

# USING ATTAINMENT SURFACE FOR COMPARING NSGA-II AND SPEA-II

## *A Case Study*

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**Abstract:** This paper presents a comparative analysis of the results obtained with two different genetic algorithms, NSGA-II and SPEA-II, in the framework of load management activities in electric power systems. The multiobjective problem deals with the identification and the selection of suitable control strategies to be applied to groups of electric loads aimed at reducing maximum power demand at the sub-station level, maximizing profits with selling of electricity and minimizing the discomfort caused to the end-users. The comparative analysis of the algorithms' performance is done based on the attainment surface approach. Besides, it is shown that this approach can be used as a vehicle to introduce the decision maker's preferences in the evaluation process.

## 1 INTRODUCTION

In single objective optimization problems, in which the merit of solutions is evaluated in just one axis, it is easy to find a metric that allows different solutions to be compared and ranked even if the optimum is not known. However, in a multiobjective (MO) environment a solution is evaluated according to its performance on each of the multiple, conflicting and incommensurate objectives under optimization. Typically, an "optimal" point, in the sense of optimizing simultaneously all objective functions, is an infeasible solution in the objective function space and it does not even exist in the decision variable space. Therefore, the decision maker (DM) must choose a final solution from the set of non-dominated solutions - the Pareto front (PF) - according to his/her preferences. However, in MOOP, although the non-dominance concept is the key one, it is also a "poor" one in the sense that it does not enable discrimination between nondominated solutions. In order to distinguish between two non-dominated solutions the DM's preference structure must be taken into

consideration. The need for taking into account the DM's preferences also arises when comparing sets of potential solutions.

In this context, an important issue is what approach to use for assessing and comparing sets of potential solutions. Several approaches have been reported in the literature regarding the evaluation of solutions and algorithms (Ang and Li, 2001) (Brockhoff et al., 2008) (Fonseca and Fleming, 1996) (Hansen and Jaskiewicz, 1998) (Knowles and Corne, 2000, 2001) (Knowles et al., 2006) (Zitzler et al., 2000, 2003, 2008). Genetic Algorithms (GA) while tools to compute potential solutions to a given problem should be able to identify diverse and well-spread solutions over the Pareto frontier and, at the same time, solutions should be as close as possible to the true Pareto optimal frontier (POF) (Zitzler et al., 2000).

The identification and choice of a metric is not easy, at least in situations where neither an optimal point nor the true Pareto front are known - that is, whenever no references exist that can be used for assessing the quality of the results obtained.

In this work the attainment surface metric (AS) is used to compare the performance of two genetic

algorithms to deal with the design and identification of load management actions in electrical power systems. It is shown that it is possible to use the AS metric also as a tool to incorporate the DM's preferences when comparing sets of solutions. In section 2 an overview about algorithm evaluation is presented. In section 3 the problem under analysis is briefly described, while the case study is introduced in section 4. The analysis of the results is done in section 5 and conclusions are drawn in section 6.

## 2 PERFORMANCE ASSESSMENT

When the true POF is not known, which is a common situation in real-world MOOP, reference alternatives have been used in order to overcome the problems resulting from not knowing exactly the Pareto front (Ang and Li, 2001)(Gomes et al., 2008). Evaluating the results of a multiobjective problem is itself a multiobjective problem, and intensive research work is being carried out in order to deal with the assessment of solutions in MOOP (Branke et al., 2001) (Knowles and Corne, 2000, 2006)(Zitzler et al., 2000, 2008). Recently some researchers focussed their work on comparing sets of solutions or populations (Zitzler et al., 2003, 2008).

In the problem under study in this work, as in many real-world problems, the POF is unknown and the diversity and the spreading of solutions are not to be taken for granted. Therefore, the assessment of the algorithms performance is carried out by using the AS concept described in Fonseca and Fleming (1996) and extended by Knowles and Corne (2000). If the non-dominated solutions resulting from a multiobjective optimizer are the points  $P_1, P_2, \dots, P_n$  (Figure 1) then the attainment surface is the surface limited by the lines joining the points. This surface divides the search space into two regions: DR is the region of the search space dominated by solutions computed by the algorithm, while NDR is the region of the search space non-dominated by the solutions computed by the algorithm. As proposed by Fonseca and Fleming (1996), if a set of lines (Figure 1.b) equally spread are drawn starting from the origin towards the AS then we can compare the distance of each AS to the origin and identify the AS that is first intersected by each line. The number of times each AS is first intersected by different lines enables to compare the sets resulting from each algorithm.

