

TEMPORAL MINING IN IMPRECISE ARCHÆOLOGICAL KNOWLEDGE

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Abstract: In this paper, we propose a new temporal data mining method considering a set of archæological objects which are temporally represented with fuzzy numbers. Our method uses an index which quantifies the anteriority between two fuzzy numbers for the construction of a weighted oriented graph. The vertices of the graph correspond to the temporal objects. Using this anteriority graph, we estimate the potential of anteriority, of posteriority and the relative temporal position of each object. We focus on excavation data from the ancient Reims stored in a Geographical Information System (GIS). We visualize the discovered temporal positions of objects and weighted relations between them in a layer of the GIS.

1 INTRODUCTION

Time is crucial for the study and the analysis of archæological knowledge. Using a Geographical Information Systems (Conolly and Lake, 2006), temporal information should be taken into consideration by analysts as the spatial configuration is. Thus, processes should consider temporal information in its complexity and quality. For instance, dates or activity periods usually results form interpretation and thus are imprecise. Furthermore, dates of some particular objects or elements are hard to estimate. Fuzzy set theory, introduced in (Zadeh, 1965), proposes a formalism to represent imprecise knowledge. In this paper, dates and activity periods are represented by fuzzy numbers (FNs).

In the analysis of archæological information, it may be natural to estimate the temporal positions of objects (*i.e.* the object ranks in a chronological view). Those positions give us new knowledge over the object information. This new knowledge helps experts and us to comprehend archæological information. As objects are temporally represented by fuzzy numbers, the method exposed in this article aims to compute the temporal position of each object related to the database objects.

In order to define those positions, a temporal relation must be used. This relation should be an order relation, but no total order relation could be defined over fuzzy numbers. The approach used when

comparing two dates, which generates a binary decision (“after” or “before”), is not suitable. Trustworthiness is an important aspect. Moreover, set ranking approaches consider reference sets defined on the studied set of fuzzy numbers (Chen, 1985; Jain, 1977; Kerre, 1982). Those approaches do not consider information given by the pairwise comparison which is essential in a temporal context.

We introduced in (de Runz et al., 2009) an anteriority index which quantifies the result of the following question “is this element anterior to this other one?”. As dates are represented with fuzzy numbers, the index value is calculated from the FNs using Kerre’s approach (Kerre, 1982) and takes value in $[0, 1]$.

Using this index, this article presents a new data mining process which allows us to define the temporal positions according to the pairwise anteriority. This archæological analysis process is based on a weighted oriented graph called “anteriority graph”. In this graph, a vertex is associated to each object and each pair of vertices is linked by two arcs weighted by the values of the anteriority index.

This graph allows us to compute the potential of anteriority and posteriority¹ of each object through its associated vertex. The difference between those potentials determines the temporal index of the object.

¹The potential of anteriority (or posteriority) of an object quantifies the way that it could be anterior (or posterior) to others in the set of objects.

A ranking of temporal index values over all objects from database assigns to each object a rank which is its temporal position in the database. The anteriority graph will allow a synthetic and formal representation of temporal structures in the set of archaeological objects stored in a spatiotemporal database.

The application context of this work is the SIGRem project (de Runz et al., 2007b; de Runz et al., 2007a). We propose in this article to use our data mining approach on the set of BDFRues objects. In this spatiotemporal database, objects represent the Roman streets found in Reims and are stored with a fuzzy representation of their activity periods. To obtain a visualization of the temporal relations and positions, the locations of objects are used to build a layer in a GIS and thus to produce some maps.

This paper is organized as follows. Firstly, the anteriority index is presented. Secondly, the definition of the anteriority graph and our data mining approach are exposed. Thirdly, our approach is illustrated on an application in an archaeological GIS. Finally, the conclusion of this work is given.

