

# A Model of an Interregional Logistic System for the Statement and Solution of Decision Problems at the Operational Level

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**Abstract.** A regional/multi-regional logistic traffic network is considered in this paper with the aim of optimizing the flows of goods which pass through the network in order to reach their final destinations. The logistic network takes into account both road and rail transportation, and it is modelled as a directed graph whose arcs represent a road or a rail link and whose nodes are not only connection points but can represent a place where some service activities (such as the change in transportation mode) are carried out. In the paper, the model of the logistic network and, in particular, the equations which formalize the dynamics of links and nodes, are described in detail. In addition, with reference to decision problems at operational level, some considerations about the degrees of freedom (decision variables) in the model, the kind and the role of decision makers, and the class of performance indicators are also outlined in the paper.

## 1 Introduction

Modelling, planning, and control of logistic systems are research streams that, in the last years, have received a significant attention by the research community due to their economic impact. An improvement of the performance of the overall logistic chain and an effective integration of the different actors of a logistic system are fundamental goals in the management of modern production/distribution systems. As a matter of fact, these systems have to be designed and planned to fulfil such relevant objectives as those related to the on-time delivery of products to final users, to the minimization of transportation costs and of costs referred to the use of infrastructures, etc.

In this context, off-line planning methodologies play a key role and a wide bibliography can be found on such subjects. Some interesting review works [1–4] define the hierarchical decisional structure to be used when dealing with systems devoted to freight intermodal transportation and, then, with logistic systems. This structure is composed of three levels: long term (or strategic) planning, medium term (or tactical) planning, and short term (or operational) planning. At the strategic level, planning problems are mainly relevant to demand forecasting, logistic nodes location [5, 6] and to the design of transportation operations between nodes [7, 8]. The tactical level consists in

the aggregate planning of operations in logistic nodes [9] and of distribution operations (Service Network Design problems [10]). Many decision problems are typically defined at the operational level and such problems require the adoption of several models and decision techniques; typical decision problems at this level are the assignment of transportation operations to transportation means [11] and the static and dynamic routing of vehicles on the transportation network or on the logistic chain [12, 13]. The model and the problems considered in this paper refer to this latter decision level.

In this paper, the model of a logistic traffic network at regional/multi-regional level is presented, being the final objective of the current research activity the statement and solution of decision problems for the management of a logistic system at operational level, such as the optimal routing of goods which pass through the logistic network in order to reach their final destinations. The proposed model is a discrete-time model and the time horizon to be considered can range from some hours to some days. The model mainly consists of a directed graph whose arcs represent a road or a rail link and whose nodes are not only connection points but can represent a place where some service activities (such as the change in transportation mode) are carried out. The model is based on some characteristics which have been introduced in [14] with reference to the macroscopic modelling of transportation networks. In particular, each link and some nodes of the logistic network are discrete-time dynamic systems whose input and output variables are represented by flows that are respectively received from and transmitted to the neighbouring links/nodes, and the basic dynamic equation is represented by the vehicle conservation equation introduced in [15, 16]. In addition, with reference to decision problems at operational level, some considerations about the degrees of freedom (decision variables) in the model, the kind and the role of decision makers, and the class of performance indicators are also outlined in the conclusions of the paper.

## 2 The Model of the Logistic Network

The model of the logistic network mainly consists of the transportation offer (i.e., the physical network where vehicles can move), the transportation demand (i.e., the requirements of moving goods over this network) and the equations that represent the dynamics of this system, both referred to nodes and links. The model is a discrete-time model; in this connection, let  $t$  and  $\Delta$  be the generic time instant and the length of one interval, respectively, with  $t = 0, \dots, T$  being  $T\Delta$  the time horizon. Note that, for the quantities considered in the model which are not referred to a time instant but to a time interval, with  $t$  we refer to the time interval  $[t, t + 1)$ .

