INFORMATION UNCERTAINTY TO COMPARE QUALITATIVE REASONING SECURITY RISK ASSESSMENT RESULTS

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Keywords: Imprecise Information, Confidence, Triage application.

Abstract: The security risk associated with malevolent acts such as those of terrorism are often void of the historical data required for a traditional PRA. Most information available to conduct security risk assessments for these malevolent acts is obtained from subject matter experts as subjective judgements. Qualitative reasoning approaches such as approximate reasoning and evidential reasoning are useful for modeling the predicted risk from information provided by subject matter experts. Absent from these approaches is a consistent means to compare the security risk assessment results. This paper explores using entropy measures to quantify the information uncertainty associated with conflict and non-specificity in the predicted reasoning results. Extensions of previous entropy measures are presented here to quantify the non-specificity and conflict associated with security risk assessment results obtained from qualitative reasoning models.

1 INTRODUCTION

In security risk assessment from malevolent actions (SRAMA) such as those of terrorism, there is an absence of quantitative historical data necessary for a conventional probabilistic risk assessment. Much of the information for SRAMA is elicited from subject matter experts (SMEs) as subjective judgements and is often available as qualitative imprecise values. An Approximate Reasoning (AR) model is a useful alternative to a probabilistic model when drawing conclusions using imprecise knowledge provided by SMEs. AR has numerous applications in engineering and control (Ross 2005, Barret and Woodall 1997, Lewis 1997) and recently has been applied to security risk assessment for malevolent actions (Bott and Eisenhawer 2006).

Important factors differentiating AR in control applications with AR of SRAMA applications is the type of information used to develop the model and in the validation of the results. This paper is focused on the validation phase. In control applications historical data can be used to validate the AR results; however, for particular terrorist attacks there is generally an absence of historical data. For example, prior to September 11, 2001, there was no historical data for successful attempts using airplanes to attack World Trade Center Towers in New York. In the absence of specific historical data, the AR results for SRAMA applications can be realistically verified by the SMEs. Apart from the SMEs verification approach there has not been a consistent means presented to quantify the difference in competing results. For example, triage studies of input values contributing to the security risk are often a necessary part of the security risk assessment model. A means to consistently measure the effect of this change in input value on the model result is critically important in sensitivity studies and result comparisons. The resulting deviation may not be sufficiently or consistently recognized when relying only on SME verification.

This study therefore proposes using entropy, i.e. information uncertainty, to sufficiently and consistently compare the AR model results. Measures of entropy have not specifically been developed for use in AR results. This study extends entropy to AR results and it is unique in that a similar approach has not been previously pursued in AR or applied in the area of SRAMA as a means to determine the confidence in the result. It is a novel approach due to its application which is distinctly different from previous approaches involving linguistic values and entropy.

Like AR, Evidential Reasoning (ER) is an alternative approach used to draw conclusions from

M. Chavez G., P. Key B., K. Zerkle D. and W. Shevitz D. (2010).
INFORMATION UNCERTAINTY TO COMPARE QUALITATIVE REASONING SECURITY RISK ASSESSMENT RESULTS.
In *Proceedings of the 2nd International Conference on Agents and Artificial Intelligence - Artificial Intelligence*, pages 398-405
DOI: 10.5220/0002748103980405

information. The major difference between the two approaches is in the uncertainty quantification. The imprecision associated with describing the state is captured with AR while the lack of certainty associated with assigning a particular state to one of several linguistic values is captured with ER. In this study AR and ER are collectively referred to as qualitative reasoning but each is treated separately. In Section 2 both AR and ER are discussed and each is illustrated with simple examples.

In Section 3 entropy as it applies to AR and ER is discussed and a general discussion on entropy can be found in Klir (Klir 2006). The utility of a methodology is measured by its applicability; therefore, the quantification of entropy using the proposed approach in AR and ER is illustrated in Section 3. The implications of quantifying entropy in AR and ER for SRAMA are discussed in Section 4.