2 ANTERIORITY INDEX

In the following, a fuzzy number is a convex and normalized fuzzy subset on the set of real numbers \mathbb{R} . According to (Wang and Kerre, 2001a; Wang and Kerre, 2001b), when, for the comparison, some of methods use valuations of involved FNs (Fortemps and Roubens, 1996), some others combine indices (Saade and Schwarzlander, 1992). Those kinds of methods are often not transitive (Wang et al., 1995). Another kind of methods exploits a reference set defined on FNs (Chen, 1985; Jain, 1977; Kerre, 1982). In this case, according to the set Ω of FNs $\{A_1, A_2, \dots, A_n\}$, the first step consists in computing the fuzzy reference set and then to value each FN A_i by the calculation of the value of an index considering the reference set and A_i . The comparison of the index values allows us to define the ranking.

For example, considering a set of n fuzzy numbers $\{A_1, A_2, \dots, A_n\}$, Kerre proposes in (Kerre, 1982) to compare two fuzzy numbers (A_i and A_j where $i, j \in [1, n]$) by comparing the Hamming distances between those fuzzy numbers and the maximum, defined by the Zadeh's extension principle (Zadeh, 1965), of $\{A_1, A_2, \dots, A_n\}$. The Hamming distance between A_i (resp. A_j) and the maximum \widetilde{max} of (A_1, A_2, \dots, A_n) is called Kerre's index $K(A_i)$ (resp. $K(A_j)$).

Thus, Kerre's index of A_i in $\{A_1, A_2, \dots, A_n\}$ is ob-

tained as follows:

$$K(A_i) = D_H(A_i, \widetilde{max}(A_1, A_2, \dots, A_n)) \quad (1)$$

Thus

$$K(A_i) = \int |A_i(x) - \widetilde{max}(A_1, A_2, \dots, A_n)(x)| dx. \quad (2)$$

For Kerre, $A_i = A_j$ according to $\{A_1, A_2, \dots, A_n\}$, with $(i, j) \in [1, n]$, iff $K(A_i) = K(A_j)$.

In this kind of approach, the meaning of pairwise comparison is not taken into consideration. Thus, we would first use the Kerre's approach for pairwise comparison, but the use of the Kerre's index in pairwise comparison can produce some non-transitive decision in the goal to rank three or more fuzzy numbers.

In order to reduce the impact of those kinds of inconsistencies during data exploitation, we build an index which quantifies the anteriority between two dates represented by fuzzy numbers.

When comparing fuzzy numbers, the key idea of Kerre's approach is, considering a set of fuzzy numbers, the higher the Kerre's index of a fuzzy number is, the lower the fuzzy number will be. We propose to use Kerre's index for a set of two FNs to define a relative index, because the goal is not only to compare a pair of dates but also to evaluate the comparison by an anteriority index

Indeed, let F and G be two fuzzy numbers, if F is equal to the maximum according the extension principle, then the proposition " F is lower than G " is true, thus the value of the anteriority index of F regarding G must be equal to 1. When G is equal to the maximum and F is not, then the proposition " F is lower than G " is false, thus the value of the anteriority index of F regarding G must be equal to 0. In other cases, the sum of the values of our index for the couple (F, G) and the couple (G, F) must be equal to 1. So we define our index on the restriction to the subset of those two fuzzy numbers as follows:

$$Ant(F, G) = \begin{cases} \frac{K(F)}{K(F)+K(G)} & \text{if } K(F) + K(G) = 1 \\ 1 & \text{if } K(F) + K(G) = 0 \end{cases} \quad (3)$$

As the Kerre's index could not take a negative value, the case $K(F) + K(G) < 0$ could not exist.

$Ant(F, G)$ is a quantification of the logical relation $F = G$. $Ant(F, G)$ is both an index of closeness between G and $\widetilde{max}(F, G)$ using Hamming distance and an index of closeness between F and $\widetilde{min}(F, G)$, where \widetilde{min} is defined by the extension principle. So, the anteriority index allows us to qualify the anteriority and the posteriority between two dates dF and dG represented by two FNs, F and G , as follows:

- $Ant(F, G) = 0 \Rightarrow$ " dF is not anterior to dG " and " dG is not posterior to dF ",

- $0 < Ant(F, G) \leq 0.5 \Rightarrow$ “ dF is rather not anterior to dG ” and “ dG is rather not posterior to dF ”,
- $0.5 \leq Ant(F, G) < 1 \Rightarrow$ “ dF is rather anterior to dG ” and “ dG is rather posterior to dF ”,
- $Ant(F, G) = 1 \Rightarrow$ “ dF is anterior to dG ” and “ dG is posterior to dF ”.