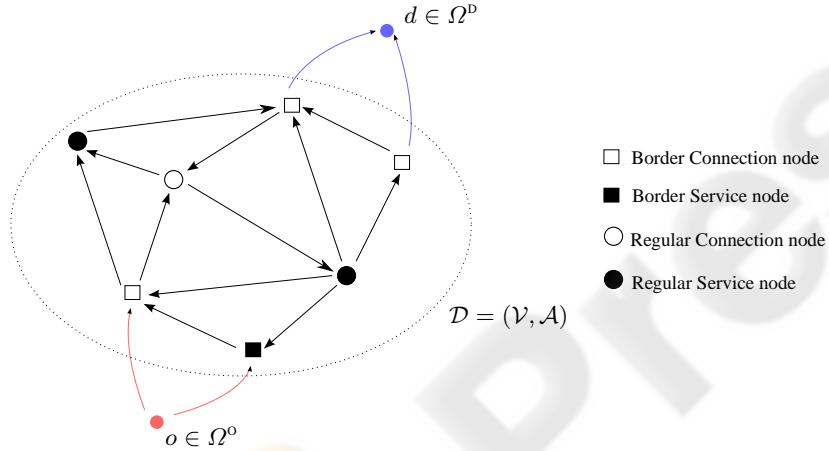
### 2.1 The Transportation Offer

The offer of transportation services is represented by means of a directed graph  $\mathcal{D} = (\mathcal{V}, \mathcal{A})$  where  $\mathcal{V}$  is the set of nodes and  $\mathcal{A}$  is the set of links. We will refer to each node as  $i \in \mathcal{V}$  and to each link as the pair of nodes it connects, i.e.  $(i, j) \in \mathcal{A}$ . For each node  $i \in \mathcal{V}$  the sets  $\mathcal{P}(i)$  and  $\mathcal{S}(i)$  gather the predecessor and successor nodes, respectively.

The graph  $\mathcal{D}$  represents an intermodal network involving two transportation modes corresponding to *road* and *rail*. Let us denote with  $\mathcal{A}^R$  and  $\mathcal{A}^T$  the set of arcs on road

and on rail, respectively. It is  $\mathcal{A}^R \cap \mathcal{A}^T = \emptyset$  since an arc corresponds univocally to a given transportation mode. Moreover, it is obvious that  $\mathcal{A}^R \cup \mathcal{A}^T = \mathcal{A}$ .

The nodes of the network are primarily divided into *connection nodes* and *service nodes*. The former are simply interconnections among different links and do not have their own dynamics, whereas the latter represent a place where some service activities are carried out (such as intermodal terminals where cargo is handled and there is a change in the transportation mode) and then are modelled as discrete-time dynamic systems. Both connection and service nodes can be either *regular nodes* or *border nodes*. Border nodes represent the access and exit points of the network. In this connection let  $\mathcal{V}^{RC}$ ,  $\mathcal{V}^{BC}$ ,  $\mathcal{V}^{RS}$ , and  $\mathcal{V}^{BS}$  be, respectively, the set of regular connection, border connection, regular service, and border service nodes. These sets are disjoint ( $\mathcal{V}^{RC} \cap \mathcal{V}^{BC} \cap \mathcal{V}^{RS} \cap \mathcal{V}^{BS} = \emptyset$ ) and their union correspond to the whole set of nodes ( $\mathcal{V}^{RC} \cup \mathcal{V}^{BC} \cup \mathcal{V}^{RS} \cup \mathcal{V}^{BS} = \mathcal{V}$ ).



