

DISTRIBUTED ARRIVAL TIME CONTROL FOR VEHICLE ROUTING PROBLEMS WITH TIME WINDOWS

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Keywords: Distributed Arrival Time Control (DATC), Distributed Control, Scheduling, Dispatching.

Abstract: Competitiveness of a supply chains depends significantly on distribution center operations because it determines responsiveness and timeliness of deliveries to customers. This paper proposes a control algorithm for routing multiple out-bound trucks to customers spread over a wide geographical area, each occupying different volume in a truck, and having a different delivery time-window. Overall operations are also constrained by geographical locations of the customers in various zones and dissimilar truck capacities. Performance of the algorithm is tested using data from a distribution center located in Latin America.

1 INTRODUCTION

Transportation is the most expensive logistics activity. The overall goal in transportation should be to connect sourcing locations with customers at the lowest possible transportation cost within the constraints of the customer service policy (Edward H. Frazelle, 2002). Most important factors which can affect transportation cost are the number of trucks to deliver customer products, shipments allocation on trucks which decide a route of a truck and shipments loading on trucks as shown Figure 1. The truck (vehicle) routing problem has been recognized for over 40 years and is one of the most important factors in distribution and logistics. In particular, the importance of on-time delivery for customers is growing up according to various and complicated customer needs. In these conditions, finding solution optimally is very hard because of dynamics of a transportation environment. Many heuristics approach which can be broadly classified into two main classes, classical heuristics and meta-heuristics, have been proposed for vehicle routing problem in a last half century (Gilbert Laporte, Michel Gendreau, Jean-Yves Potvin, Frederic Semet, 2000).

Several meta-heuristic methods have been proposed to solve the vehicle routing problem. The important issues of meta-heuristics for the vehicle routing problems is how they can diversify search space and intensify routing solution to reduce

transportation cost. For example, tabu search and simulated annealing algorithm tried to jump out of the local minimum by search its neighborhood space. To improve limitation of their neighborhood search space, some advanced methods were developed and plugged in search logic to diversify neighborhood search space (Haibing Li, Andrew Lim, 2003, J-F Cordeau, G Laporte and A Mercier, 2001).

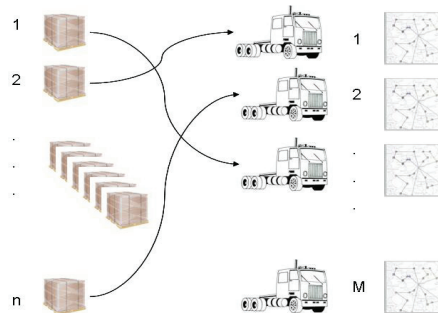


Figure 1: Transportation problem.

Furthermore, research for system dynamism has been conducted recently. Some measurements were proposed to explain how dynamic vehicle routing system (A Larsen, O Madsen and M Solomon, 2002, Larsen A, 2000). These measurements can play a crucial role in determining proper models or algorithms to solve vehicle routing problem according to the dynamic characteristics of the system.

In this paper, truck assignment and routing algorithm (TARA) is proposed to meet not only transportation cost needs but also customer service needs. Core of TARA algorithm is constructed based on distributed arrival time control (DATC) which is a feedback control-based scheduling approach that attempts to minimize the average of the square of the due-date deviation for Just-in-Time system (Hong, J., Prabhu, V. V., 2003).

2 PROBLEM DEFINITION

The problem, in terms of the distribution center and customer delivery, could be described as follow. The distribution center run 24 hours to process customers' transportation orders, but trucks operate loading and control jobs from 4 AM to 10 PM at the distribution center. During this time, there is no out-of-order of trucks which is used for delivery. All trucks are assumed to return to the distribution center and to be ready for loading at 4 AM.

