

# A GENERAL MODEL FOR JOB SHOP PROBLEMS USING IMMUNE-GENETIC ALGORITHM AND MULTIOBJECTIVE OPTIMIZATION TECHNIQUES

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**Abstract:** We define a global model to simulate the characteristics of three kinds of the manufacturing systems with transport resources. Based on this model, we use an immune-based genetic algorithm to solve the associated scheduling problems. We take the makespan and minimum storage as the two objectives and use modified Pareto ranking method to solve this problem. We show how to choose the best solutions for the studied systems. Though not all the constraints of the real systems are considered until now, the computational results show that our proposed model and algorithm have efficiencies in solving scheduling problems.

## 1 INTRODUCTION

During the past decade, problems in production planning have been arisen dramatically in automated manufacturing systems. A well planed synchronization between the machines and the transportation resources are crucial to improve their efficiency. Most of the classical works do not consider transport operations constraints. However, material handling systems may become critical resources. Moreover numerous practical constraints have to be taken into account, and several objectives have to be considered. Multiobjective optimization no doubt plays a very important role to get a more realistic solution for the decision maker.

In this paper, we consider the manufacturing systems with transportation resources which can be classified into mainly three main classes: flexible manufacturing systems (FMS), robotic cells (RC), and treatment surface facilities (TSF). A classification can be found in the literature for each of these systems (Hall et al., 1998, Tacquard & Martineau, 2001, Manier & Bloch, 2003, Brauner et al., 2005). In each system, the associated scheduling problems can be considered as specific ones. Nevertheless, there also exist similarities among them. In fact, there is few works related to the general problems which link the scheduling of

product operations and transportation together. Knust (Knust, 1999) integrated the transportation issues into classical scheduling models. In the same way, we try to define a global model suitable for any of those systems with transportation constraints (section 2). An improved immune-based genetic algorithm and a modified Pareto-compliant ranking method are applied as the main solving methods to solve the scheduling problem with two objectives (minimization of the makespan and the storage) (section 3). The computational results for the proposed algorithm show that our model and the adopted algorithm are efficient enough to schedule the activities of production (section 4).

## 2 GENERAL MODEL

### 2.1 Notation

Our notations consider the following four aspects:

#### 1) Job/task

$n$  : total number of jobs.

$O_i$ : number of the operations of job  $i$  ( $i \in [1, n]$ ).

$P(i,j)$ : operation  $j$  of job  $i$  ( $j \in [1, O_i]$ ).

$p_{ijk}$ : processing time for  $P(i,j)$ , on machine  $MP_k$ .

$p_{ijk}^-$ : minimal processing time of  $P(i,j)$  on machine

$MP_k$ .

$p_{ijk}^+$ : maximal processing time of  $P(i,j)$  on machine

$MP_k$ .

$d_i$ : due date of job  $i$ ,  $i \in [1, n]$ .

$t_{ij}$ : starting date of  $P(i,j)$ . ( $i \in [1, n], j \in [1, O_i]$ )

$C_i$ : completion time of job  $i$ .

## 2) Processing Resources

$MP$ : total set of the machines (processing resources)

$MP_k$ : machine  $k$  ( $k \in [1, |MP|]$ ) (unitary capacity).

$PR_{ij}$ : is the total set of the processing resources that can perform operation  $j$  of job  $i$ . ( $i \in [1, n], j \in [1, O_i]$ ).

$PJ_{ijk}$ :  $PJ_{ijk} = 1$ , if the operation  $j$  of job  $i$  is performed by machine  $MP_k$ ;  $PJ_{ijk} = 0$ , otherwise

$YP_{iji'j'k}$ :  $= 1$ , if  $P(i, j)$  is performed right before  $P(i', j')$  on the machine  $MP_k$ ;  $= 0$ , otherwise

$S_{iji'j'k}$ : setup time on  $MP_k$  between  $P(i, j)$  and  $P(i' j')$ .

## 3) Transportation Resources

$MT$ : total set of the transportations.

$MT_h$ : transportation resource  $h$  (Unitary capacity).

$T(i, j)$ : transportation task between  $P(i, j)$  and  $P(i, j+1)$ .

$TR_{ij}$ : total set of the transportation resources that can transport  $T(i, j)$ .

$l_{kh}^-, l_{kh}^+$ : needed time for a transportation resource  $MT_h$  to unload (respectively to load) machine  $MP_k$ .