**Fig. 1.** A sketch of the logistic network.

## 2.2 The Transportation Demand

In the considered model, we suppose that the real origins and destinations of the demand are outside the transportation network  $\mathcal{D}$ . However, all goods must pass through the proposed regional/multi-regional logistic traffic network in order to reach their final destinations. At this purpose, let  $\Omega^0$  and  $\Omega^D$  represent, respectively, the set of origins and the set of destinations for the whole demand (see Fig. 1). Note that there can be some geographic areas that are both the origin and the destination of logistic flows, then in general  $\Omega^0 \cap \Omega^D \neq \emptyset$ . Goods coming from a certain origin may enter the network through one or more “compatible” border nodes; in the same way goods can reach their destination by exiting the network from one or more “compatible” border nodes. Then, let  $\mathcal{V}_o^{IN} \subseteq \mathcal{V}^{BC} \cup \mathcal{V}^{BS}$  (resp.,  $\mathcal{V}_d^{OUT} \subseteq \mathcal{V}^{BC} \cup \mathcal{V}^{BS}$ ) be the set of border nodes associated with origin  $o \in \Omega^0$  (resp., destination  $d \in \Omega^D$ ). Moreover, for each destination  $d \in \Omega^D$  and for each node  $\mu \in \mathcal{V}_d^{OUT}$ , we denote with  $\tau_{\mu,d}(t)$  the time necessary to reach  $d$  from  $\mu$  if the logistic units are in  $\mu$  at time  $t$ .

The transportation demand is defined for each different network user, i.e., road carrier, shipper and so on, that needs to transport some logistic units from a certain origin to a certain destination. Each network user is denoted with  $n = 1, \dots, N$  and it has a set of  $\Gamma_n$  transportation requests to satisfy. The  $l$ -th request of user  $n$ ,  $n = 1, \dots, N$ ,  $l = 1, \dots, \Gamma_n$ , is characterized by: *origin*  $o_{n,l} \in \Omega^0$ , *destination*  $d_{n,l} \in \Omega^D$ , *number of logistic units*  $\delta_{n,l}$ , *due date*  $dd_{n,l}$ , *release time*  $rt_{n,l}$ , i.e., the time instant in which the logistic units are available to enter the network. In addition, let  $st_{n,l}$  be the time instant in which the logistic units actually enter the network; moreover,  $\lambda_{n,l}^{\nu\mu}$ ,  $\nu \in \mathcal{V}_{o_{n,l}}^{\text{IN}}$ ,  $\mu \in \mathcal{V}_{d_{n,l}}^{\text{OUT}}$  represents the percentage of  $\delta_{n,l}$  that enter the network in  $\nu$  and exit from  $\mu$ . Note that these last two terms are decision variables whose values depend on the choices taken by the network user.

Finally, in order to associate the request  $l$  of network user  $n$  with the considered time horizon, let the function of time  $\delta_{n,l}(t)$  be defined as follows:

$$\delta_{n,l}(t) = \begin{cases} \delta_{n,l} & \text{if } t = st_{n,l} \\ 0 & \text{otherwise} \end{cases} \quad n = 1, \dots, N \quad l = 1, \dots, \Gamma_n \quad t = 0, \dots, T \quad (1)$$

### 3 The Dynamics of the Logistic Network

Links and nodes are considered as discrete-time dynamic systems whose state is represented by the number of logistic units which are in the link or node at a certain time instant. Each state variable is updated according to a state equation (*conservation equation*) which takes into account the number of logistic units entering and exiting the link or node in the time interval between two subsequent time instants. Moreover, in order to separately consider all requests of all network users and all exiting nodes, an approach similar to the one proposed in [14], which considers destination-oriented variables (composition and splitting rates), is adopted.

#### 3.1 Links

The dynamics of links involves road links only, since trains transporting a finite number of logistic units over a rail link are not explicitly modelled. As it will be clear in the following, the dynamics of trains is implicitly considered in the dynamics of service nodes. Then, in the following, it is assumed  $(i, j) \in \mathcal{A}^R$ .

