ON MODIFICATION OF THE GENERALISED CONDITIONING TECHNIQUE ANTI-WINDUP COMPENSATOR

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Abstract: New anti-windup scheme is presented in application to pole-placement control, with a complete analysis of its behaviour for a class of second-order minimumphase stable plants of oscillatory and aperiodic characteristics with different dead-times. The classical generalised conditioning technique anti-windup compensator performance is compared with its three proposed modifications, arising in a new GCT compensation scheme. A critical discussion of the necessity of compensation is also given.

1 INTRODUCTION

Constraints are ubiquitous in real-world environment. As the result of their presence or the presence of some nonlinearities in the control loops, arises the difference in between computed and applied (i.e. constrained) control signal. In such a case, the performance of the closed-loop system degrades in comparison with the performance of the linear system, when constraints are not active. Such a degradation is defined as windup phenomenon (Rundqwist, 1991; Walgama and Sternby, 1990; Walgama and Sternby, 1993).

This can be also viewed from the point of discrepancy in between internal controller states and its output. When there is no correspondence in between controller's output and its internal controller states, the controller does not have any information what the current value of the constrained control signal is, and windup phenomenon arises.

The windup phenomenon has been, at first, defined for controllers comprising integral terms, as the so-called integrator windup (Rundqwist, 1991). For such controllers, control constraints may cause excessive integration of the error signal, giving rise to longer settling of the output signal and overshoots. There are two strands in compensating windup phenomenon (in AWC, anti-windup compensation) – taking constraints into account during the design procedure of the controller or assuming the system is linear, designing the controller for the linear case, and, subsequently, imposing constraints and applying AWCs (Horla, 2007; Horla and Krolikowski, 2003a; Horla and Krolikowski, 2003b).

The simplest anti-windup compensators have been based on the idea of integrator clamping, i.e. they referred to the controllers comprising integral terms only (Visioli, 2003). The proposed AWCs avoided integration of the error signal whenever some conditions were met, e.g., the control signal saturated, or error exceeded some predefined threshold, etc.

Such an approach was simple enough to be easily implemented, but as it has been already said, applicable to some controllers only.

The advanced anti-windup compensators have been designed for the case of general controller, which input-output equation is written in the RST form. Among the proposed AWCs one can find in the literature deadbeat, generalised, conditioning technique, modified conditioning technique and generalised conditioning technique anti-windup compensators (Horla and Krolikowski, 2003a; Horla and Krolikowski, 2003b). The three latter AWCs are based on the idea of back-calculation, i.e. modification of the signal that the output signal of the plant is to track, with respect to current saturation level.

The paper focuses on the generalised conditioning technique AWC (GCT-AWC), being a compromise solution in between the simplicity of the advanced AWC and compensation capabilities of the conditioning algorithm, what will be explained later.

The main idea of the paper is to present a modification of the GCT-AWC that can arise from the idea of integrator clamping methods, and to show that it can result in better control performance than performance

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of the system with original GCT-AWC. The presented results refer to the research carried for a set of stable minimumphase second-order discrete-time plants and different constraint levels.

There are no remarks in the literature how to improve the performance of the GCT-AWC, apart from (Horla and Krolikowski, 2003a). The proposed method limits the number of modifications, with the same excess. By introducing the proposed modifications one can improve the performance of the most appealing AWC technique.

2 PLANT MODEL

Let the discrete-time CARMA model be given

$$A(q^{-1})y_t = B(q^{-1})u_{t-d}\,, \tag{1}$$

where y_t is the plant output, u_t is the constrained control input, $d \geq 1$ is a dead-time and:

$$\begin{aligned} A(q^{-1}) &= 1 + a_1 q^{-1} + a_2 q^{-2}\,, & (2)\\ B(q^{-1}) &= b_0 + b_1 q^{-1} & (3) \end{aligned}$$

are relatively prime. The control signal $u_t = \mathrm{sat}(v_t;\alpha)$ is the computed control signal after saturation by symmetrical cut-off function at level $\pm\alpha$.