$\sigma_{kk'h}$ : empty travel time between machine  $MP_k$  and  $MP_{k'}$  by transportation resource  $MT_h$ ,  $k, k' \in [1, |MP|]$ ,  $h \in [1, |MT|]$ .

$\tau_{kk'h}$ : loaded travel time between machines  $MP_k$  and  $MP_{k'}$  by transportation resource  $MT_h$ . (it includes  $l_{kh}^-$  and  $l_{kh}^+$ )

$TJ_{ijh}$ :  $= 1$ , if  $T(i, j)$  is performed by  $MT_h$ ;  $= 0$ , otherwise.

$YT_{iji'j'h}$ :  $= 1$ , if  $MT_h$  performs  $T(i, j)$  right before  $T(i', j')$ ;  $YT_{iji'j'h} = 0$ , otherwise.

## 4) Storage Configuration:

$\gamma_{ijk}^{s-}$ : time of the input buffer for  $P(i, j)$  treated on  $MP_k$ .

$\gamma_{ijk}^{s+}$ : time of the output buffer for  $P(i, j)$  on  $MP_k$ .

## 2.2 Mathematical Model

The objectives of the general model are to minimize the makespan and the minimal storage:

$$\text{Min } C \text{ max} = \text{Max}_{i=1 \text{ to } n} (C_i), C_i = t_{iO_i} + \sum_{k \in PR_{i_j}} p_{ijk} \times PJ_{ijk}$$

$$\text{Min } \sum_{k \in MP} \sum_{i \in n} \sum_{j \in O_i} (PJ_{ijk} \times \gamma_{ijk}^{s-} + PJ_{ijk} \times \gamma_{ijk}^{s+})$$

And the following constraints of the problem are

respected.  $\forall i \in [1, n], \forall j \in [1, O_i - 1]$ ,

$$t_{ij} + \sum_{k \in PR_{ij}} p_{ijk} \times PJ_{ijk} \leq t_{ij} \quad (1)$$

$\forall i \in [1, n], \forall j \in [1, O_i], \forall k \in PR_{ij}$ ,

$$p_{ijk}^- \leq p_{ijk} \leq p_{ijk}^+ \quad (2)$$

$$\forall i \in [1, n], \forall j \in [1, O_i], \sum_{k \in PR_{ij}} PJ_{ijk} = 1 \quad (3)$$

$$\forall i \in [1, n], \forall j \in [1, O_i - 1], \sum_{h \in TR_{ij}} TJ_{ijh} = 1 \quad (4)$$

$\forall (i, i') \in [1, n]^2, \forall j \in [1, O_i], \forall j' \in [1, O_{i'}]$ ,

$\forall k \in PR_{ij} \cap PR_{i'j'}$ , and  $M$  is a very large fixed number.

$$t_{ij} + p_{ijk}^- + l_{kh_2}^- + l_{kh_3}^+ + s_{iji'j'k} \leq t_{i'j'} + (1 - YP_{iji'j'k}) \times M \quad (5)$$

$$PJ_{ijk} \times PJ_{i'j'k} (t_{i'j'} + p_{i'j'k} + l_{kh_3}^- + l_{kh_4}^+ + s_{i'j'ijk}) \leq t_{ij} + YP_{iji'j'k} \times M \quad (6)$$

$$\forall (i, i') \in [1, n]^2, \forall j \in [1, O_i], \forall j' \in [1, O_{i'}], \forall h \in TR_{ij} \cap TR_{i'j'}, PJ_{ijk} = 1, PJ_{i(j+1)k} = 1, PJ_{i'j'k''} = 1, PJ_{i'(j'+1)k''} = 1,$$

$$t_{ij} + p_{ijk} + \gamma_{ijk}^{s+} + \tau_{kk'h} + \sigma_{k'k'h} \leq t_{i'j'} + p_{i'j'k''} + \gamma_{i'j'k''}^{s+} + (1 - YT_{iji'j'h}) \times M \quad (7)$$

$$(t_{i'j'} + p_{i'j'k''} + \gamma_{i'j'k''}^{s+} + \tau_{k''k'h} + \sigma_{k''k'h}) \times TJ_{ijh} \times TJ_{i'j'h} \leq t_{ij} + p_{ijk} + \gamma_{ijk}^{s+} + YT_{iji'j'h} \times M \quad (8)$$

$$\forall i \in [1, n], \forall j \in [1, O_i - 1], TJ_{ijh} = 1, PJ_{ijk} = 1,$$

$$PJ_{i(j+1)k} = 1,$$

$$t_{ij} + \sum_{k \in PR_{ij}} p_{ijk} \times PJ_{ijk} + \gamma_{ijk}^{s+} + \sum_{k \in PR_{ij}, k' \in PR_{ij+1}} \tau_{kk'h} \times PJ_{ijk} \times PJ_{ij+1k'} + \gamma_{ij+1k'}^{s-} + \gamma_{ij+1k'}^{s-} \leq t_{i, j+1} \quad (9)$$

