ON ADAPTIVE MODELING OF NONLINEAR EPISODIC REGIONS IN KSE-100 INDEX RETURNS

Rosheena Siddiqi and Syed Nasir Danial
Bahria University, Karachi, Pakistan

Keywords: Financial forecasting, Bicorrelation test, Transient nonlinearity, Neural network.

Abstract: This paper employs the Hinich portmanteau bicorrelation test with the windowed testing method to identify nonlinear behavior in the rate of returns series for Karachi Stock Exchange indices. The stock returns series can be described to be comprising of few brief phases of highly significant nonlinearity, followed by long phases in which the returns follow a pure noise process. It has been identified that major political and economic events have contributed to the short bursts of nonlinear behavior in the returns series. Finally, these periods of nonlinear behavior are used to predict the behavior of the rest of the regions using a feedforward neural network and dynamic neural network with Bayesian Regularization Learning. The dynamic neural network outperforms the traditional feedforward networks because Bayesian regularization learning method is used to reduce the training epochs. The time-series generating process is found to closely resemble a white noise process with weak dependence on value at lag one.

1 INTRODUCTION

The nonlinear behavior of financial time series data has intrigued researchers for more than a decade and it has seen further growth in recent years. Among the factors, which induce nonlinearity, are difficulties in executing arbitrage transactions, market imperfections, irrational behavior of investors, diversity of belief in agents, and heterogeneity of objectives with investors. Research has produced encouraging results, with more and more empirical evidence emerged to suggest that nonlinearity is a universal phenomenon (Kantz and Schreiber 1997). This has received a wide empirical support across different financial markets. However, presence of nonlinearity in stock exchange index, returns, or in the data which shows direction of stocks, forms serious consequences for the well known Efficient Market Hypothesis (EMH) of finance, which states that markets are efficient in that opportunities for profit are discovered so quickly that they cease to be opportunities. EMH effectively states that no system can continually beat the market because if a system becomes public everyone will use it thus negating its potential gain. A direct corollary of this hypothesis is that stock prices form a stochastic process — a Markovian process of certain order — and that any profit derived from timing the market are due entirely to chance.

Now detecting nonlinearity, in general, is considered a difficult problem in dynamical systems theory, or nonlinear time-series analysis, to be specific. There are several standard ways to assess nonlinearity in a given time-series data, e.g., determining estimates of intrinsic dimension, such as box-counting and correlation dimension (using Grassberger and Procaccia (1983), (1983a)), finding the maximum Lyapunov exponent, etc., see (Kantz and Schreiber 1997). All these estimates are invariants and can only be estimated in a phase-space of some appropriate embedding dimension. Moreover, a possible attractor in which to confine our attention for discovering nonlinear behavior requires abundance of data points, *N*. Methods which deal with estimation of invariant quantities in a nonlinear setting are not only difficult for application purpose but also that they require huge amount of data. There are several criteria one can exploit to check for the fulfillment of this condition (we refer to Kantz and Schreiber (1997) for details about such criteria). In general, an attractor with a high (unknown) dimension needs larger number of data points for the estimation of its dimension as compared to that whose dimension is low.

Siddiqi R. and Danial S. (2009).
ON ADAPTIVE MODELING OF NONLINEAR EPISODIC REGIONS IN KSE-100 INDEX RETURNS .
In *Proceedings of the International Joint Conference on Computational Intelligence*, pages 402-407
DOI: 10.5220/0002276804020407

Besides the difficulties of employing standard nonlinear time-series methods such as those discussed in Barnett et al. (1996), there are some statistical techniques, which are helpful in deciding if a given financial time-series is nonlinear. Lim and Hinch (2005) argue that the detected nonlinear behavior, i.e., the linear and nonlinear serial dependence which can be estimated by computing portmanteau correlation, bicorrelation and tricorrelation, is episodic in that there are long periods of pure noise process, only to be interrupted by relatively few brief episodes of highly significant non-linearity. Such nonlinearities may be detected by employing a windowing approach proposed by Hinich and Patterson (1995) to detect major political and economic events that may have contributed to the short burst of non-linear dependencies. Their study advocated a form of event studies that is data dependent to determine endogenously those events that trigger non-linear market reactions. We use this technique to find out regions in our data where nonlinearity is significant; political events are listed which may be responsible for such a behavior. Based on this information, we construct neural network models for each frame. It is further observed that inside a given region the forecast error is found to be less as compared to what is observed when we forecast values outside the region. This finding is of utmost importance to us because it suggests insignificance of global models.