3 CONTROLLER

The plant is controlled by the pole-placement controller that ensures tracking of a given reference signal r_t by the plant output y_t with given dynamics,

$$\begin{aligned} v_t &= k_R r_t - k_P y_t + k_I \frac{q^{-1}}{1-q^{-1}}(r_t - y_t) - \\ &\quad - k_D \frac{1-q^{-1}}{1-\gamma q^{-1}} y_t\,, \end{aligned} \tag{4}$$

where $k_R = r k_P, r > 0$. The above controller equation can be obtained by discretisation of a continuous-time PID controller (Rundqwist, 1991), and it can be rewritten into the RST structure (Horla and Krolikowski, 2003b)

$$R(q^{-1})v_t = -S(q^{-1})y_t + T(q^{-1})r_t\,. \tag{5}$$

Coefficients of polynomials $R(q^{-1})$, $S(q^{-1})$, $T(q^{-1})$ can be determined by solving the following Diophantine equation

$$\begin{aligned} A(q^{-1})R(q^{-1}) + q^{-d}B(q^{-1})S(q^{-1}) &= \\ = A_M(q^{-1})A_o(q^{-1})\,, \end{aligned} \tag{6}$$

where polynomials $A_o(q^{-1})$ and $A_M(q^{-1})$ are stable, and given polynomial $A_M(q^{-1})$ is of second order in the chapter.

Controller polynomials $R(q^{-1})$, $S(q^{-1})$, $T(q^{-1})$ are of order $d + nB$, nA, nA_o, respectively, and have forms as follows:

$$\begin{aligned} R(q^{-1}) &= (1-q^{-1})R'(q^{-1})\,, \\ S(q^{-1}) &= s_0 + s_1 q^{-1} + s_2 q^{-2}\,, \\ T(q^{-1}) &= k_R A_o(q^{-1})\,. \end{aligned} \tag{7}$$

From the controller equation (5), given structure $R(q^{-1})$, $S(q^{-1})$, $T(q^{-1})$ (7) and (4) it follows that:

$$\begin{aligned} s_0 &= k_P + k_D\,, \\ s_1 &= k_I - 2k_D - k_P(1+\gamma)\,, \\ s_2 &= k_D - \gamma(k_I - k_P)\,, \end{aligned} \tag{8}$$

$$\begin{aligned} A_o(q^{-1}) &= \left(1-\gamma q^{-1}\right)\left(1-q^{-1}\left(1-\frac{k_I}{k_R}\right)\right) = \\ &= 1 + a_{o1}q^{-1} + a_{o2}q^{-2}\,, \end{aligned} \tag{9}$$

where $\gamma = -\frac{b_1}{b_0}, k_R = rk_P,\ a_{o1} = \frac{k_I}{k_R} - (1+\gamma), a_{o2} = \gamma\left(1-\frac{k_I}{k_R}\right)$. As the polynomial $A_o(q^{-1})$ has to be stable, $0 < \frac{k_I}{k_R} < 2$ must hold what can be ensured by a proper choice of r.

The controller algorithm is assumed to be altered by anti-windup compensator presented in the next Section, in order to assure better control performance of the closed-loop system subject to constraints. It is to be borne in mind that the compensation is based on back-calculation, i.e., it does not require the controller to have integral terms in general.

4 GENERALISED CONDITIONING TECHNIQUE AWC

In GCT, the filtered set-point signal is conditioned, instead of the set-point r_t, and given as

$$r_{f,t} = \frac{Q(q^{-1})T_1(q^{-1})}{L(q^{-1})}\, r_t\,, \tag{10}$$

with

$$\begin{aligned} T(q^{-1}) &= T_2(q^{-1})T_1(q^{-1})\,, \\ Q(q^{-1}) &= q_0 + q_1 q^{-1} + \cdots + q_{nQ} q^{-nQ}\,, \\ L(q^{-1}) &= 1 + l_1 q^{-1} + \cdots + l_{nL} q^{-nL} \end{aligned} \tag{11}$$

and $T_2(0) = t_{2,0}$.