Constraint (1) is the precedence constraints between two operations of job  $i$ ; constraint (2) is the processing time constraints for  $(i, j)$  on machine  $MP_k$ ; Constraint (3) makes sure that one operation can only be assigned to one machine; constraint (4) makes sure that one operation can only be assigned to one transportation resource; constraints (5) and (6) are the capacity constraints for each processing resource  $MP_k$ ; constraints (7) and (8) are the capacity constraints for each transportation resource  $MT_h$ ; constraint (9) is the travelling constraint, which expresses that a transportation resource  $MT_h$  must have enough time to move a job  $i$  between two successive operations.

### 3 RESOLUTION

We use an improved immune-based genetic algorithm as the training method to find nondominated solutions of the  $n$ -objective optimization problem.

In our case, we code the antibody into two parts. The first part is a permutation of the  $s$  transportation tasks ( $s = \sum_{i \in n} (O_i - 1)$ ). The second part is a permutation of the  $m$  operation tasks ( $m = \sum_{i \in n} O_i = s + n$ ).

#### 3.1 Selection Operation

In the algorithm (Zhang et al., 2006), the affinity between antigen and antibody  $v$ , is defined by  $ax_v = opt_v$ , where  $opt_v$  is the fitness of antibody  $v$ .

The expected selection probability  $e_v$  of antibody  $v$  is calculated as:  $e_v = ax_v/c_v$ , where  $c_v$  is the density of antibody  $v$ . It can be seen from the above equation that the antibody with both high fitness and low density would have more chances to survive.

We define that antibody  $v$  and antibody  $w$  have the affinity when the following inequality is satisfied  $f(v, w) < L$ , where  $f(v, w) = d(v, w) + |ax_v - ax_w|$ , and  $|ax_v - ax_w|$  is the Euclidean distance,  $L = L_0 \times \exp(b \cdot T)$ ,  $L_0 > 0$ ,  $b > 0$ , and  $T > 0$  is the number of evolution generations.  $L$  is an increasing function of evolution generations. The antibody's diversity and density would be increased efficiently with the increase of the evolution generations and that the suppression would be more powerful to preserve high diversity. So the algorithm would have strong ability to control the reproducing process.

#### 3.2 Learning Procedure

The whole learning process of the Pareto-immune-genetic algorithm can be described as follows:

**Step 1. Initialization of the Population.** All the gene bits of each antibody in the first generation are generated randomly within the feasible domain. In the initialization stage, we calculate the time windows  $[\alpha_{ij}^-, \alpha_{ij}^+]$  and  $[\beta_{ij}^-, \beta_{ij}^+]$ , which are the earliest and the latest starting dates of each operation  $P(i, j)$  and  $T(i, j)$  respectively. Then, according to all the constraints, we narrow down the time windows for all the operations and transportation tasks. Firstly, we update the earliest starting dates forwardly.

Secondly, we update latest starting dates backwardly.

**Step 2. Calculation of the Time Windows.** Then we allocate the tasks on each transport resource by randomly sequence. We do the same for each machine according to constraint (1).

After that, we verify all the time windows. If an individual is not eligible we generate a new one. We do this until we obtain an eligible individual. The initial individual is replaced with this new one.

**Step 3. Fitness Calculation.** We change the calculation of the new fitness as follows:  $f_k = \exp(-(k - 1))$ , which makes the value of the first objective varies according to the rank. And we take the Pareto ranking method (Goldberg, 1989) to calculate the rank. In this paper, the two objective fitness values are defined as the makespan and the minimum storage.

**Step 4. Evolution of the Population.** The algorithm starts with the initial population that is generated randomly. The reproduction, crossover and mutation operators are used to produce the filial generation superior to their parents. Because it has improved the affinity calculation and makes the threshold value a dynamic parameter, it has strong ability to overcome the shortage of the tendency towards local optimum value and premature. We take the single point for crossover and the single bit for mutation. The reproduction operator is based on not only the fitness but also the density which plays an important role in diversity maintenance in immune system. The aforementioned steps are performed repeatedly until all the training data are trained completely.