Let us denote with  $n_{i,j}^{n,l,\mu}(t)$ ,  $n = 1, \dots, N$ ,  $l = 1, \dots, \Gamma_n$ ,  $\mu \in \mathcal{V}_{d_{n,l}}^{\text{OUT}}$ ,  $t = 0, \dots, T$ , the number of logistic units, belonging to the  $l$ -th transportation request of network user  $n$ , which are in link  $(i, j)$ , at time  $t$ , and have to reach border node  $\mu$ . In the following, the triple  $(n, l, \mu)$  will be referred to as a whole. The state equation is then given by:

$$n_{i,j}^{n,l,\mu}(t+1) = n_{i,j}^{n,l,\mu}(t) + q_{i,j}^{n,l,\mu}(t) - Q_{i,j}^{n,l,\mu}(t) \quad (2)$$

where  $q_{i,j}^{n,l,\mu}(t)$  and  $Q_{i,j}^{n,l,\mu}(t)$  are, respectively, the number of logistic units of  $(n, l, \mu)$  which enter and exit  $(i, j)$  in the time interval  $[t, t+1]$ .

$Q_{i,j}^{n,l,\mu}(t)$  is given by:

$$Q_{i,j}^{n,l,\mu}(t) = \gamma_{i,j}^{n,l,\mu}(t) \cdot Q_{i,j}(t) \quad (3)$$

being the overall number of logistic units exiting from  $(i, j)$ , namely  $Q_{i,j}(t)$ , obtained from

$$Q_{i,j}(t) = v_{i,j}(t) \cdot \rho_{i,j}(t) \cdot \Delta \quad (4)$$

where  $v_{i,j}(t)$  and  $\rho_{i,j}(t)$  indicate the mean speed and the density on link  $(i, j)$  in the time interval  $[t, t + 1]$ . If we suppose that the density is uniformly distributed along  $(i, j)$  and constant in  $[t, t + 1]$ , we can define the density as:

$$\rho_{i,j}(t) = \frac{n_{i,j}(t) + \bar{m}_{i,j}(t)}{L_{i,j}} \quad (5)$$

where  $L_{i,j}$  is the length of  $(i, j)$  and  $\bar{m}_{i,j}(t)$  represents the number of other vehicles (such as cars or other logistic vehicles which are not matter of decision in the considered system) present in  $(i, j)$  at time  $t$ . The value of  $\bar{m}_{i,j}(t)$  is supposed to be known, at least as an average value, and then it is an input to the problem. However, note that  $\bar{m}_{i,j}(t)$  must be taken into account because it affects the traffic behaviour and, then, the evolution of the state variable.

Moreover, the mean speed on the link is defined as  $v_{i,j}(t) = f[\rho_{i,j}(t), (i, j), t]$ , i.e., it is a function of the density on the link (as well as function of the link itself and of the time instant). This relation is generally known as the *steady state speed-density characteristic* [17].

The *link composition rate*  $\gamma_{i,j}^{n,l,\mu}(t)$  specifies the fraction of logistic units, which are actually in link  $(i, j)$ , belonging to  $(n, l, \mu)$ , with respect of the overall number of logistics units in  $(i, j)$ . It is computed as

$$\gamma_{i,j}^{n,l,\mu}(t) = \frac{n_{i,j}^{n,l,\mu}(t)}{n_{i,j}(t)} = \frac{n_{i,j}^{n,l,\mu}(t)}{\sum_{n=1}^N \sum_{l=1}^{\Gamma_n} \sum_{\mu \in \mathcal{V}_{d_{n,l}}^{\text{OUT}}} n_{i,j}^{n,l,\mu}(t)} \quad (6)$$

The equation providing  $q_{i,j}^{n,l,\mu}(t)$  depends on the kind of node  $i$ . If node  $i$  is a regular connection node, then

$$q_{i,j}^{n,l,\mu}(t) = \sum_{h \in \mathcal{P}(i)} \beta_{h,i,j}^{n,l,\mu}(t) \cdot Q_{h,i}(t) \quad i \in \mathcal{V}^{\text{RC}} \quad (7)$$

where  $\beta_{h,i,j}^{n,l,\mu}(t)$  is the *link splitting rate* from link  $(h, i)$  to link  $(i, j)$ , in the time interval  $[t, t + 1]$ , with reference to  $(n, l, \mu)$ . The link splitting rates are given by