2 QUALITATIVE REASONING

SMEs may indicate that the occurrence of a particular result is “highly likely”, “somewhat likely”, or "negligible" and the resulting consequences are “extremely costly”, “moderately costly”, or “insignificant”. These expressions are called propositions and the kind of uncertainty associated with these propositions can be from vagueness, imprecision, a lack of information regarding a specific state of the system, or lack of certainty when assigning a specific state a particular value. While a combination of all these uncertainties can also be encountered this study does not address the combination of these uncertainties. Uncertainty due to vagueness, imprecision, and/or lack of information is collectively referred to as fuzzy uncertainty while a lack of uncertainty associated with assigning a specific state to one of several linguistic values is referred to as assignment uncertainty (Klir 2006). Fuzzy set theory provides a means for representing fuzzy uncertainty contained in these propositions while evidence theory provides a means for representing assignment uncertainty. Both fuzzy set theory and evidence theory as they apply to AR and ER, respectively, are discussed in this section. The reader is referred to (Ross 2004) for an in depth description of fuzzy set theory and evidence theory.

2.1 Fuzzy Set Theory

Natural language tends to be interpreted differently by various individuals. The linguistic values used by SMEs are no different and have a tendency to be vague and imprecise. For example, an SME may indicate that the process to construct a weapon device is “extremely difficult” or that it is “somewhat difficult”. The precise meaning of these linguistic values may be interpreted slightly differently by various individuals; however, linguistic values may often be the values the SME is most confident in and comfortable providing. There is vagueness and imprecision associated with a linguistic value which has been termed fuzzy uncertainty. Fuzzy uncertainty is different from random uncertainty, where random uncertainty arises due to chance and deals with specific and well defined values such as the number on the top face of a die that is thrown. Random uncertainty is referred to as an aleatoric uncertainty and fuzzy uncertainty is referred to as an epistemic uncertainty. In some cases epistemic uncertainty may be reduced to aleatoric uncertainty but aleatoric uncertainty is non reducible uncertainty (Oberkampf et al. 2004, Zadeh 1995). Linguistic values such as “high”, “medium”, and “low” describe several specific states or conditions and are considered sets. The boundary that defines any one of these sets is unclear or fuzzy and thus these sets are called fuzzy sets.

A collection of elements having similar characteristics defines a universe of discourse, X. The individual elements, i.e. states, in X are denoted as x_i, with the same notations used for Y and y_j, and Z and z_k, respectively. The elements can be grouped into various sets, such as: $\tilde{A}$, $\tilde{B}$, or $\tilde{C}$. The set value of $\tilde{A}$, $\tilde{B}$, or $\tilde{C}$ may represent something like “high” which has a fuzzy boundary. The individual states of a fuzzy set can be mapped to a universe of membership values using a function theoretic form. If a specific state x_i is a member of the set $\tilde{A}$, then this mapping is given by Equation (1). A typical mapping of $\tilde{A}$ is shown in Figure 1.

$$\mu_{\tilde{A}}(x_i) \in [0, 1] \quad (1)$$

The complement of $\tilde{A}$ is defined as:

$$\mu_{\bar{\tilde{A}}}(x_i) = 1 - \mu_{\tilde{A}}(x_i) \quad (2)$$

The mapping for the complement is also shown in Figure 1. The mapping is known as a membership function and the membership of a specific state is x_i is referred to as the degree of membership. The degree of membership of x_i provides an indication of the fuzzy set's ability to describe the state.