Section 2 explains some of the previous work. In section 3, we give basic definitions of the concepts being used in this analysis. Section 4 presents our computations regarding windowing approach. We also construct neural network models of our computed frames. Finally, section 5 opens with a discussion and concludes the entire present work.

2 PREVIOUS WORK

The Karachi Stock Exchange (KSE-100 index) is the main stock exchange in Pakistan. In a recent study by Danial et al. (2008), the authors have tested the daily stock returns of the entire history of KSE-100 index against nonlinearity. A nonlinear dynamical system invariant, viz., correlation dimension is attempted to be computed but they have concluded that correlation dimension can not be estimated due to either insufficient data or insufficient information content within available data so as to be framed as a dynamical system. However, Danial et al. (2008) demonstrates modeling of KSE-100 index returns using feedforward neural network with a comparison to ARMA/ARIMA modeling. In Burni, Jilani and Ardil (2004), the author use neural network to model KSE-100 direction of index data. Here only very short time series is used for modeling purpose in contrast with the work of Danial et al. (2008). Antoniou, Ergul, and Holmes (1997) and Sarantis (2001) list several possible factors which might induce nonlinearity in stock returns.

Much of the earlier evidence of the presence of nonlinearity was drawn from stock markets of developed countries. Hinich and Patterson (1985) establish the presence of nonlinear non-Gaussian process generating daily stock returns by estimating a bispectrum of time series data of fifteen common stocks chosen at random from the set of stocks listed continuously on the New York Stock Exchange and American Stock Exchange, and describe a test of nonlinearity based on skewness. Similar findings regarding nonlinearity observed in Latin America and UK are respectively reported by Bonilla, Romero-Meza, and Hinich (2006), Abhyankar, Copeland and Wong (1995) and Opong et al. (1999). However in recent years, more and more evidence of nonlinearity from emerging stock markets are documented by Brooks and Hinich (1998), Ammermann and Patterson (2003), Lim, Hinich, Liew (2003), Lim and Hinich (2005, 2005a). In particular, Hinich and Patterson (1985), discuss the parameter instability of GARCH models and the transient nature of ARCH effects. It has been shown that the GARCH model cannot be considered a full representation of the process generating financial market returns. In particular, the GARCH models fails to capture the time-varying nature of market returns, and treats coefficients as fixed and being drawn from only one regime.

In all these aforementioned studies, the detected nonlinear behavior is also episodic in that there were long periods of pure noise process, only to be interspersed with relatively few brief episodes of highly significant nonlinearity as shown by Wild, Hinich and Foster (2008).

3 FUNDAMENTAL CONCEPTS

3.1 The Portmanteau Bicorrelation Tests

The 'windowing' approach and the bicorrelation test statistic proposed in Hinich and Patterson (1995) (denoted as H statistic) are briefly described in this section. Let there be a sequence $\{y(t)\}$ which denotes the sampled data process, where the time unit, t, is an integer. The test procedure employs non-overlapped data window, thus if n is the window length, then k-th window is $\{y(t_k), y(t_k+1), \ldots, y(t_k+n-1)\}$. The next non-overlapped window is $\{y(t_{k+1}), y(t_{k+1}+1), \ldots, y(t_{k+1}+n-1)\}$, where $t_k+1 = t_k+_n$. The null hypothesis for each window is that $y(t)$ are realizations of a stationary pure noise process that has zero bi-covariance.