Knowles and Corne (2000) extended this analysis in the following manner. Having  $m$  runs from each algorithm each line has  $2m$  intersections and a statistical univariate analysis on the

distribution of the intersections can be done providing a measure about the performance of the algorithms in the region of the space represented by each line. These authors use a pair of values  $(a, b)$ , in which  $a$  represents the percentage of the space (lines) in which the first algorithm performs better than the second algorithm and  $b$  represents the percentage of the space (lines) in which the second algorithm performs better than the first one. If the set of lines cover the whole front then the attainment surface method allows to deal with the three issues raised in the performance assessment: distance to the POF, distribution and diversity of the solutions.

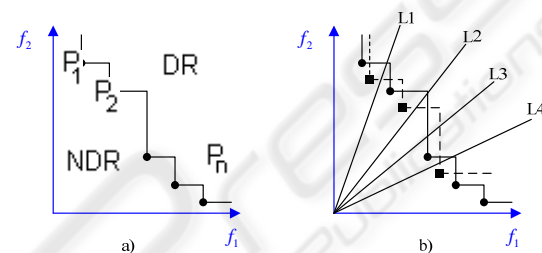


Figure 1: (a) Attainment surface, in a problem with two objective functions. (b) Two different attainment surfaces resulting from 2 different runs (two different algorithms or the same algorithm with a different set of parameters).

The analysis done using the AS is carried out after the simulations of the GAs, and two different analyses are possible: one carried out in a generational basis all over the simulation and the other based on the last generation populations only.

Very often the POF computed is very large and may present many solutions that are not interesting from the DM's perspective. An interesting situation is the one in which it is possible to take into account the DM's preferences when comparing the populations. It is possible to introduce thresholds representing the DM's preferences and then computing the AS metric taking into account those threshold levels. That is, the AS can be used as a tool allowing both to compare the performance of GAs and at the same time to incorporate the DM's preferences in the analysis, thus resulting in a reduced set of solutions more in accordance with the DM's preferences. The DM is asked to provide aspiration and/or reservation levels and the AS takes these levels into account. The aspiration level represents the value that the DM would like to attain in each objective function, thus leading to the identification of a region in the search space containing the "better" solutions according to the DM's preferences. The reservation levels represent the worst values the DM is willing to accept for each

objective function, according to his/her knowledge about the problem domain. Solutions that do not attain these levels may be eventually penalized and would be hardly chosen. A third type of threshold may be considered denoting a “non-feasible” level, meaning that a solution is “unfeasible” according to the DM’s preferences (that may evolve over the simulation/generations). These non-feasible levels can be taken into consideration in the MO model by imposing hard constraints at the outset, which may later be removed or revised. In some real-world problems the best way to deal with non-feasible solutions is through the penalization of their fitness, and thus give them low reproduction probability. These solutions can exist (preferably in very low number) in the population. These preference-driven thresholds influence the assessment of the algorithms done by the AS (Figure 2 and Figure 3). In figure 2, algorithm B performs better than algorithm A in 60% of the search space. However, when taking into consideration the DM preference levels the amount of space in which algorithm B performs better than algorithm A changes drastically. One can say that A performs better in 85% of the “space” that is more interesting from the DM perspective, that is, 85% of the space delimited by the reservation levels of the DM.

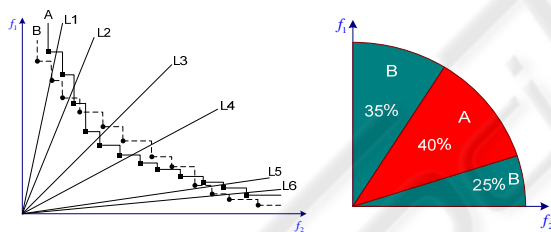


Figure 2: AS metric as a way to compare the algorithm performance.