The problem has three major constraints related with truck loading, delivery time and order processing. For truck loading constraints, each truck has own pallet capacity limit and maximum number of the customer order in a truck is four. Also, there are exclusive shipping requests which cannot share trucks with other shipments. For delivery time constraints, there are three different types of time window in this problem. In a sense, a time window implies the open time of a distribution center. A customer can request three type of time window. At first, for specific time of a specific date, delivery should be as punctual as possible. This is similar to Just-in-Time strategy with minimizing earliness and tardiness. Secondly, delivery could be done within a time window (w^{\min} , w^{\max}). Lastly, there could be no time requirement from customers. It means that a time window is within (0, 24) hour. Travel time among each customer's location including distribution center is shown in Table 1. For order processing constraints, customer orders are received every day, except on Sunday, until 6 PM. The scheduler generates the shipments of the next day. In other words, every customer order must be shipped the day after its arrival. Hence, if the distribution center cannot ship an order due to no available trucks, the customer order will be shipped the next earliest truck-available date, which will turn out to be a large deviation from the requested time window.

The truck assignment and routing algorithm is evaluated by two objective functions which are

related with the trucking cost and customer service. The first objective function for trucking cost can be determined as follow:

$$\text{Minimize } \sum_{j=1}^m n_j \cdot C_j + \sum_{j=1}^m n_j \cdot F_j \quad (1)$$

where $\forall j \in S$, S is the index set of trucks which are used for delivery, n_j is the number of trucks type j , F_j is the fixed cost for operating one truck and C_j is the trucking cost of truck type j . The second objective function related with customer service can be represented by time window violation cost and formulated as follow:

$$\text{Minimize } \sum_{i=1}^n ETC_i \quad (2)$$

where $ETC_i = \alpha \cdot \max\{0, w_i^{\min} - c_i\} + \beta \cdot \max\{0, c_i - w_i^{\max}\}$ and c_i is completion time of customer order i . In this equation, α is the penalty cost for earliness and β represents the penalty cost for tardiness of time window for customer i .

Table 1: Customer location and travel time.

Location Code	L10001	L10002	L10003	•
L10001	0.00	0.42	0.44	•
L10002	0.42	0.00	0.50	•
L10003	0.44	0.50	0.00	•
L10004	2.05	2.16	1.83	•
L10005	0.75	0.80	0.49	•
L10006	0.84	0.92	0.59	•
•	•	•	•	•

3 DISTRIBUTED TIME CONTROL FOR TRUCK ASSIGNMENT

DATC is a closed-loop distributed control algorithm for manufacturing shop floor in which each part controller uses only its local information to minimize deviation from its part's due-date (Hong, J., Prabhu, V. V., 2003). In DATC, the integral control law is represented as follow:

$$a_i(t) = k_i \int_0^t (d_i - c_i(\tau)) d\tau + a_i(0) \quad (3)$$

where k_i is the controller gain, $a_i(0)$ is the arbitrary initial arrival time, d_i is the due-date and $c_i(\tau)$ is the predicted completion time for the i th job in the

system. In vehicle routing problem with time window, the controller gain value is defined by function of relationship between completion time and time window and it can be decided according to two cases, earliness and tardiness. When earliness occurs, the next arrival time of a job moves toward minimum value of time window by above integral control law. Similarly, in case of tardiness, the next arrival time moves toward maximum value of time window as shown in Figure 2.



Figure 2: Controller gain adaptation.

The momentum of each arrival time is controlled by the controller gain, k_i which is calculated differently according to the earliness and tardiness as described in equation (4) and (5).

$$k_i = KE_i = \frac{w_i^{\max} - c_i}{w_i^{\min} - c_i} \cdot k, \quad c_i \leq w_i^{\min} \quad (4)$$

$$k_i = KT_i = \frac{c_i - w_i^{\min}}{c_i - w_i^{\max}} \cdot k, \quad c_i \geq w_i^{\max} \quad (5)$$

As a result, in case of earliness, equation (3) can be converted by equation (4) as follow:

$$a_i(t) = a_i(t-1) + KE_i \cdot \Delta \cdot \{w_i^{\min} - c_i(t-1)\} \quad (6)$$

Similarly, equation (3) is changed by equation (5) for the tardiness case as follow:

$$a_i(t) = a_i(t-1) + KT_i \cdot \Delta \cdot \{w_i^{\max} - c_i(t-1)\} \quad (7)$$

where $a_i(t)$ is the arrival time at t th time step, Δ is the time step and $c_i(t-1)$ is the completion time at $(t-1)$ th time step.