$$\beta_{h,i,j}^{n,l,\mu}(t) = \gamma_{i,j}^{n,l,\mu}(t) \cdot \alpha_{h,i,j}^{n,l,\mu}(t) \quad (8)$$

where  $\alpha_{h,i,j}^{n,l,\mu}(t)$  are *route choice parameters*. If node  $i$  is a border connection node and represents one of the access points for the logistic units belonging to  $(n, l, \mu)$  (that is,  $i \in \mathcal{V}_{o_{n,l}}^{\text{IN}}$ ), then

$$q_{i,j}^{n,l,\mu}(t) = \beta_{i,j}^{n,l,\mu}(t) \cdot \lambda_{n,l}^{i,\mu} \cdot \delta_{n,l}(t) \quad i \in \mathcal{V}_{o_{n,l}}^{\text{IN}} \subseteq \mathcal{V}^{\text{BC}} \quad (9)$$

where  $\beta_{i,j}^{n,l,\mu}(t)$  is the *node splitting rate* from node  $i$  to link  $(i, j)$ , in the time interval  $[t, t + 1]$ , with reference to  $(n, l, \mu)$ . Finally, if node  $i$  is a service node, both regular and border, the dynamics of the node must be taken into account, thus

$$q_i^{n,l,\mu}(t) = \beta_{i,j}^{n,l,\mu}(t) \cdot \tilde{Q}_i(t) \quad i \in \mathcal{V}^{\text{RS}} \cup \mathcal{V}^{\text{BS}} \quad (10)$$

where  $\tilde{Q}_i(t)$  is the number of logistic units exiting the node  $i$  (see next subsection).

### 3.2 Nodes

The dynamics of nodes is related to the possibility of queuing logistic units inside the node and thus it involves service nodes only (both regular and border). Let us denote with  $n_i^{n,l,\mu}(t)$ ,  $n = 1, \dots, N$ ,  $l = 1, \dots, \Gamma_n$ ,  $\mu \in \mathcal{V}_{d_{n,l}}^{\text{OUT}}$ ,  $t = 0, \dots, T$ , the number of logistic units, belonging to the  $l$ -th transportation request of network user  $n$ , which are in node  $i$ , at time  $t$ , and have to reach border node  $\mu$ . As before, in the following, the triple  $(n, l, \mu)$  will be referred to as a whole. The state equation is then given by:

$$n_i^{n,l,\mu}(t + 1) = n_i^{n,l,\mu}(t) + q_i^{n,l,\mu}(t) - Q_i^{n,l,\mu}(t) \quad (11)$$

where  $q_i^{n,l,\mu}(t)$  and  $Q_i^{n,l,\mu}(t)$  are, respectively, the number of logistic units of  $(n, l, \mu)$  which enter and exit  $i$  in the time interval  $[t, t + 1]$ .

$Q_i^{n,l,\mu}(t)$  is given by

$$Q_i^{n,l,\mu}(t) = \tilde{Q}_i^{n,l,\mu}(t) + \hat{Q}_i^{n,l,\mu}(t) \quad (12)$$

where  $\tilde{Q}_i^{n,l,\mu}(t)$  (resp.,  $\hat{Q}_i^{n,l,\mu}(t)$ ) represents the overall number of logistic units, belonging to  $(n, l, \mu)$ , exiting from node  $i$  and entering a road link (resp., rail link).  $\tilde{Q}_i^{n,l,\mu}(t)$  is provided by

$$\tilde{Q}_i^{n,l,\mu}(t) = \tilde{\gamma}_i^{n,l,\mu}(t) \cdot \tilde{Q}_i(t) \quad (13)$$

where  $\tilde{\gamma}_i^{n,l,\mu}(t)$  is the *node-to-road composition rate*, and  $\tilde{Q}_i(t)$  is the overall number of logistic units exiting  $i$  and entering a road link; this last term is given by