Similarly to the conditioning method (see (Horla and Krolikowski, 2003a)), the modified filtered reference signal is given by

$$r^r_{f,t} = r_{f,t} + \frac{q_0(u_t - v_t)}{t_{2,0}}\,, \tag{12}$$

and the control signal is defined as

$$\begin{aligned} v_t &= (1-Q'(q^{-1})R(q^{-1}))u_t + \frac{t_{2,0}}{q_0} r_{f,t} + \\ &+ \frac{1}{q_0}((T_2(q^{-1})L(q^{-1}) - t_{2,0}) r^r_{f,t} - \\ &- Q'(q^{-1})S(q^{-1})y_t\,, \end{aligned} \tag{13}$$

where $Q'(q^{-1}) = \frac{Q(q^{-1})}{q_0}$.

The GCT method enables additional tuning of the performance by reference signal filter design. Because its parameters should correspond to model parameters, saturation level and set-point values, a special choice of parameters of the filter (10) for minimumphase second-order model is proposed (Horla and Krolikowski, 2003a). Let ρ_1 and ρ_2 denote poles of stable $A(z^{-1})$, then

$$\rho = \max(|\rho_1|, |\rho_2|)\,, \tag{14}$$

$$Q(q^{-1}) = 1 + \left((1-\rho)^{\xi} - 1\right) q^{-1}\,, \tag{15}$$

$$L(q^{-1}) = 1 - (1-\rho)^{\xi} q^{-1}\,, \tag{16}$$

where $0 < \xi \leq 1$ is the damping factor obtained from classical root locus theory for the second-order systems. The suggested filter (14–16) takes into consideration model parameters and set-point values only, forcing the initial values of the filtered reference signal for slow models and reducing the amplitude and rate of transients for oscillatory ones.

The inherent property of the conditioning technique is the so-called short sightedness phenomenon, resulting in consecutive resaturations of the control signal because of excessive modification of the reference signal. In order to improve the performance of the compensation three modifications will be considered as in the next Section.

5 MODIFIED GENERALISED CONDITIONING AWCS

In order to combine classical conditional integration methods that work for controllers comprising integrators with back-calculation AWC presented in the previous Section, the following three back-calculation modifications have been proposed – the modification of the filtered reference input is applied when:

M1 $|e_t| > e_1$,
M2 $u_{t-1} \neq v_{t-1}$,
M3 $u_{t-1} \neq v_{t-1}$ and $e_t u_{t-1} > 0$,

where e_1 is a threshold value for reference modification clamping.

By applying the modifications to the GCT-AWC one assures that modification of the filtered reference signal is performed only when necessary.

6 SIMULATED PLANTS

The simulations have been performed for a set of stable, second-order, minimumphase plants with $B(q^{-1}) = 1 + 0.5q^{-1}$ and:

- P1 type

$$A(q^{-1}) = (1 - q^{-1}(\sigma + \omega i))(1 - q^{-1}(\sigma - \omega i))\,,$$

where:

$$\begin{aligned} -1 < \;&\sigma < 1\,, \\ -1 < \;&\omega < 1\,, \\ |\sigma \pm \omega i| &< 1\,, \end{aligned}$$

what corresponds to oscillatory behaviour of the plant,

- P2 type

$$A(q^{-1}) = (1 - q^{-1} z_1)(1 - q^{-1} z_2)\,,$$

where:

$$\begin{aligned} 0 < \;&z_1 < 1\,, \\ 0 < \;&z_2 < 1\,, \end{aligned}$$

what corresponds to aperiodic behaviour of the plant.

The simulations have been run for square wave reference signal of period 40 samples and symmetrical amplitude ± 3 with $\frac{k_I}{k_R} = 0.5$, $A_M(q^{-1}) = 1 - 0.5q^{-1} + 0.06q^{-2}$ and $e_1 = 3$.

In order to evaluate the quality of regulation process, the performance index is introduced

$$J = \frac{1}{N} \sum_{t=0}^{N} (r_t - y_t)^2\,, \tag{17}$$

where $N = 150$ denotes the simulation horizon.

The simulations have been performed for the same constraint hardness for each of the plants, denoted by relative constraint level α_r (i.e., the multiplicity of the minimum constraint level $\alpha_{\min} = 3\frac{|A(1)|}{|B(1)|}$ allowing asymptotic tracking). The absolute value of the constraint is $\alpha = \alpha_r \alpha_{\min}$ and changes as the plants change.