We can also note that iff $Ant(F, G)$ is equal to 0.5 then $Ant(G, F) = 0.5$. In this case, each fuzzy number is as close to the minimum as to the maximum. Thus, the decision “ dF is rather anterior to dG ” is as possible as the decision “ dG is rather anterior to dF ”.

3 ANTERIORITY GRAPH

In this section, the construction of the anteriority graph is studied. This study is illustrated using the following example: let Ω a set of archaeological objects $\{A_1, A_2, A_3\}$ which are temporally represented by respectively $A_1.fDate$, $A_2.fDate$ and $A_3.fDate$ presented in Figure 1.

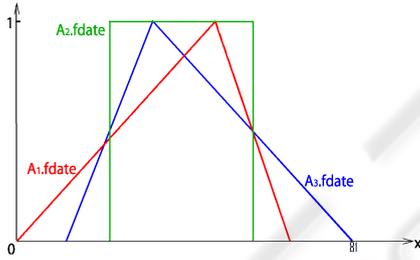


Figure 1: Membership functions of $A_1.fDate$, $A_2.fDate$ and $A_3.fDate$.

3.1 Graph Construction

Let Ω be a set of elements, \mathfrak{R} a binary relation over Ω and App an application from \mathfrak{R} to \mathbb{R} .

A weighted directed graph $G_{\mathfrak{R}}(L_S, L_A, L_C)$ can be used to provide a schematic representation of \mathfrak{R} where L_S is the set of vertices ($A \in \Omega \Leftrightarrow S_A \in L_S$), L_A is the set of edges ($A \mathfrak{R} B \Leftrightarrow (S_A, S_B) \in L_A$) and L_C is the set of costs ($C(S_A, S_B) = App(A, B) \in L_C$).

The exploratory analysis of our work is based on such a graph. Ω is a set of archaeological data with a temporal feature $fDate$ expressed as a fuzzy number. The application is the anteriority index defined previously ($App \equiv Ant$).

As all the elements of Ω are pairwise connected, the obtained graph is complete. To each element A_i , with activity period $A_i.fDate$, a vertex S_{A_i} is associated. The weight of arc (A_i, A_j) is the value of the

anteriority index $Ant(A_i.fDate, A_j.fDate)$ and it represents the anteriority of A_i with regard to A_j . As example, let us consider the set $\Omega = \{A_1, A_2, A_3\}$. On this set, the values of the anteriority index are:

- $Ant(A_1.fDate, A_2.fDate) = 0.44$,
- $Ant(A_2.fDate, A_1.fDate) = 0.56$,
- $Ant(A_1.fDate, A_3.fDate) = 0.51$,
- $Ant(A_3.fDate, A_1.fDate) = 0.49$,
- $Ant(A_2.fDate, A_3.fDate) = 0.50$,
- $Ant(A_3.fDate, A_2.fDate) = 0.50$.

On this set, we build the anteriority graph $G_{Ant}(\Omega, \Omega \times \Omega, [0; 1])$ illustrated in Figure 2.

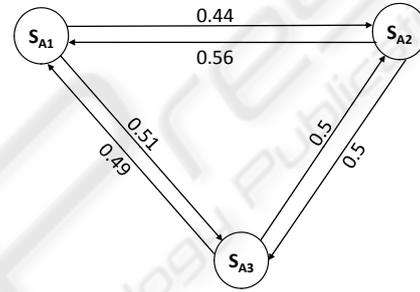


Figure 2: Anteriority graph with $\Omega = \{A_1, A_2, A_3\}$.

3.2 Potential of Anteriority, Posteriority and Temporal Position

Let S_{A_i} and S_{A_j} two vertices of the anteriority graph which corresponds respectively to the A_i and A_j objects, the cost $C(S_{A_i}, S_{A_j})$ is the quantification of the anteriority of A_i to A_j .