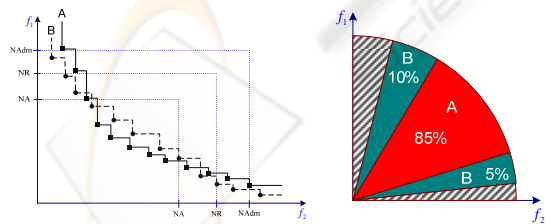


Figure 3: AS metric, taking into consideration the DM’s preferences.

### 3 CASE STUDY

With the restructuring and unbundling of the electricity sector, a common scenario is the one in

which electricity retailers buy the electricity in the wholesale market and sell the electricity to the end-users in the retail markets. Very often, the electricity prices at the wholesale market change more frequently and more intensely than at the retail market. As most consumers buy electricity at fixed costs (over a period of time) if retailers have the ability to appropriately change their customer’s load then they can take advantage of the difference of prices between the wholesale market and the retail market. As energy profits depend on the amount of energy sold and the maximum peak demand, the application of control strategies to some end-use loads changes the demand patterns and thus changes the profits per unit of energy sold. The aim is to identify adequate on/off periods (load control actions that change the regular working cycle of end-use loads) to be implemented on a daily basis that allow, at the same time, to reduce the maximum power demand and to increase profits without imposing a severe discomfort to the end-users. These competing objectives make the design and selection of the direct load control actions a hard combinatorial multiobjective problem that can be tackled by EAs.

The objective of this work is to compare the performance of NSGA-II and SPEA-II when used in the identification of the direct load control action to be applied over several groups of air conditioners in order to reduce the maximum demand of a substation. The topmost issue in these activities is the identification of suitable on/off periods to be applied over some end-use loads (Heffner and Goldman, 2001)(Hirst and Kirby, 2001)(Gomes et al., 2004) (Molina et al., 2004).

In this case, the pursued objectives are minimization of maximum peak power demand in a given substation (PD); maximizing profits with selling of electricity (Pr); and minimizing the eventual discomfort caused to the consumers, measured as the maximum interval of time (MI), in minutes, in which the room temperature was above the comfort temperature.

Usually, the implementation of activities involving the use of demand-side resources requires combining several end-use loads in groups and applying a given control strategy (set of all on/off control periods) over each group of loads 800 air conditioners were identified as available for control and have been grouped as shown in table 1.

The maximum peak demand at the substation is about 17769 kW, at 15:00h, and the maximum demand of all air conditioners under control is about 2280 kW also occurring at 15:00h.

Some characteristics of the algorithm are:

- Size of the population: 30
- Stop condition: 7000 generations
- Crossover probability
  - NSGA-II: 0.1; SPEA-II: 0.7
- Mutation probability: adaptive control.

Table 1: Groups of air conditioners under control.

Groups	1	2	3	4	5	6	7
# loads	40	60	50	100	75	75	100
Groups	8	9	10	11	12	13	14
# loads	15	20	30	100	50	10	75

The values for the crossover probability in the two algorithms are the ones that allowed obtaining the best results in each case.

The implementation of the algorithms is slightly different from the original implementations. The main difference is the way the mutation operator is constructed. In our implementation, instead of being a fixed value the mutation operator presents an adaptive control behaviour, in such a way that the mutation probability of one gene may be different from the mutation probability of other genes in the same individual. Also, the mutation probability of genes in one individual may be different from the mutation probability of the genes in other individuals in the population. Moreover, as the binary alphabet has been used for the encoding of individuals two different mutations can occur ( $0 \rightarrow 1$  and  $1 \rightarrow 0$ ), and these two mutations may occur with different probabilities (Figure 4).

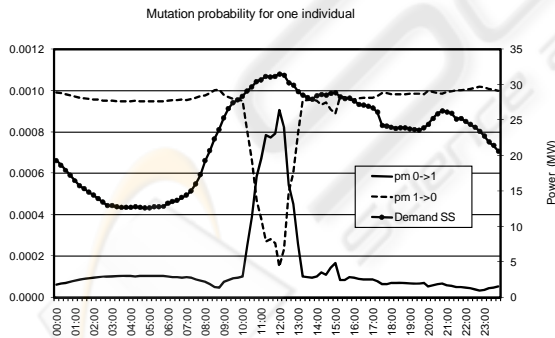


Figure 4: Mutation probability for one individual (showing the values for the two mutations that can occur).