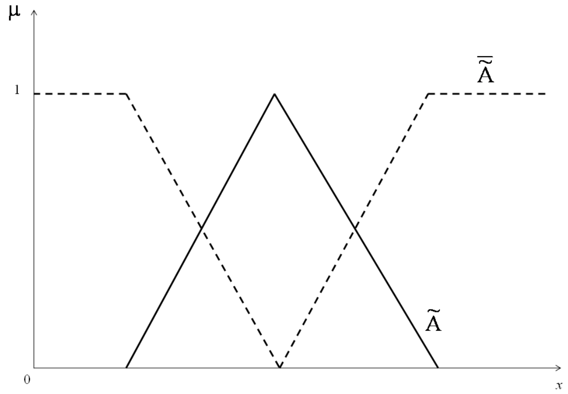

Figure 1: Mapping of $\tilde{A}$ and its complement $\bar{\tilde{A}}$.

2.2 Fuzzy Set Theory and Approximate Reasoning

An AR model uses the degrees of membership of states in fuzzy sets to draw conclusions about a system, such as risk of attack on a facility. The AR result is comprised of a vector of various fuzzy sets used to describe a specific state of risk and a respective degree of membership in each fuzzy set. Now suppose that an SME indicates that values $\tilde{A}$ and $\tilde{B}$ for states x_i and y_j, respectively, infers a particular value $\tilde{E}$ for z_k. The information provided is considered a rule governing the outcome z_k and can be represented as follows:

Rule 1: IF x_i is $\tilde{A}$ and y_j is $\tilde{B}$ THEN z_k is $\tilde{E}$

These IF-THEN rules consist of an antecedent and a consequence portion. The conditional portion of the rule, i.e. the IF x_i is $\tilde{A}$ and y_j is $\tilde{B}$ of Rule 1, forms the antecedent and the consequence of the antecedent includes THEN z_k is $\tilde{E}$. All the rules governing the particular outcome z_k involving values for x_i and y_j can be grouped together into a rule base, see Table 1. Now consider the situation when both x_i and y_j can be described by more than one value. In such a situation, x_i and y_j have a degree of membership in each value that describes them. The values of x_i and y_j are used to identify the governing rule and infer the value of z_k. The inferred value of z_k will have an associated degree of membership which results from the conjunction $\wedge$, i.e. taking the minimum value, of the degree of membership for x_i *AND* y_j included in the governing rule. Take for example the rule specified above with $\mu_{\tilde{A}}(x_i) = 0.3$ and $\mu_{\tilde{B}}(y_j) = 0.6$, which results in a $\mu_{\tilde{E}}(z_k) = 0.3$. Another applicable governing rule may be:

Rule 2: IF x_i is $\tilde{B}$ and y_j is $\tilde{B}$ THEN z_k is $\tilde{E}$

with $\mu_{\tilde{B}}(x_i) = 0.7$ and $\mu_{\tilde{B}}(y_j) = 0.6$, which results in $\mu_{\tilde{E}}(z_k) = 0.6$. Both Rule 1 and Rule 2 result in the value $\tilde{E}$ for z_k but there are now two different values for the degree of membership in $\tilde{E}$. That is, either Rule 1 *OR* Rule 2 is applicable and the disjunction ($\vee$), i.e. taking the maximum value, of $\mu_{\tilde{E}}(z_k) = 0.3$ and $\mu_{\tilde{E}}(z_k) = 0.6$, results in $\mu_{\tilde{E}}(z_k) = 0.6$. The conjunction and disjunction operations are used when the logical *AND* and *OR* are encountered, respectively. In each of the rules the logical *AND* is encountered and the conjunction operation is used to determine the resulting degree of membership. The logical *OR* is encountered in the example because either Rule 1 *OR* Rule 2 result in $\tilde{E}$. Additional logical operations can be found in (Ross 2005) as well as the axioms involved in fuzzy sets. It is important to note that the excluded middle axiom is not required for fuzzy sets; therefore, the resulting degree of membership for AR need not sum to 1.

Table 1: Rule Base.