We state without proof and derivation that the H statistic is defined as:

$$\mathrm{H} = \sum_{s=2}^{L} \sum_{r=1}^{s-1} G^2(r,s) \sim \chi^2 (L-1)(L/2) \quad (1)$$

Where

$$G(r,s) = \sqrt{n-s}\, C_{zzz}(r,s) \quad (2)$$

and

$$C_{zzz}(r,s) = \frac{\sum_{t=1}^{n-s} Z(t)\, Z(t+r) Z(t+s)}{(n-s)^{-1}} \quad (3)$$

where $Z(t)$ are the standardized observations, obtained by subtracting the sample mean of the window and dividing by its standard deviation. The number of lags L is specified as $L= n^b$ with $0 < b < 0.5$, where b is a parameter under the choice of the user. Based on the results of Monte Carlo simulations, Hinich and Patterson (1995) recommended the use of $b = 0.4$ in order to maximize the power of the test while ensuring a valid approximation to the asymptotic theory even when n is small. In this test procedure, a window is significant if the H statistic rejects the null of pure noise at the specified threshold level.

3.2 Neural Network

Parameterized nonlinear maps, capable of approximating arbitrary continuous functions over compact domains are known as neural networks. It has been proven by Cybenko (1989) and Hornik, Stinchcombe, & White (1989) that any continuous mapping over a compact domain can be estimated as accurately as required by a feedforward neural network with a single hidden layer.

In the context of neural network literature, the term neuron refers to an operator that maps $\Re^n \rightarrow \Re$ and can be illustrated by the equation

$$y = \Gamma(\sum_{j=1}^{n} w_j u_j + w_0) \quad (4)$$

where $U^T = [u_1, u_2, \ldots u_n]$ is the input vector, $W^T = [w_1, w_2, \ldots w_n]$ is referred to as the weight vector of the neuron, and w_0 is the bias. $\Gamma(.)$ is a monotone continuous function such that $\Gamma(.):\Re \rightarrow (-1, 1)$. The function $\Gamma(.)$ is commonly called a 'sigmoidal function'; tanh(.), and $(1+\exp(-(.)))^{-1}$ are some widely used examples. The neurons can be arranged in a variety of layered architecture with layers $l = 0, 1, \ldots L$, such as, feedforward, dynamic, ARX networks, Vector Quantization networks, etc. A neural network, as defined above, represents a specific family of parameterized maps. If there are n_0 input elements and n_L output elements, the network defines a continuous mapping NN: $\Re^{n_0} \rightarrow \Re^{n_L}$.

4 ANALYSIS OF KSE-100 INDEX RETURNS

The present work is an attempt for analyzing economic conditions for the current fiscal decade when Pakistan has undergone serious changes in its financial and political policies. Literature describing such policies and their impact on economy abounds. In a sense, the purpose of this work is three-fold; first, we attempt to find out regions of significant transient nonlinearity. Second, we use this information to construct models which may be used for short-term forecasting. Finally, we try to find out those events which might have been the reason for the observed transient nonlinearity.

A time-series T is obtained using $\{y_i\}_{1 \le i \le 2195} = \log(p_i / p_{i-1})$, where p_i represents the i^{th} day stock index, from October ′99 to August ′08. The autocorrelation function of T shows clearly that the series is first-order stationary. The partial autocorrelation function (PACF) plot of T reveals significant peak at lag 1, thus we compute H statistic for the residuals obtained after fitting an autoregressive model of unit order, AR(1). The minimum AICC Yule-Walker equation for the series T is given here as under:

$$y_t = 0.07 y_{t-1} + \xi_t \quad (5)$$

The value of AICC statistic is –12988.8. Based on this information, the residuals of the series T are used to construct frames each of length thirty as described in the previous section. Our results show

Table 1: Significant Frames with nonlinear behavior along with Major Events; Total Number of windows: 74; Significant H windows: 12 (16.2%).