## 4 SOME RESULTS

The analysis is based on 7 runs for each algorithm. The solutions computed by NSGA-II and SPEA-II are displayed in Figures 5 - 8.

A 2D graph enables to qualitatively evaluate each non-dominated front. As the results obtained for objective function MI are integer values that are multiple of 5 (5, 10, 15...), the non-dominated fronts for different values of MI (5, 10, 15 and 20 minutes) are shown in order to use 2D graphs. Only NSGA-II was able to identify solutions with MI=5 minutes. Regarding the situation MI=10 minutes, solutions computed by SPEA-II dominate all the solutions computed by NSGA-II. The opposite happens for MI=15 and MI=20 minutes.

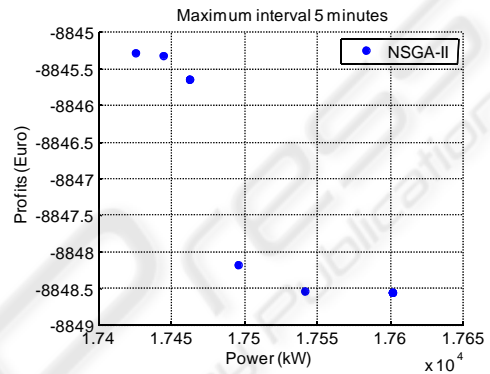


Figure 5: Results for profits and power demand (considering maximum interval 5 minutes).

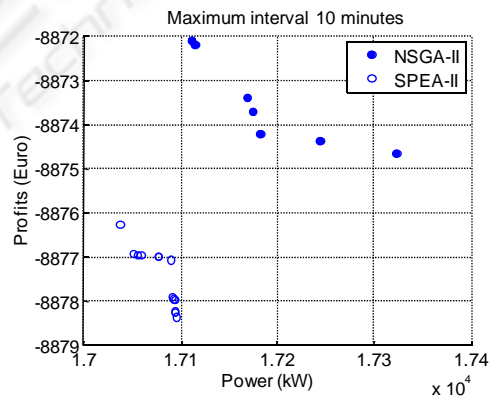


Figure 6: Results for profits and power demand (considering maximum interval 10 minutes).

With the non-dominated solutions resulting from the 7 runs of each algorithm, the percentage of space in which the solutions computed by NSGA-II dominate the solutions computed by SPEA-II (Figure 9) was calculated. NSGA-II is better in about 35% of the space while SPEA-II performs better in about 24%. In the remaining 41% one cannot say that one algorithm is better than the other.

The introduction of reservation levels (inferior values the DM is willing to accept) leads to



“removing” from the analysis some regions of the search space that are not interesting to the DM when computing the AS. In this problem the DM choose as reservation levels the following values: 17300 kW for peak power demand, 8860 k€ for profits and 10 for maximum interval objective. The new results for the AS metric are shown in Figure 10.

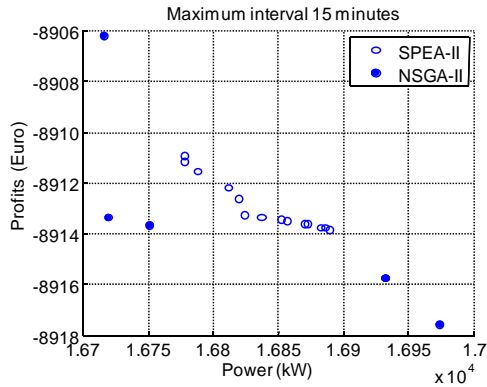


Figure 7: Results for profits and power demand (considering maximum interval 15 minutes).

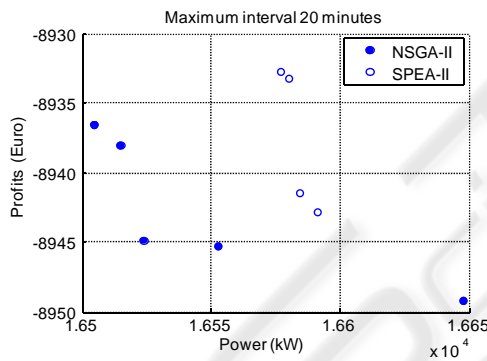


Figure 8: Results for profits and power demand (considering maximum interval 20 minutes).