Rule Base		Universe of Discourse X		
		$\tilde{A}$	$\tilde{B}$	$\tilde{C}$
Universe of Discourse Y	$\tilde{A}$	$\tilde{F}$	$\tilde{E}$	$\tilde{E}$
	$\tilde{B}$	$\tilde{E}$	$\tilde{E}$	$\tilde{E}$
	$\tilde{C}$	$\tilde{E}$	$\tilde{G}$	$\tilde{G}$

2.2.1 Application of AR in Risk

This section illustrates the use of AR in SRAMA using a simple example to determine the risk of attack from success likelihood and the economic consequences of the attack. Table 2 provides the rule base used to infer the risk given the success likelihood and the consequences.

Table 2: AR Risk Rule Base.

Risk		Economic Consequence				
		Very Low	Low	Medium	High	Very High
Success Likelihood	Negligible	Very Low	Very Low	Very Low	Very Low	Very Low
	Extremely Unlikely	Very Low	Very Low	Very Low	Very Low	Low
	Very Unlikely	Very Low	Very Low	Very Low	Low	Medium
	Unlikely	Very Low	Low	Low	Medium	**Medium**
	Somewhat Likely	Very Low	Low	Low	Medium	**Medium**
	Likely	Low	Low	Medium	High	Very High
	Nearly Certain	Low	Low	Medium	High	Very High

An attack scenario S1 has the following input vector of membership values for success likelihood and economic consequences:

S1(success likelihood): [0, 0, 0, 0.57, 0.43, 0, 0]

S1(economic consequences): [0, 0, 0, 0, 1]

The leftmost entry for degree of membership in the vector of success likelihood corresponds to "negligible", followed by "extremely unlikely", "very unlikely", "unlikely", somewhat likely", "likely" and the rightmost entry corresponds to "nearly certain". The leftmost entry for degree of membership in the vector of economic consequences corresponds to "very low" and so on to the rightmost entry corresponding to "very high". Using the rule base of Table 2 and AR operations of Section 2.2, "very high" economic consequences AND an "unlikely" success likelihood results in a "medium" risk with a degree of membership of 0.57. While a "very high" economic consequences AND a "likely" success likelihood results in a "medium" risk with a degree of membership of 0.43. Since either of these two rules, shown in bold in Table 2, result in "medium" risk, the maximum of the resulting degree of membership values is used to determine the final degree of membership for a "medium" risk. The resulting vector of membership values for risk in scenario 1 are:

S1(risk): [0, 0, 0.57, 0, 0]

Corresponding to linguistic risk values of "very low", "low", "medium", "high", and "very high" from left to right. Inference trees, consisting of a complex sequence of inference rules leading up to risk are used to assess risk for each attack scenario (see Bott and Eisenhawer 2006). Here only a simplified portion is provided.

2.3 Evidential Reasoning

This paper is concerned with a particular aspect of evidence theory which involves the uncertainty associated with assigning a specific *x* to a particular crisp value *A*. The SMEs' degree of belief that *x* is *A* is called a basic evidence assignment (*bea*). A crisp set value has a precise well defined boundary and precisely describes *x*. The ER model uses the *bea* in the antecedent of the rule, to determine the *bea* for the consequence of the rule. That is, the SMEs *bea* quantifies the evidence supporting a particular claim, i.e. x is A, which can be used to form other belief, plausibility, and probability measures (see Ross 2005). The *bea* does not account for the uncertainty associated with imprecisely describing *x* with *A*. The degree of membership is used to assess the uncertainty involved in describing a specific state using an imprecise linguistic value. There have been recent attempts to combine AR and ER for SRAMA applications which have been termed *fuzzy evidential reasoning* (Yang et al. 2009) and *belief measures on fuzzy sets* (Darby 2007). However, the simultaneous quantification of fuzzy and assignment uncertainty was not addressed by Yang et al. and Darby and the reader is referred to Chavez (Chavez 2007). In this paper, AR and ER are recognized as distinct methods and discretely applied.