Overall TARA procedure is described as follow:

STEP 1 : Initialize customer and truck parameters

a_i = minimum value of time window - t_{li}

$c_i, u_j = 0, p_j = P_j$

$l_j = 1$ (Location 1 implies DC)

STEP 2 : Sort customer based on FCFS rule

STEP 3 : Truck assignment for each customer

For $i = 1$ to n Do

For $j = 1$ to m Do

$j = \text{Arg}(\min(E_j))$, subject to $v_i \leq p_j$

$e_j = E_j, p_j = p_j - v_i, l_j = l_i$

$u_j = e_j + v_j/R$, where R is the unloading rate in pallets/hour

$c_i = u_j$

$a_i = a_i + k_i \cdot \Delta \cdot (d_i - c_i)$

$E_j = u_j + t_{ji}$

STEP 4 : Compute summation of total earliness and tardiness for each i

STEP 5 : Initialize customer and truck parameters

STEP 6 : Go to STEP 2:

In these TARA steps, a_i is arrival time of customer i , c_i is delivery completion time of customer i , p_j is the number of pallets it can be loaded based on the current load of truck j , p_j is the maximum number of pallets it can be loaded by truck j , l_j is the current location of the truck j , l_i is the location of customer i , u_j is the last unloading time of truck j , v_i is the number of pallet of customer i and E_j is time consumption of truck j from current location to customer i . Also, by equation (6) and (7), k_i is KE_i in case of earliness or KT_i when the completion time is greater than maximum value of time window.

4 TRUCK ASSIGNMENT AND ROUTING ALGORITHM PERFORMANCE

4.1 Performance Comparison

To measure the TARA performance, we used the following scalar equation (8) for mean squared due date deviation (MSD) which is used to characterize the global dynamics (Prabhu, V. V., 2003).

$$MSD = \sqrt{\frac{\sum_{i=1}^n (c_i - d_i)^2}{n}} \quad (8)$$

Four customer demand sets which have 24, 31, 43 and 54 orders in them were used for test. These customer order data were real-world data used by one of global health and hygiene companies. The number of trucks was fixed as 12 and they have equal capacity to load 60 pallets. According to experiments, in case of set 1, 2 and 4, MSD became zero within 20th iteration. For set 3, time violation became the minimum value at 9th iteration. The MSD results for four kinds of data sets are described in Figure 3.

Furthermore, by using these experimental data, TARA performance was compared to dispatch rules, such as earliest due date (EDD), shortest processing

time (SPT) and latest processing time (LPT) as shown in Table 2. For the EDD rule, customer orders are arranged in ascending order by amount of time difference between each order’s maximum time window and distance from the distribution center. Then each order is loaded in trucks one after another and finally, total MSD is calculated. For the SPT rule, orders are arranged in ascending order by distance between the distribution center and each order. Then, similar with the EDD rule, each order is loaded in trucks and total MSD is calculated. In case of the LPT rule, customer orders are arranged in descending order by same measurement with the SPT rule. The number of trucks was fixed as 12.

As a result, for average MSD of four data sets, TARA obtained 196% better result than the EDD rule. For the SPT and LPT rules, TARA showed approximately 199.6% improved results.

Experimental Results of minimum travel distance for each experiment set are shown in Table 3. Travel distance is estimated by assuming that the average trucking distance is 60 miles per hour. For set 1 and set 2 which have relatively small amount of customer orders, TARA with static controller gain have same or better performance than dynamic controller gain. In case of large amount of customer orders, TARA with dynamic controller gain gives relatively better performance. However, for almost all cases, dispatch rules have relatively better performance than TARA. This is because, basically, TARA controller proposed in this paper is designed to minimize tardiness and earliness of customer orders and it does not contain any device to consider travel distance.

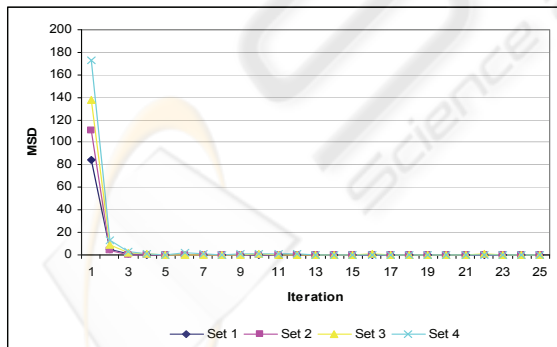


Figure 3: Mean squared due date deviation.