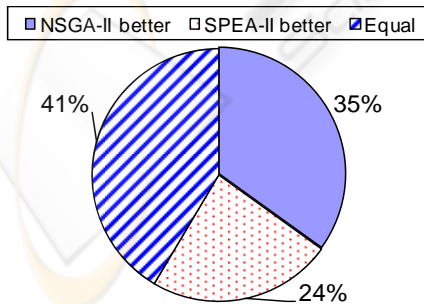


Figure 9: Percentage of the search space in which each algorithm performs better.

It can be seen that solutions identified by SPEA-II are never dominated by solutions computed by

NSGA-II and dominate the solutions computed by NSGA-II in about 85% of the space. Introducing the DM’s preferences in the calculation of the AS metric causes changes in the initial results of the metric. The two algorithms perform distinctly in different regions of the search space, being the SPEA-II able to compute solutions more in accordance with the DM preferences. Probably, NSGA-II is able to compute solutions that are very good in some objectives but really bad in the other objectives.

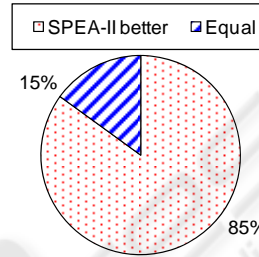


Figure 10: Percentage of the search space in which each algorithm performs better taking into consideration the DM preferences.

Besides applying the AS metric on the results of the last generation of each algorithm, we have done a generational analysis by computing the AS in each generation by using one run of each algorithm (Figure 11).

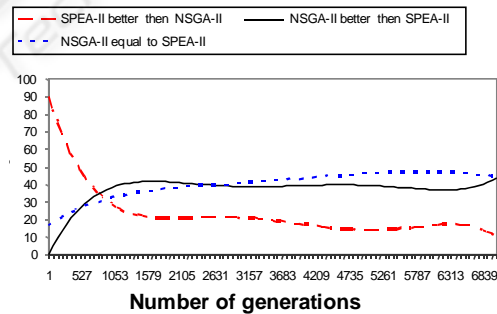


Figure 11: AS computed in each generation.

We can see that after the first one thousand generations the SPEA-II algorithm performs better than NSGA-II only in about 15%-20% of the objective search space while in about 40% of the space no algorithm performs better than the other.

A generational analysis has been done based on the minimal Euclidean distance between the population and a reference point whose coordinates can be the values the DM would like to attain in each objective (aspiration levels) or, as in this case, the best values obtained in each objective (16 500 kW; 5 minutes; 8950 k Euros) (Figure 12). The population generated by NSGA-II always contains

the individual(s) closer to the reference point (Fig. 12).

The impacts of the direct load control actions on the demand are displayed in Table 1. It was possible to reduce the maximum power demand at the sub-station level without decreasing profits.

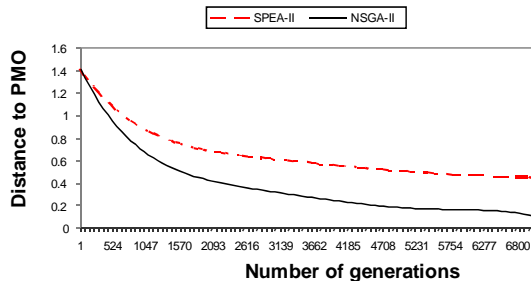


Figure 12: Minimal distance in each generation between the population and the reference point.

Table 1: Impacts of direct load control actions on the demand at the sub-station and on the profits.

	Original	SPEA-II
Demand at SE (kW)	17769,1	17055,8
Profits (€)	8837,74	8876,96

## 5 CONCLUSIONS

In this work the AS has been used to compare the results obtained with NSGA-II and SPEA-II in the identification of load control actions to be implemented over groups of air conditioners. AS allows to dealing with diversity of solutions, distribution and proximity to the true Pareto front. Moreover, it was possible to introduce the DM preferences and thus reduce the number of non-dominated solutions that the DM has to screen in order to select one solution for implementation.

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