$$\tilde{Q}_i(t) = \min \{ \tilde{\sigma}_i(t) \cdot n_i(t), \tilde{s}_i(t) \cdot \Delta \} \quad (14)$$

with

$$\tilde{\sigma}_i(t) = \sum_{n=1}^N \sum_{l=1}^{\Gamma_n} \sum_{\mu \in \mathcal{V}_{d_{n,l}}^{\text{OUT}}} \tilde{\sigma}_i^{n,l,\mu}(t) \quad (15)$$

being  $\tilde{\sigma}_i^{n,l,\mu}(t)$  the fraction of logistic units of  $(n, l, \mu)$  which are in node  $i$  at time  $t$  and leave, in the subsequent time interval, namely  $[t, t + 1]$ , the node towards a road link or leave the network, and

$$n_i(t) = \sum_{n=1}^N \sum_{l=1}^{\Gamma_n} \sum_{\mu \in \mathcal{V}_{d_{n,l}}^{\text{OUT}}} n_i^{n,l,\mu}(t) \quad (16)$$

Then, the node-to-road composition rate can be computed as

$$\tilde{\gamma}_i^{n,l,\mu}(t) = \frac{\tilde{\sigma}_i^{n,l,\mu}(t) \cdot n_i^{n,l,\mu}(t)}{\tilde{\sigma}_i(t) \cdot n_i(t)} \quad (17)$$

In (14),  $\tilde{s}_i(t)$  represents the *node-to-road service rate* (expressed as number of logistic units per time unit) in the node  $i$  in the time interval  $[t, t+1]$ . Note that it is assumed that every logistic unit entering a service node in a given time interval cannot exit the node itself in the same time interval.

Before introducing the equation providing  $\hat{Q}_i^{n,l,\mu}(t)$ , it is necessary to briefly describe the behaviour of logistic units on rail links. A rail link  $(i, j) \in \mathcal{A}^T$  is assumed to be served by one or more trains which transport logistic units from  $i$  to  $j$ . It is assumed that one train begins a transportation in  $i$  at each time instant and the number of logistic units that are transported by the train depends on the state of the node. However, such a number is upper-bounded by a value  $C_{i,j}(t)$  which represents the capacity (maximum number of logistic units that can be transported) of the train leaving  $i$  towards  $j$ , at time instant  $t$ . Moreover, let  $\Lambda_{i,j}$  be the travel time of a train travelling from  $i$  to  $j$ , expressed as number of time intervals; such a value is assumed fixed and a-priori known.

Because of the finite capacity of trains, some of the logistic units that concluded their service and that have to proceed with their travel in a rail link, may be not allowed to exit the node. Then, it is necessary to distinguish between the “potential” number of logistic units which leave from the node and the “actual” number. In (12),  $\hat{Q}_i^{n,l,\mu}(t)$ , is the actual number. The potential number is provided by

$$\hat{Q}_i^{\text{POT } n,l,\mu}(t) = \hat{\gamma}_i^{n,l,\mu}(t) \cdot \hat{Q}_i^{\text{POT}}(t) \quad (18)$$

where  $\hat{\gamma}_i^{n,l,\mu}(t)$  is the *node-to-rail composition rate*, and  $\hat{Q}_i^{\text{POT}}(t)$  is the overall number of logistic units which potentially exit  $i$  and enter a rail link; this last term is given by

$$\hat{Q}_i^{\text{POT}}(t) = \min \{ \hat{\sigma}_i(t) \cdot n_i(t), \hat{s}_i(t) \cdot \Delta \} \quad (19)$$

with

$$\hat{\sigma}_i(t) = \sum_{n=1}^N \sum_{l=1}^{\Gamma_n} \sum_{\mu \in \mathcal{V}_{d_{n,l}}^{\text{OUT}}} \hat{\sigma}_i^{n,l,\mu}(t) \quad (20)$$

being  $\hat{\sigma}_i^{n,l,\mu}(t) = 1 - \tilde{\sigma}_i^{n,l,\mu}(t)$ ,  $\forall(n, l, \mu)$ , the fraction of logistic units of  $(n, l, \mu)$  which are in node  $i$  at time  $t$  and leave the node towards a rail link. Then, the node-to-rail composition rate can be computed as

$$\hat{\gamma}_i^{n,l,\mu}(t) = \frac{\hat{\sigma}_i^{n,l,\mu}(t) \cdot n_i^{n,l,\mu}(t)}{\hat{\sigma}_i(t) \cdot n_i(t)} \quad (21)$$