Table 2: Performance comparison – MSD.

Num of Order	TARA		EDD	SPT	LPT
	Static	Dynamic			
24	0.394	0.000	0.676	5.720	5.367
31	0.000	0.000	0.601	5.636	4.912
43	0.015	0.086	0.544	5.377	4.552
54	0.225	0.000	0.637	5.368	4.153

Table 3: Performance comparison – Travel Distance.

Num of Order	TARA		EDD	SPT	LPT
	Static	Dynamic			
24	1504	1504	1606	1409	1560
31	1635	1749	1861	1591	1905
43	2454	2427	2387	2105	2699
54	2959	2953	3320	2553	2926

4.2 Trucking Cost

Average total tardiness and average trucking cost were measured by varying the number of truck for TARA. To calculate trucking cost, fuel efficiency was estimated by 15 miles per gallon and fuel price was assumed by \$3.5 per gallon. Also, fixed cost for a truck which has 1-ton capacity was assumed \$0.25 per mile based on the Excel software program developed by the Texas A&M university (Ron Torrell, Willie Riggs and Duane Griffith). The number of customer orders and the controller gain value was set to 43 and 0.6. Experimental results for these two measurements are described in Figure 4. As the number of truck increases, average total tardiness increases. On the contrary, average trucking cost decreases as the number of truck increases.

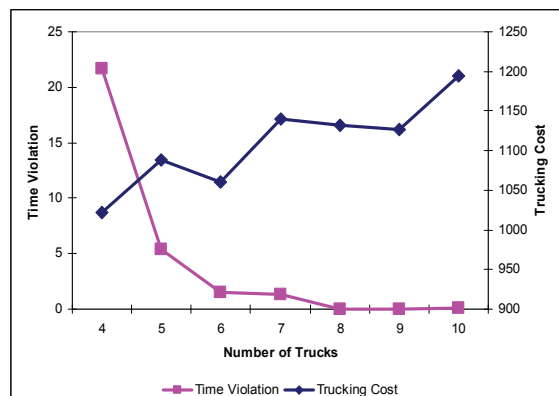


Figure 4: Trucking cost & time violation.

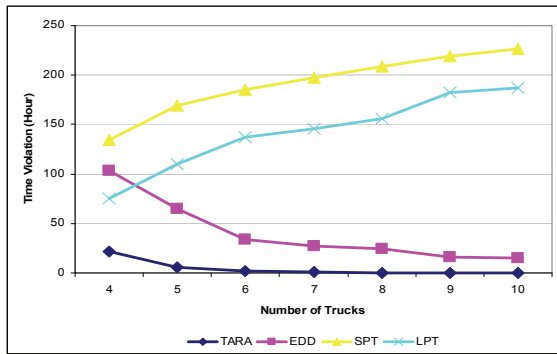


Figure 5: Time violation comparison.

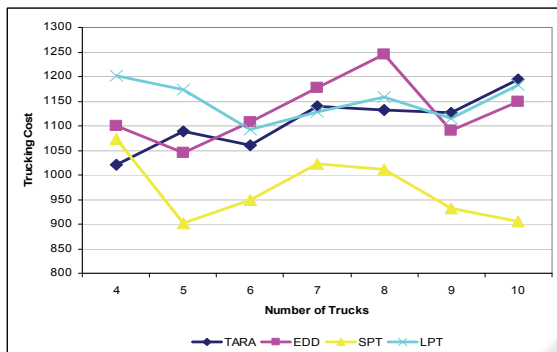


Figure 6: Trucking cost comparison.

Also, time violation and the trucking cost comparison among TARA and dispatch rules are shown in Figure 5 and Figure 6. In case of time violation, TARA and EDD had smaller time violation value as increasing the number of trucks. SPT and LPT, however, had increasing time violation value as increasing the number of trucks. In case of the trucking cost, it is hard to capture the relationship between the number of trucks and the trucking cost as shown Figure 6.