Frame	Date	Major Events
F_1	11/16/99 - 12/28/99	A month after Military Coup (12th October 1999)
F_2	3/21/01 - 5/7/01	Ex-President Rafiq Tarar resigns, Ex- President Pervez Musharraf takes over (a month after the frame – this does not induce nonlinearity)
F_3	6/10/03 - 7/21/03	Khalid Sheikh Mahmood (suspected mastermind of 9/11) arrested (3 months before the nonlinear region)
F_4	4/15/04 - 5/27/04	• National Security Council Bill passed • EU decides to improve trade with Pakistan • Pakistan is back in Commonwealth
F_5	12/31/04 - 2/14/05	• Tsunami hits far-eastern countries • Ex-President Pervez Musharraf decides to keep his post as army • Pakistan Army opens fire on insurgents in Baluchistan, in the first armed uprising since General Rahimuddin Khan's stabilization of the province in 1978 (A month after this nonlinear region)
F_6	9/16/05 - 10/27/05	• Bombs exploded in KFC and McDonalds in Karachi • October 2005 Earthquake killing over 175, 000 people in Northern regions of Pakistan • Ex President Pervez Musharraf shakes hands with Israeli Prime Minister Ariel Sharon and later addresses American Jewish Society later in the month, creating a row
F_7	6/16/06 - 7/27/06	Waziristan war heats up (suicide bombings escalated)
F_8	12/13/06 - 1/26/07	Waziristan War (military retaliated)
F_9	6/11/07 - 7/20/07	• Operation on Lal Masjid (Islamabad) • Torrential rains in Sindh, Balochistan causing flood • Talks between Ex-President Pervez Musharraf and opposition leader Ex-Prime Minister (late) Benazir Bhutto take place in Dubai causing speculations of potential coalition
F_{10}	10/22/07 - 12/3/07	• Right after first assassination attempt on Ex-Prime Minister (late) Benazir Bhutto 18th Oct 2007 • Emergency imposed 3rd Nov 2007 • Ex-Prime Minister Nawaz Sharif makes a failed attempt to return to Pakistan • Ex-President Pervez Musharraf stands down as the head of Pakistan army
F_{11}	12/4/07 - 1/18/08	• Emergency lifted by Ex-President Pervez Musharraf • Ex-Prime Minister (late) Benazir Bhutto's assassinated on 27th Dec 2007, riots • Rumors of assassination of Ex-President Pervez Musharraf • General Elections delayed till February • General Elections held (exactly a month after the nonlinear region ends)
F_{12}	4/17/08 - 5/29/08	• Changing policy towards militants, talks with local Taleban • Ministers of PML(N) resign from Federal government • Lawyers movement continue

that out of around seventy three frames twelve are those in which significant nonlinearity is observed (see Table 1). The frames which are subjected to further analysis are those which are taken as common after estimating H statistic of residuals of AR(1) and AR(9) models, because the PACF at lag 1 and 9 show peaks beyond statistical significance. Assuming that F_i represents the i^{th} frame with $i = \{1, 2, \ldots, m=12\}$ containing values from T such that individual frame F_i takes consecutive values from T but frames F_i and F_j with $i \neq j$ do not necessarily come adjacent to each other in T. Thus, we can write $F_1 = \{y_{11}, y_{12}, y_{13}, \ldots, y_{1k}\}$, $F_2 = \{y_{21}, y_{22}, y_{23}, \ldots, y_{2k}\}$, and in general $F_m = \{y_{m1}, y_{m2}, y_{m3}, \ldots, y_{mk}\}$, where k is the frame length which is 30 in our case.

Each of these sets F_i is used as input to a feedforward neural network with lag 1. Obviously, each F_i is divided into training, validation and testing sets for modeling with neural network. We use gradient descent backpropagation algorithm for training with varying number of neurons and hidden layers. This way, we have come across having several neural network models for each of the frame F_i, however, for brevity, Table 1 describes only those neural networks which traced the behavior of input data comparatively well. On average, all the

Table 2: The results of *NN*-modeling.