In (19),  $\hat{s}_i(t)$  represents the *node-to-rail service rate* in the node  $i$  in the time interval  $[t, t+1]$ . The actual number of logistic units which leave from the node is then computed as

$$\hat{Q}_i^{n,l,\mu}(t) = \sum_{\substack{j \in \mathcal{S}(i) \\ (i,j) \in \mathcal{A}^T}} \hat{\xi}_{i,j}^{n,l,\mu}(t) \cdot \hat{Q}_i^{\text{POT } n,l,\mu}(t) \quad (22)$$

where  $\widehat{\xi}_{i,j}^{n,l,\mu}(t)$  represents the fraction of logistic units of  $(n, l, \mu)$  which actually leave the node  $i$  towards rail link  $(i, j)$ , with respect to the relative potential number. It is worth noting that the meaning of  $\widehat{\xi}_{i,j}^{n,l,\mu}(t)$  is different from that of splitting rates introduced in the link dynamics. Moreover, such quantities must satisfy the following constraint

$$\sum_{n=1}^N \sum_{l=1}^{\Gamma_n} \sum_{\mu \in \mathcal{V}_{d_{n,l}}^{\text{OUT}}} \widehat{\xi}_{i,j}^{n,l,\mu}(t) \cdot \widehat{Q}_i^{\text{POT } n,l,\mu}(t) \leq C_{i,j}(t) \quad (23)$$

It is worth finally observing that, when  $i = \mu$ , all logistic units belonging to  $(n, l, \mu)$  leave the network; in this case, it turns out  $\widehat{\sigma}_{\mu}^{n,l,\mu}(t) = 1$ ,  $\widehat{\sigma}_{\mu}^{n,l,\mu}(t) = 0$ ,  $t = 0, \dots, T$ .

Coming back to (11),  $q_i^{n,l,\mu}(t)$  is given by

$$q_i^{n,l,\mu}(t) = \begin{cases} \lambda_{n,l}^{i,\mu} \cdot \delta_{n,l}(t) & i \in \mathcal{V}_{o_{n,l}}^{\text{IN}} \subseteq \mathcal{V}^{\text{BS}} \\ \widehat{q}_i^{n,l,\mu}(t) + \widetilde{q}_i^{n,l,\mu}(t) & i \in \mathcal{V}^{\text{RS}} \cup \mathcal{V}^{\text{BS}}, i \notin \mathcal{V}_{o_{n,l}}^{\text{IN}} \subseteq \mathcal{V}^{\text{BS}} \end{cases} \quad (24)$$

where, in case of service nodes that are not an access point for logistic units belonging to  $(n, l, \mu)$  (bottom expression of (24)),  $\widehat{q}_i^{n,l,\mu}(t)$  (resp.,  $\widetilde{q}_i^{n,l,\mu}(t)$ ) represents the overall number of logistic units, belonging to  $(n, l, \mu)$ , coming from a road link (resp., rail link) and entering node  $i$ .  $\widehat{q}_i^{n,l,\mu}(t)$  and  $\widetilde{q}_i^{n,l,\mu}(t)$  are provided by

$$\widehat{q}_i^{n,l,\mu}(t) = \sum_{\substack{h \in \mathcal{P}(i) \\ (h,i) \in \mathcal{A}^{\text{R}}}} Q_{h,i}^{n,l,\mu}(t) \quad (25)$$

$$\widetilde{q}_i^{n,l,\mu}(t) = \sum_{\substack{h \in \mathcal{P}(i) \\ (h,i) \in \mathcal{A}^{\text{T}}}} \widehat{\xi}_{h,i}^{n,l,\mu}(t - \Lambda_{i,j}) \cdot \widehat{Q}_h^{\text{POT } n,l,\mu}(t - \Lambda_{i,j}) \quad (26)$$