4.2 Dynamic Controller Gain Effects

As we explained in equation (4) and (5), TARA used dynamic controller gain which is changed by status of completion time. Actually, the basic distributed arrival time control updates the arrival time continuously with the static controller gain through fixed iteration. However, by changing the controller gain dynamically according to the status of completion time, earliness and tardiness, the convergence velocity and quality of MSD were improved as shown in Figure 7-10. In case of the 24 orders set (set 1), MSD from the dynamic controller gain reached zero at 14th iteration, but MSD from the static controller gain was greater than zero and it

was not converged in zero. Similar results could be observed in other data sets. For the 31 (set 2), 54 (set 4) order sets, MSD from the dynamic controller gain were converged in zero at 8th and 21th iteration. In case of the 31 order set, however, MSD from the static controller gain was converged at 20th iteration and the other one was greater than zero and not converged. Although MSD by the dynamic controller gain of the 43 order set (set 3) was not converged, it was less than the result of the static controller gain and reached at minimum value faster.

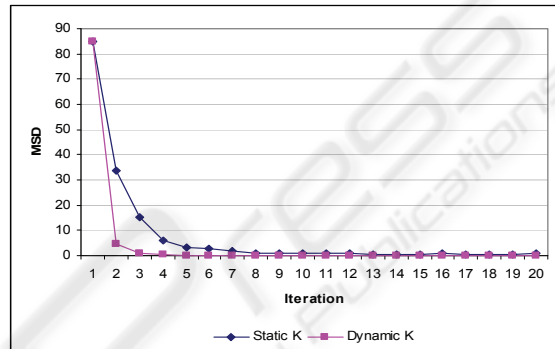


Figure 7: Dynamic controller gain effect – set 1.

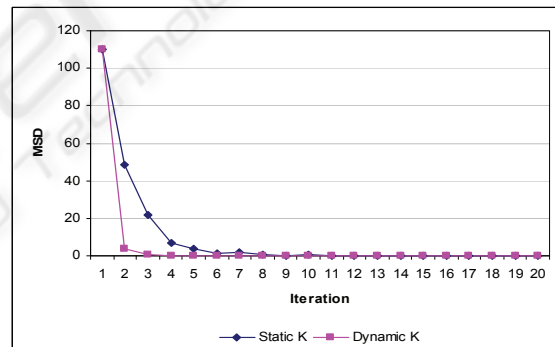


Figure 8: Dynamic controller gain effect – set 2.

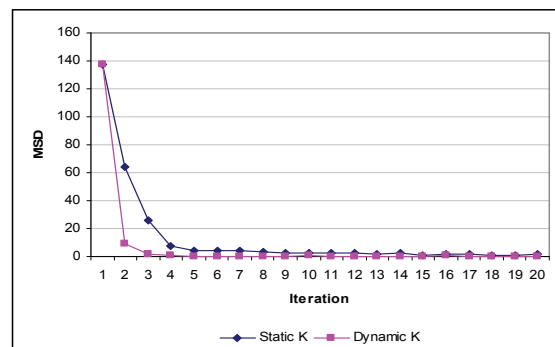


Figure 9: Dynamic controller gain effect – set 3.

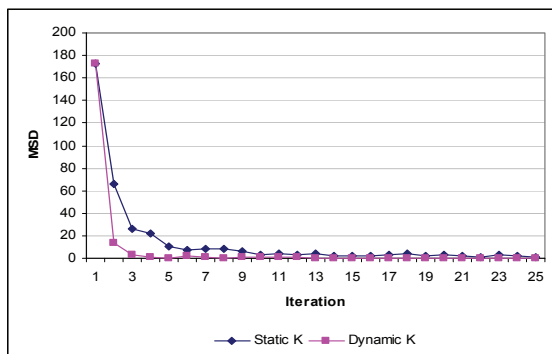


Figure 10: Dynamic controller gain effect – set 4.

5 CONCLUSIONS

Truck assignment and routing algorithm is an effective algorithm based on distributed arrival time control to solve the vehicle routing problem which has various delivery time windows of customers. In this work, TARA using the dynamic controller gain has been developed to determine the best vehicle routing plan for maximizing customer service level. Basically, the controller gain used in basic DATC is maintained static values through the whole algorithm processes. The dynamic controller gain, however, is updated continuously through whole iteration according to the result of the completion time, earliness and tardiness. Thus, we can improve not only the convergence velocity of the solution but also the quality of the solution compared with dispatch rules simultaneously.

ACKNOWLEDGEMENTS

The research was partially supported by the Marcus PSU/Technion Exchange Partnership.

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