7 PERFORMANCE SURFACES

The results of the simulation tests are shown as performance surfaces. Each of the axes has been divided into 101 values, thus all simulation results refer to a grid of 101×101 different plants. The idea of such surfaces is as follows – let J_0 denote the value of the performance index of the control system with some plant and given α_r and no AWC. Let J_1 denote the value of the performance index of the same control system with the same plant but with classical GCT-AWC. Let J_2 denote the value of the performance index of, again, the same control system with the same plant but with modified GCT-AWC (M1, M2 or M3).

For each of the plants and constraints level the following face is plotted:

- (magenta) $J_0 = J_1 = J_2$,
- (red) modification is of the worst performance, $J_0 < J_1 < J_2$ or $J_1 < J_0 < J_2$, the intensity of the red level is proportional to $J_2 - J_0$ or $J_2 - J_1$,
- (white) modification improves the performance of the GCT-AWC, $J_0 < J_2 < J_1$,
- (black) it is not worth to modify GCT, $J_1 < J_2 < J_0$, the intensity of the black level is proportional to $J_2 - J_1$,
- (blue) modification improves the performance where GCT fails to, $J_2 < J_0 < J_1$, the intensity of the blue level is proportional to $J_0 - J_2$,
- (green) modification is of the best performance, the intensity of the green level is proportional to $J_0 - J_2$ or $J_1 - J_2$.

8 SHOULD ONE MODIFY GCT?

The performance surfaces have been obtained for P1 and P2 type plants with different dead-times and presented in Figs 1 and 2, where consecutive rows for different dead-times refer to M1, M2 and M3.

In all the cases of P1 and $d = 1$ it is visible that all modifications can improve the performance of the GCT for slow plants, i.e., with small natural frequency, whereas in the case of M1 and M3 there is an improvement visible for such plants near stability border. In the case of M1 and M3, one can see region of the best improvement (green). By comparing the given surfaces one can say that M3 is of the best AWC performance, because of the green regions and brighter red regions than in other cases, what refers to less performance degradation.

It is not advisable to modify the GCT algorithm when the region is red, it is advisable to improve where it is white and definitely advisable when green.

In the case of $d = 3$ one can see that red regions have almost disappeared and the improvement is best in the case of M3.

For P2 type plants a performance improvement can be observed for slow plants (green) with M1 and M3. Because of the size of white and green regions one can say that the best performance is assured by M1, mainly because of the $\alpha_r > 1$, that is visibility of green regions for greater α_rs. The vast areas of red color suggest that it is inadvisable to modify the original GCT when plant is moderately slow (expressed by absolute values of its poles).

In the case of $d = 3$ because of the area of white region and brightness of the red region, it can be said that M1 is the best choice, then M2 and M3.

9 SUMMARY

It has been shown that it can be advantageous to modify the algorithm of well-known GCT-AWC in order to obtain high control performance. Such a modification can be implemented with the use of lookup table, where the information is stored what GCT algorithm should be used when plant parameters vary in time, e.g. due to aging or set-point change. A similar approach has been presented for continuous system, PID controllers and integrator clamping (Visioli, 2003).

REFERENCES

Horla, D. (2007). Simple anti-integrator windup compensators – performance analysis. *Studies in Control and Computer Science*, 32:85–102.

Horla, D. and Krolikowski, A. (2003a). Anti-windup circuits in adaptive pole-placement control. In *Proceedings of the 7th European Control Conference*.

Horla, D. and Krolikowski, A. (2003b). Anti-windup compensators for adaptive pid controllers. In *Proceedings of the 9th IEEE International Conference on Methods and Models in Automation and Robotics*, pages 575–580.

Rundqwist, L. (1991). *Anti-reset Windup for PID Controllers*. PhD thesis, Lund University of Technology.

Visioli, A. (2003). Modified anti-windup scheme for pid controllers. *IEE Proceedings-D*, 150(1):49–54.

Walgama, K. and Sternby, J. (1990). Inherent observer property in a class of anti-windup compensators. *International Journal of Control*, 52(3):705–724.