The sum of the costs of the arcs with A_i as initial node is called the potential of anteriority:

$$PotAnt_{\Omega}(A_i) = \sum C(S_{A_i}, S_{A_j}), \forall S_{A_j} \neq S_{A_i}. \quad (4)$$

In the example of $\Omega = \{A_1, A_2, A_3\}$, using the anteriority graph presented in Figure 2, we obtain: $PotAnt_{\Omega}(A_1) = 0.94$, $PotAnt_{\Omega}(A_2) = 1.06$, $PotAnt_{\Omega}(A_3) = 0.99$.

The sum of the costs of the arcs with A_i as terminal node is called the potential of posteriority:

$$PotPost_{\Omega}(A_i) = \sum C(S_{A_j}, S_{A_i}), \forall S_{A_j} \neq S_{A_i}. \quad (5)$$

In the example of $\Omega = \{A_1, A_2, A_3\}$, using the anteriority graph presented in Figure 2, we obtain: $PotPost_{\Omega}(A_1) = 1.05$, $PotPost_{\Omega}(A_2) = 0.94$, $PotPost_{\Omega}(A_3) = 1.01$.

The goal of this work is to propose a data mining process to obtain the temporal positions and relations

of archaeological objects stored in a spatiotemporal database. In order to obtain the position, the temporal index ($TempInd_{\Omega}$) is defined as the difference between the posteriority and anteriority potential:

$$TempInd_{\Omega}(A_i) = PotPost_{\Omega}(A_i) - PotAnt_{\Omega}(A_i). \quad (6)$$

In the example $\Omega = \{A_1, A_2, A_3\}$, the temporal index values are: $TempInd_{\Omega}(A_1) = 0.10$, $TempInd_{\Omega}(A_2) = -0.12$, $TempInd_{\Omega}(A_3) = 0.02$.

Using the temporal index, we propose to temporally rank the archaeological objects. Those ranks are called temporal position ($TempPos_{\Omega}$) and are obtained using the following principle:

$$\begin{aligned} \text{If } TempInd_{\Omega}(A_i) > TempInd_{\Omega}(A_j) \\ \text{then } TempPos_{\Omega}(A_i) > TempPos_{\Omega}(A_j). \end{aligned} \quad (7)$$

In the example $\Omega = \{A_1, A_2, A_3\}$, the temporal positions are: $TempPos_{\Omega}(A_1) = 2$, $TempPos_{\Omega}(A_2) = 0$, $TempPos_{\Omega}(A_3) = 1$.

3.3 Object Analysis through Anteriority Graph

Using the anteriority graph, we can extract three particular temporal objects: the most anterior object, the most posterior object and the temporally median object.

The oldest object in the application, i.e. the most anterior object (MA), is the one with the lowest temporal index value in Ω . The temporal position of MA is then the minimal temporal position of objects in Ω . In the example $\Omega = \{A_1, A_2, A_3\}$, $MA = A_2$.

The most recent object - the most posterior (MP) - is the one with the highest temporal index value in Ω . The temporal position of MP is the maximal temporal position of objects in Ω . In the example $\Omega = \{A_1, A_2, A_3\}$, $MP = A_1$.

From the ranking process used to define the temporal positions of archaeological objects, it is trivial to extract the median temporal object (MT). This object has the median value of temporal index in the set of temporal index values obtained on Ω . In the example $\Omega = \{A_1, A_2, A_3\}$, $MT = A_3$.

Moreover, an object of Ω with a negative temporal index value may be considered as a ‘‘rather anterior’’ object in Ω . A positive temporal index value could be interpreted as a ‘‘rather posterior’’ object. Thus, we propose to split the set of elements into two subsets: ‘‘rather anterior’’, ‘‘rather posterior’’. In the example $\Omega = \{A_1, A_2, A_3\}$, A_2 is ‘‘rather anterior’’ but A_1 and A_3 are ‘‘rather posterior’’.

The anteriority graph construction is an original approach to rank fuzzy numbers and also archaeological objects according to the fuzzy numbers representing their activity periods. This graph offers a global

vision of temporal relations between archaeological objects. It gives information for the classification objective and for the analysis at a local scale (the excavation site) or a global (the city) scale.