	Feedforward - Backpropagation			Dynamic Network, Bayesian regularization						
Frame	MSE	Neurons	Epochs	SSE	Neurons	Epochs	Forecast		Original	
							Day-1	Day-2	Day-1	Day-2
F_1	3.65×10^{-4}	1	208	3.52×10^{-3}	4	11	0.012	0.007	0.021	0.0336
F_2	5.56×10^{-5}	1	34	1.08×10^{-3}	4	10	0.0051,	–0.0049	–0.0133	–0.0079
F_3	1.24×10^{-4}	1	1500	4.14×10^{-3}	4	9	0.004	0.003	–0.007	0.013
F_4	9.77×10^{-4}	1	81	1.13×10^{-3}	4	47	–0.001	–0.006	0.0047	–0.0009
F_5	9.93×10^{-4}	4	316	1.59×10^{-3}	4	9	0.014	0.0063	0.008	0.014
F_6	9.74×10^{-4}	1	78	1.8×10^{-3}	4	104	0.00994	0.0051	–0.0085	0.0227
F_7	9.91×10^{-4}	3	105	6.3×10^{-3}	4	8	0.0084	–0.0038	–0.0074	0.014
F_8	9.70×10^{-4}	1	104	1.6×10^{-3}	4	9	0.0067	0.0091	0.015	0.0067
F_9	9.43×10^{-4}	1	16	1.05×10^{-3}	4	10	0.0119	–0.0030	0.0228	–0.0079
F_{10}	9.65×10^{-4}	1	78	3.39×10^{-3}	4	13	–0.0054	–0.0066	0.0081	0.0072
F_{11}	9.99×10^{-4}	1	145	2.5×10^{-3}	4	9	0.0146	–0.0032	–0.0066	0.0020
F_{12}	9.77×10^{-4}	1	86	2.0×10^{-3}	4	9	–0.0016	–0.0084	0.0123	0.0160

neural networks are found to have mean square error ~ 10^{-4}. To improve the Day-1 and Day-2 out-of-sample forecast we employ a dynamic neural network with Bayesian Regularization Learning. This has greatly improved our results and at several points we find a very little difference between the observed and the forecasted values. Besides there are deviations of long magnitude which are probably due to less amount of training data, only around 10 points in each frame.

5 CONCLUSIONS

The Hurst Exponent (*Hu*) of the series *T* is found to be 0.6 which shows a slight effect of long memory with persistence but the process, in general, may also be considered as a first-order autoregressive process. However, the regression is contributed by only a very less amount of previous value (see equation 5), and as a whole the process should be treated to be governed by white noise. And that is the reason why we obtain the *Hu* close to 0.5, on contrary to 1.0.

The windowing-approach is found to give satisfactory results as the most effective and significant events (see Table 2) that have affected the country's economical growth, political stability and international relations happened during our detected nonlinear regions.

The forecasting results obtained after applying a dynamic neural network with Bayesian regularization learning supersedes the conventional feedforward-backpropagation network. We think that the major obstacle against good forecasting is the amount of data which is limited to ten in a single frame due to the frame length, or actually the length of the transient period in which nonlinearity is significant. Finding the global parameters of the dynamics of the involved process should be an interesting problem to attempt.

REFERENCES

Kantz, H & Schreiber, T 1997, *Nonlinear time series analysis*, University Press, Cambridge

Grassberger, P, & Procaccia, I 1983, 'Measuring the strangeness of strange attractors'. *Physica D,* vol. 9, pp. 189.

Grassberger, P. & Procaccia, I. 1983a, Characterization of strange attractors. *Phys. Rev. Lett.,* vol. 50*,* pp. 346.

Barnett, W.A., Gallant, A.R., Hinich, M.J., Jungeilges, J.A., Kaplan, D.T. & Jensen, M.J. 1996, An Experimental Design to Compare Tests of Nonlinearity and Chaos, Nonlinear Dynamics and Economics, in *Proceedings of The Tenth International Symposium in Economic Theory and Econometrics*, Eds. W. A. Barnett, A.P. Kirman, and M. Salmon, Cambridge University Press, New York, NY

Lim, K.P. & Hinich, M. J. 2005, Cross-temporal universality of non-linear dependencies in Asian stock markets, *Economics Bulletin* vol. 7, no. 1, pp. 1-6.

Hinich, M. J. & Patterson D. M. 1995, Detecting epochs of transient dependence in white noise. *Mimeo, University of Texas at Austin*.

