 0\}$.

Recalling that $\tau_2(t) = h_2 + \eta_2(t), 0 \leq \eta_2(t) \leq \mu_2, \dot{\eta}_2(t) \leq d_2 < 1$, the proof of this theorem use the same Lyapunov-Krasovskii as given by (11) with a single delay $\tau_2(t)$:

$$V(t) = V_n(t) + V_a(t)$$

where $V_n(t)$ is a nominal LKF corresponding to the nominal system (25) with $h_2 \neq 0$ and $\eta_2(t) = 0$, and such that (see (Fridman, 2004))

$$\begin{aligned} V_n(t) &= \bar{e}(t)^T E P \bar{e}(t) + \int_{-h_2}^{0} \int_{t+\theta}^{t} z(\beta)^T R_2 z(\beta) d\beta d\theta \\ &\quad + \int_{t-h_2}^{t} e(s)^T S_2 e(s) ds \end{aligned} \quad (26)$$

and $V_a(t)$ is an additional term of the following form:

$$\begin{aligned} V_a(t) &= \int_{-\mu_2}^{0} \int_{t+\theta-h_2}^{t} z(s)^T R_{a_2} z(s) ds d\theta \\ &\quad + \int_{t-\tau_2(t)}^{t} e(s)^T S_{a_2} e(s) ds \end{aligned} \quad (27)$$

Then, by differentiating of $V(t)$ along the trajectories of system (25), and posing $P = P_2$, $P_3 = \varepsilon P_2$, where $\varepsilon \in \mathbb{R}$ is a tuning scalar parameter, then the proof is achieved by noting that $X = P^T L$. ■

5 ILLUSTRATIVE EXAMPLE

This section aims at illustrating the theoretical results of section 4, through an example related to the remote control of a "ball and beam" system. Regarding to Figure 2, this plant mainly consists in a steel ball rolling on two parallel tensioned wires. These are mounted on a beam, pivoted at its center, such that the beam angle may be controlled by a servo-motor and sensed by transducers to provide measurements of the beam angle and ball position.

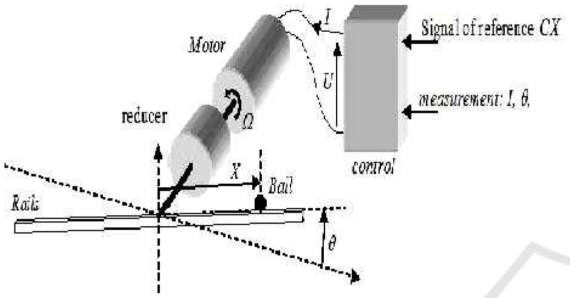

Figure 2: The Ball and Beam system to be controlled.

Regarding to the control scheme of Figure 3, the fast dynamics of the plant are regulated by two inner loops (with PI and PD controllers located in the Slave systems), so that the remaining control problem is to regulate the ball position by varying the beam angle.

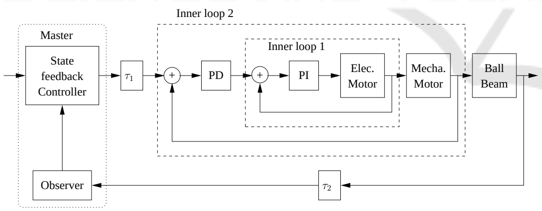

Figure 3: Control scheme of the Slave system.

According to this, the system dynamics to be controlled by means of the remote observer-based state-feedback controller, can then be defined by:

$$\begin{aligned} \dot{x}(t) &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ k_b \end{bmatrix} u(t-\tau_1(t)) \\ y(t) &= \begin{bmatrix} k_x & 0 \end{bmatrix} x(t) \end{aligned} \qquad (28)$$

where $x(t) = [x_x(t) \;\; x_v(t)]^T \in \mathbb{R}^2$ is the state-vector, $x_x(t)$ and $x_v(t)$ correspond respectively to the position and the speed of the ball. $u(t-\tau_1(t))$ is the control input (with input delay $\tau_1(t)$)), $y(t)$ is the measured output (corresponding to the ball position) which is forwarded to the Master system. k_b and k_x are two constant parameters (with $k_b = 6.1\ ms^{-2}rad^{-1}$ and $k_x = 7\,V/m$).

Now, let us consider non-symmetric delays $\tau_1(t) \neq \tau_2(t)$, with, according to (2): $h_1 = 0.3s$, $h_2 = 0.25s$, $\mu_1 = \mu_2 = 0.1s$ (recalling that h_1 and h_2 are constant values, while $\eta_1(t)$ and $\eta_2(t)$ are time-varying perturbations bounded by μ_1 the and μ_2 respectively). Moreover, let us consider $d_1 = d_2 = 0.1$. By applying Theorem 2 to (6) for $\varepsilon = 9$, we find the LMI (18) is feasible for symmetric, positive-definite matrices:

$$\bar{P}_1 = \begin{bmatrix} 1.98 & 0 \\ 0 & 1.98 \end{bmatrix} \bar{R}_1 = \begin{bmatrix} 2.29 & -0.64 \\ -0.64 & 1.38 \end{bmatrix}$$

$$\bar{R}_{a_1} = \begin{bmatrix} 2.13 & -0.45 \\ -0.45 & 1.48 \end{bmatrix} \bar{S}_1 = \begin{bmatrix} 0.12 & -0.08 \\ -0.08 & 0.11 \end{bmatrix}$$

$$\bar{S}_{a_1} = \begin{bmatrix} 0.1934 & -0.1241 \\ -0.1241 & 0.5671 \end{bmatrix}$$

and,

$$\bar{P} = \begin{bmatrix} 0.5138 & -0.2327 \\ -0.2327 & 0.3694 \end{bmatrix} \quad W^T = \begin{bmatrix} -0.0088 \\ -0.0699 \end{bmatrix}$$