4 APPLICATION

During an archaeological analysis process, the study of object temporal positions according to their locations is essential. In order to exploit the spatial aspect of data, Conolly and Lake propose to record the information from archaeological excavations using GIS (Conolly and Lake, 2006).

In this section, we will use a GIS for the spatial visualization of the object temporal positions and labels.

4.1 Context

In the perspective of the promotion and the management of Reims archaeological patrimony, the SIGRem project, carried out by the University of Reims Champagne-Ardenne, the INRAP (National Institute in Preventive Archaeological Research) and the Culture Ministry, integrates the geo-informatics tools and takes in consideration the archaeological information in the urban and regional analysis.

To achieve this objective, the first goal of the SIGRem project is to develop a geographical information system to manage archaeological knowledge. This system should propose and present some ad-hoc spatiotemporal analysis tools. The temporal mining process presented in this paper is now applied in this context and specifically to the BDFRues database.

This database stores data about the Reims roman streets and is based on excavation. The activity periods of stored objects are represented with fuzzy numbers. A visualization of those fuzzy numbers is proposed in Figure 3.

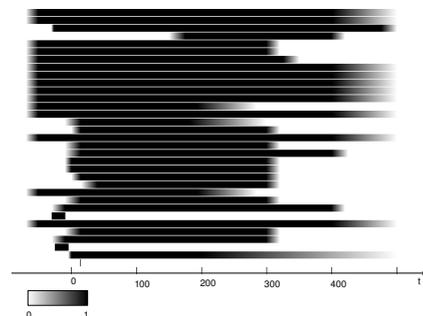


Figure 3: Fuzzy numbers representing BDFRues object activity periods.

4.2 Temporal Positions and Labels

According to the anteriority graph construction and the object locations, we obtain for the BDFRues elements the temporal positions visualized by a layer of the GIS (Figure 4). In this figure, the higher the temporal position, the higher the value of temporal index, according to all others.

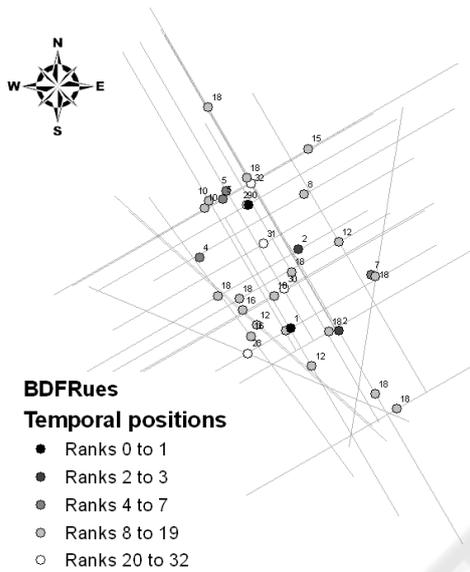


Figure 4: Temporal positions of BDFRues objects.

We can remark that the ranking from temporal positions are in some cases different to the Jain's and Kerre's ranking (Figure 5 and Figure 6).

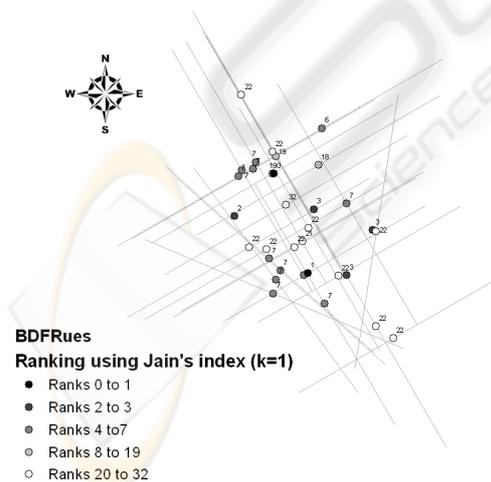


Figure 5: Jain's ranking, with $k = 1$, of BDFRues objects according to their activity periods.