## 4 Conclusions and Further Research Directions

In the previous section the model of an intermodal logistic network has been presented. The dynamic evolution of the elements (links and nodes) of this network has been represented by means of discrete-time state equations where the state variables indicate the number of logistic units present in a link or in a node. The main decisions to be taken concern the splitting of these logistic units over the alternative paths in the network (and consequently the choice of transportation mode) and the time instant in which they enter the network. Different approaches can be defined in order to determine these decisions and they depend on which decision makers are considered and, for each decision maker, the decision power, the available information and the performance indexes.

Three classes of decision makers can be considered in general. First of all, *network users* are decision makers that must move goods from given origins to given destinations, characterized by specific due dates. These network users work in a competitive environment, therefore each of them is characterized by a specific objective (i.e. minimizing costs and/or travel times in order to deliver goods within a given due date). Another class of decision makers is given by *infrastructure managers*, such as managers

of links (e.g. highways) or managers of nodes (e.g. terminal operators) or managers of trains. Each of them has, again, a specific objective (i.e. minimizing risk factors, maximizing profits, and so on) that can be in conflict with the objectives of other decision makers. A third class of decision makers is represented by the *local authorities* or *territory managers* devoted to manage the territory with social objectives (such as assuring security, minimizing traffic congestion, and so on). These three classes of decision makers are involved in a decision framework that is, in general, a hierachic structure. The territory manager is at the top of this decision structure, it decides on the basis of its social objectives and it can act on the system in two ways, by advising the other decision makers about how to act or by imposing to them some policies (e.g. forbidding to cover a given link in a certain time period, imposing the number of specific cargo units that can move in a part of the network, and so on). The decisions taken by the territory manager affect the decisions of the network managers that, again, can be applied by advisory or coercive policies and, in their turn, affect the decisions of the network users. Therefore, the network users make their decisions by taking into account the social policies of the territory managers and the cost/incentive policies provided by the infrastructure managers.

The main decisions of the proposed system, i.e. the definition of the path followed by the logistic units, the transportation mode and the time instant in which they enter the network, are taken by network users and this can be obtained as the solution of a specific optimization problem. The considered objective function concerns the minimization of some cost terms concerning the network users (travel costs, also including highway or rail fares, deviations from due dates, and so on), possibly weighted in a different way for each network user. In the considered optimization problem, the constraints include the discrete-time state equations of nodes and links, as well as some other specific constraints. Note that the decisions taken by infrastructure managers and territory managers can affect the optimization problem both in the objective function and in the constraints. For instance, if the manager of a node/link applies different fares in different time slots, this is considered in the problem objective function. Otherwise, if the territory manager imposes a limit to the number of logistic units that can move in a certain area in a given time slot, this is considered in the problem by adding a constraint.

The proposed model is very general and can refer to different real applications, by adding specific constraints and/or decision variables. If a completely centralized system is considered, a single large optimization problem must be solved. Since such a problem generally has a nonlinear form, if real applications are considered, the problem dimensions are probably too large to be solved with nonlinear solvers. For this reason, it could be more reasonable to state different separate problems for each network user or for groups of network users, in order to obtain smaller instances of the problem. Anyway, in this case, it is necessary to model the interaction among the network users (either in a competitive or in a cooperative environment) such that an overall solution can be obtained by considering the single solutions that each of them has found by solving its specific optimization problem. The present research activity is devoted to the analysis of some real situations and the statement of ad-hoc optimization problems, in order to evaluate the effectiveness of different management policies in logistic networks.

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