### 4 RESULTS

Here, we take a simple example of five jobs ( $n=5$ ), with:  $O_1 = 4$ ,  $O_2 = 3$ ,  $O_3 = 2$ ,  $O_4 = 4$ ,  $O_5 = 4$ ,  $\forall i = [1, 5]$ ,  $d_i = 15$ ,  $r_i = 0$ ,  $MT = \{MT_1, MT_2, MT_3\}$ ,  $\forall i = [1, n]$ ,  $\forall j \in [1, O_i]$ ,  $\forall k \in PR_j$ ,  $\forall h \in [1, |MT|]$ ,  $\forall k' \in [1, |MP|]$ ,  $p_{ijk}^- = 1$ ,  $p_{ijk}^+ = 3$ ,  $\sigma_{kk'h} = \tau_{kk'h} = 1$ , and  $l_{kh}^-, l_{kh}^+ = 0$ . The colony size is taken from 10 to 100 respectively, the max evolution generation is 9000, crossover probability is 0.8 and mutation probability is 0.15. Other parameters are:  $b=0.01$ ,  $l_0=0.8$ . We run the program for 100 times, we got the pareto solutions sets, among which the best makespan is 9 and the minimum storage is 0.

Fig.2 shows results for a population size 100,

and a evolution generation 1000. The Pareto solutions are the rectangular solutions; the others are the dominated ones. For manufacturing systems that required no storage (like in the TSF), the solutions correspond to makespans 12, 13, 14 or 15. For other systems that allow storage, we obtain solutions with better makespan. For this example the best makespan is 9 with storage 1.

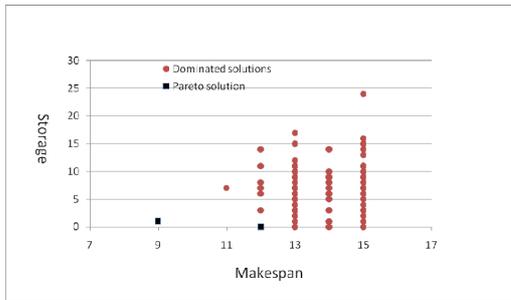


Figure 2: A resolution set with the population size 100, and with the evolution generation 1000.

Fig. 3 and 4 respectively present a solution with and without storage. The dotted (resp. blanked) squares are transportation tasks (resp. operations); their width represents the associated times. In Fig. 3, the dotted line for P(4,2) on MP<sub>3</sub> means that P(4,2) can start between time 3 and time 5. The blank spaces between two transportation tasks represent the empty movements or waitness of the resource. The minimum storage corresponds to the time between T(1,3) and P(1,4) with time windows [6,7]. In Fig. 4, as all the processing times are bounded, the minimal storage for this solution can reach 0.

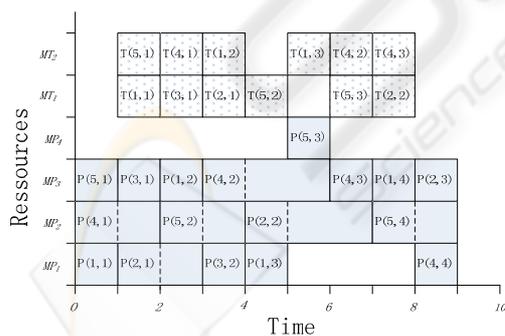


Figure 3: The time windows for a solution with makespan 9 and minimal storage 1.

## 5 CONCLUSIONS

We define a general model which enables us to solve several kinds of manufacturing schedule problems with transportation constraints. To reach this goal,

we use pareto-immune-genetic algorithm to schedule both processing and transport operations. In this paper, we report our first results for a simplified model of a production system with or without storages, and with bounded processing times. In the future, we will complete this model with the additional constraints (the configuration of the transport network and the conflicts between transport resources). We will also try to improve our solving algorithm and to compare it with efficient algorithms developed for each of the considered systems.

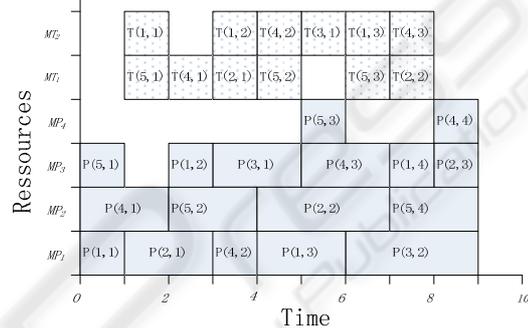


Figure 4: The time windows for a solution with makespan 9 and minimal storage 0.

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