However, the temporal position has a more temporal interpretability than Jain's or Kerre's rank, which allows us to assign temporal labels to objects such as,

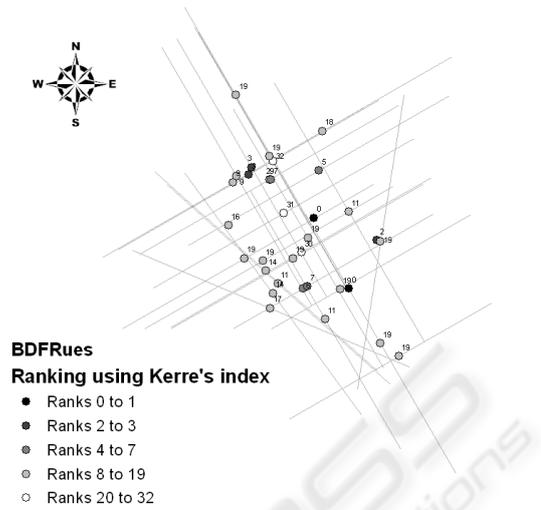


Figure 6: Kerre's ranking of BDFRues objects according to their activity periods.

for instance, the most anterior, the most posterior and the temporally median objects localized in Figure 7.

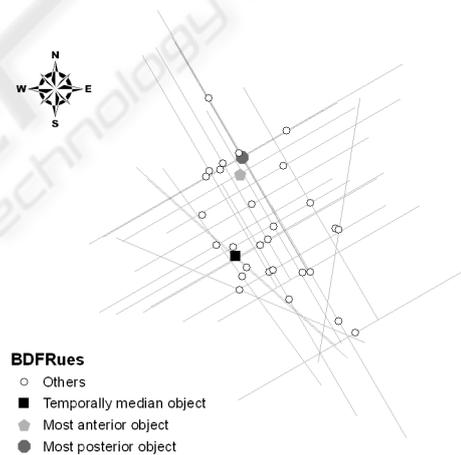


Figure 7: Particular temporal positions of BDFRues objects.

The Figure 8 presents the map of BDFRues elements grouped in two groups: "rather anterior" and "rather posterior".

In this application, there are two times more objects in the group "rather posterior" than in the group "rather anterior". Indeed, the study of the temporal index values gives the following information:

- the value of temporal index of the most anterior object is -29.8,
- the highest value of temporal index of "rather anterior" objects is -5.4,
- more than 72% of "rather posterior" objects have a temporal index value higher than 6.

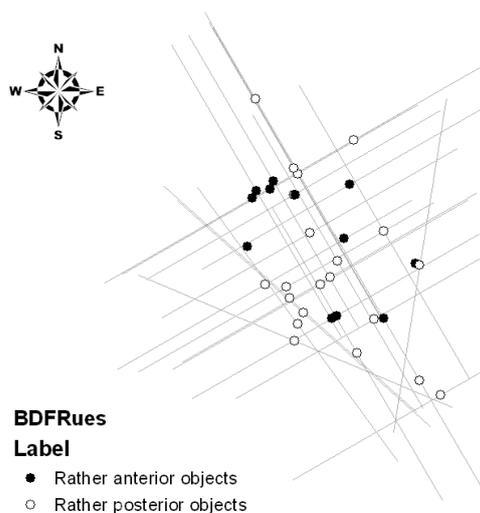


Figure 8: BDFRues objects grouped in “rather anterior” and “rather posterior”.

According to the comparison between Figure 7 and Figure 8, we remark that the temporally median object is a member of the “rather posterior” objects. Moreover, considering that the anteriority index values between “rather anterior” objects and “rather posterior” objects are very often close to 1, the bipolarization of objects seems pertinent.

5 CONCLUSIONS

In this article, in order to analyze the temporal relations between archaeological objects, we proposed a new data mining process. It is based on the construction of a weighted oriented graph over the database objects using the anteriority index values pairwise linking all the objects. According to the graph, the process computes the temporal index value for each object and, using them, the temporal position of objects. By the visualization of results in a layer of a GIS, the combination of spatial distribution and temporal ranking facilitates the spatiotemporal analysis of objects.

During the expertise, the archaeologists estimate the functions, the activity periods of objects which are found in an excavation site. They need to study the temporal relation between objects in databases in order to (i) look if the temporal logic is respected and (ii) analyze the city temporal evolution. Thus, in this objective, the data mining process proposed in this article may be used.

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