With respect to (20), the state-feedback controller gain K is then given by,

$$K = \begin{bmatrix} -0.1440 & -0.2800 \end{bmatrix} \qquad (29)$$

Now, by applying Theorem 3 to (21) for $\varepsilon = 5.5$ (tuned by trial and error), we find the LMI (22) is feasible for symmetric, positive-definite matrices:

$$P_1 = \begin{bmatrix} 8.27 & 0 \\ 0 & 8.27 \end{bmatrix} R_2 = \begin{bmatrix} 1.05 & -1.46 \\ -1.46 & 10.29 \end{bmatrix}$$

$$R_{a_2} = \begin{bmatrix} 4.22 & -2.16 \\ -2.16 & 15.84 \end{bmatrix} S_2 = \begin{bmatrix} 0.62 & -0.23 \\ -0.23 & 0.70 \end{bmatrix}$$

$$S_{a_2} = \begin{bmatrix} 0.157 & -0.112 \\ -0.112 & 0.373 \end{bmatrix}$$

with,

$$P = \begin{bmatrix} 0.963 & -2.240 \\ -2.240 & 9.964 \end{bmatrix} \quad X = \begin{bmatrix} -0.069 \\ -0.097 \end{bmatrix}$$

Then, from (24), we finally obtain the observer gain:

$$L = [-0.198 \quad -0.054]^T \qquad (30)$$

By considering the control scheme of figure 3 (meaning that the two inner loops are taking in account in simulating the dynamical behavior of the closed-loop Master-Slave system), and numerical results (29)–(30) for the controller and observer gains respectively, we then obtain the simulation results of figures 4 and 5 (for delays $h_1 = 0.3s$, $h_2 = 0.25s$). Figure (4) represents the ball position on the beam axis when dealing with a step response of the closed-loop system

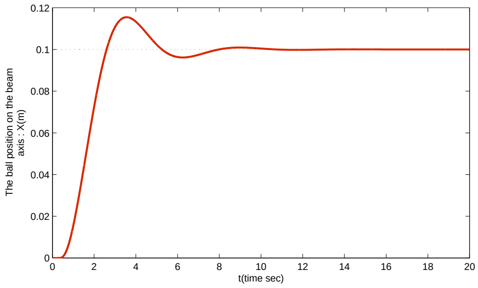

Figure 4: Step response of the closed-loop system.

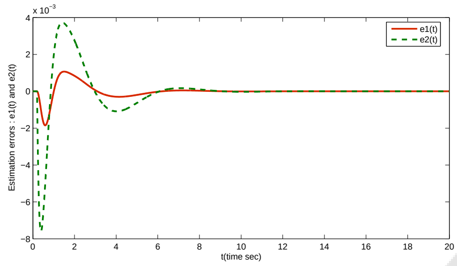

Figure 5: Estimation error $\hat{x}(t) - x(t)$.

with a step magnitude 0.1 m, while Figure (5) represents the observations errors $e_1(t) = \hat{x}_x(t) - x_x(t)$ and $e_2(t) = \hat{x}_v(t) - x_v(t)$. Moreover, Figure (6) shows the corresponding delayed control input.

By looking at these simulations results, we can see that the Luenberger-type observer insure the asymptotic convergence of the estimation error towards zero, while the state-feedback control guarantees the asymptotic stability of the closed-loop system, whatever the presence non-symmetric delays $\tau_1 \neq \tau_2$ in the control and feedback loops.

6 CONCLUSIONS

This paper has dealt with the stabilization problem of a Networked Control System with a TCP network as communication media. In particular, our attention was focusing on a Master-Slave setup with uncertain, time-varying, ”non-small”, non-symmetric transmission delays affecting the Slave control input and its transmitted (scalar) output. A main feature of our work was the use of a Lyapunov-Krasovskii functional derived from a descriptor model transformation, to give rise to some conditions for the design of an observer-based state-feedback control. In future works, we will study the stability of Networked Control Systems with both delays and packet dropping.

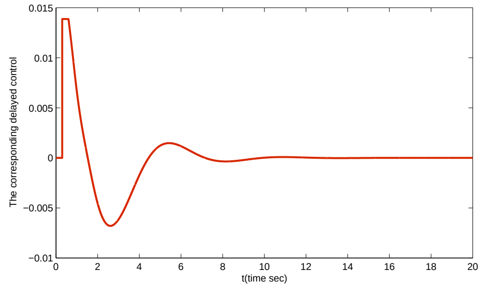

Figure 6: The corresponding delayed control input.

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