Danial, S. N., Noor, S. R., Usmani, B. A., & Zaidi, S.J.H. 2008, A Dynamical System and Neural Network Perspective of Karachi Stock Exchange Returns. in *International Multi – Topic Conference IMTIC 2008* , Dr. M. Akbar Hussain, Abdul Qadeer Khan Rajput, Bhawani Shankar Chowdhry and Quintin Gee (Eds.), Mehran University Jamshoro, Pakistan. April 11-12, 2008. Revised Selected Papers. Wireless Networks, Information Processing and Systems, *Communications in Computer and Information Science* vol. 20, pp. 88-99, Springer-*Verlag*, Heidelberg. For Online version:

http://www.springerlink.com/content/978-3-540-89852-8?sortorder=asc&p_o=10

Burni, S. M. A., Jilani, T. A. & Ardil C. 2004, Levenberg-Marquardt Algorithm for Karachi Stock Exchange Share Rates Forecasting. *Proceedings of World Academy of Science, Engineering and Technology* vol. 3**,** (ISSN1307-6884).

Antoniou, A., Ergul N. & Holmes P. 1997, Market efficiency, thin trading and non-linear behavior: evidence from an emerging market. *European Financial Management,* vol. 3, no. 2, pp. 175-190.

Sarantis, N. 2001, Nonlinearities, cyclical behaviour and predictability in stock markets: international evidence. *International Journal of Forecasting,* vol. 17, pp. 459-482.

Hinich, M.J. & Patterson, D. M. 1985, Evidence of nonlinearity in daily stock returns. *Journal of Business and Economic Statistics,* vol. 3, pp. 69-77.

Bonilla, C. A., Romero-Meza, R. & Hinich, M. J. 2006, Episodic nonlinearity in Latin American stock market indices. *Applied Economics Letters,* vol. 13, no. 3, pp. 195-199.

Abhyankar, A.H., Copeland L. S. & Wong W. 1995, Nonlinear dynamics in real-time equity market indices: evidence from the United Kingdom. *Economic Journal,* vol. 105, pp. 864-880.

Opong, K.K., Mulholland, G., Fox, A. F. & Farahmand, K. 1999, The behavior of some UK equity indices: an application of Hurst and BDS tests. *Journal of Empirical Finance,* vol. 6, pp. 267-282.

Brooks, C. & Hinich M. J. 1998, Episodic nonstationarity in exchange rates. Applied Eco*nomics Letters,* vol. 5, pp. 719-722.

Ammermann, P.A. & Patterson D. M. 2003, The cross-sectional and cross-temporal universality of nonlinear serial dependencies: evidence from world stock indices and the Taiwan Stock Exchange. *Pacific-Basin Finance Journal,* vol. 11, pp. 175-195.

Lim, K.P., Hinich, M. J. & Liew, V. K. S 2003, Episodic non-linearity and non-stationarity in ASEAN exchange rates returns series. *Labuan Bulletin of International Business andFinance,* vol. 1, no. 2, pp. 79-93.

Lim, K.P. & Hinich, M. J. 2005, Cross−temporal universality of non−linear dependencies in Asian stock markets. *Economics Bulletin***,** vol. 7, no. 1, pp. 1−6

Lim, K.P. & Hinich, M. J. 2005a, Non−linear Market Behavior: Events Detection in the Malaysian Stock Market. *Economics Bulletin,* vol. 7, no. 6, pp. 1−5

Wild, P., Hinich, M. J., Foster, J. 2008, Are Daily and Weekly Load and Spot Price Dynamics in Australia's National Electricity Market Governed by Episodic Nonlinearity, Discussion Paper, University of Queensland, pp. 6-10.

Cybenko, G. (1989), Approximation by Supervisions of a Sigmoidal Function. *Mathematics of Control, Signals, and Sys.,* vol. 2, pp. 303–314.

Hornik, K, Stinchcombe, M, & White, H 1989, Multilayer feed-forward networks are universal approximators, *Neural Net.,* vol. 2, pp. 359–36