Walgama, K. and Sternby, J. (1993). On the convergence properties of adaptive pole-placement controllers with anti-windup compensators. *IEEE Transactions on Automatic Control*, 38(1):128–132.

Figure 1: Performance surfaces for P1.

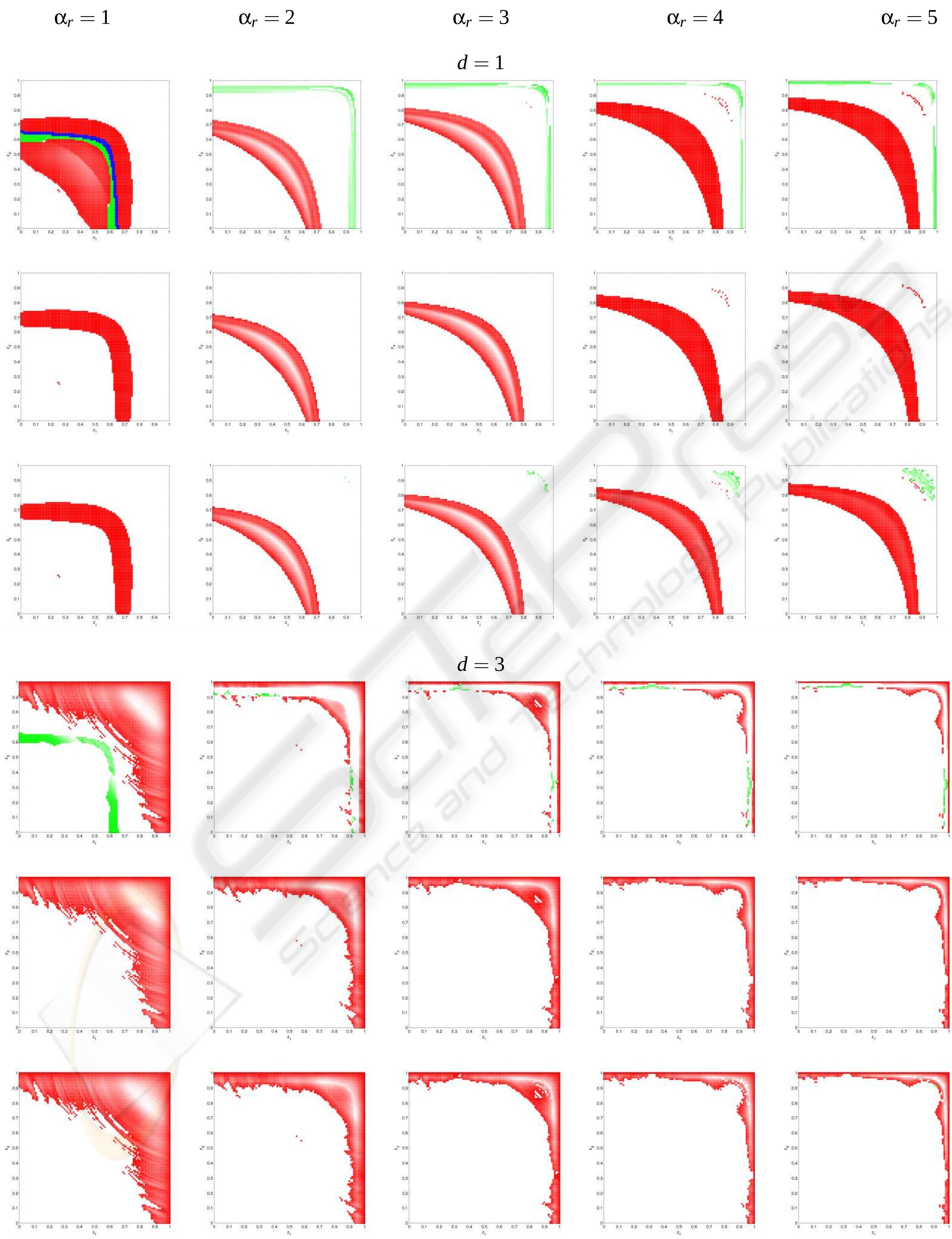

Figure 2: Performance surfaces for P2.