An ER result is comprised of a vector of *bea* values for *x* is A_j, where A_j ...A_n are the available crisp linguistic sets in the outcome. Comparing one resulting vector to another is the focus of this paper. Here we briefly discuss the operations used to obtain an ER vector result in SRAMA. A simple method of determining the *bea* associated with the inferred linguistic value for each rule is to take the product of the *bea* values involved in the antecedent of the rule. This process is performed for all the inferred linguistic values in the result. Two or more rules in the rule base may result in the same linguistic value, in such a case these resulting *bea* values are summed to determine the resulting *bea* value for the linguistic value. It is important to note that the *bea* (*m*) must satisfy the following boundary conditions:

$$m(\emptyset) = 0 \tag{3}$$

$$m(A) = \sum_{A_j \in P(X)}^{j=1,2,3,\dots,n} m(A_j) = 1 \tag{4}$$

Equation 3 indicates that a *bea* value cannot be assigned to the proposition that x_i is defined by the null set, ∅, because the null set defines no states. Equation 4 indicates that the sum of the *bea* values for x_i is A_j is equal to 1 where, A_j are crisp subsets of the power set *P(X)*. The power set *P(X)* is the set if all subsets of *X*.

2.3.1 Application of ER

This section demonstrates the use of ER using a simple example to determine the *effectiveness of physical inventory* from the *material inventory frequency* and *effectiveness of inventory verification*. Table 3 provides the rule base used to infer the effectiveness of physical inventory from the material inventory frequency and effectiveness inventory verification. A processing facility F1 has the following vector of *bea* values for a specific material inventory frequency and a specific effectiveness of inventory verification:

F1(material inventory frequency): [0, 0.1, 0.9, 0]
F1(effectiveness of inventory verification):
[0, 0, 1, 0]

The leftmost entry for *bea* in the vector of material inventory frequency corresponds to “not applicable” (NA), followed by “occasionally”, “regularly”, and the rightmost entry corresponds to “continuously”. The leftmost entry for the *bea* in the vector of effectiveness of inventory verification corresponds to “not applicable” (NA), followed by “low”, “moderate”, and the rightmost entry corresponding to “excellent”. Using the rule base of Table 3 and ER operations of Section 2.3, a *bea* value of 0.1 in “occasionally” for material inventory frequency AND a *bea* value of 1.0 in “moderate” for effectiveness of inventory verification results in a *bea* value of 0.1 in “low” for effectiveness of physical inventory. While a *bea* value of 0.9 in “regularly” for physical inventory frequency AND a *bea* value of 1.0 in “moderate” for effectiveness of inventory verification results in a *bea* value of 0.9 in “moderate” for effectiveness of physical inventory.

F1(effectiveness of physical inventory):
[0, 0.1, 0.9, 0],

The resulting vector of values for effectiveness of physical inventory of: “not applicable”, “low”, “moderate”, and “excellent” from left to right.

Table 3: Effectiveness of Physical Inventory ER Rule Base.

Effectiveness of Physical Inventory		Effectiveness of Inventory Verification			
		NA	Low	Moderate	Excellent
Material Inventory Frequency	NA	NA	NA	NA	NA
	Occasionally	NA	Low	**Low**	Low
	Regularly	NA	Low	**Moderate**	Moderate
	Continuously	NA	Low	Moderate	Excellent

3 QUANTIFICATION OF INFORMATION UNCERTAINTY

Decision makers are interested in the confidence associated with each of the competing alternatives. The quantity of uncertainty present in a result is related to the confidence (Devore 1999). That is, the less uncertainty present in the resulting alternative the more confidence one can have in the result. Thus, by measuring the information uncertainty present in each resulting alternative, the possible alternatives can be compared and the alternative with the most confidence can be determined.

The quantification of entropy for random uncertainty was addressed by Shannon (Shannon 1948). The term entropy is defined as a measured quantity of information uncertainty related to non-specificity and conflict (Klir and Wierman 1999). The measure of entropy proposed by Shannon measures conflict and works as follows: there exists a regular die with six faces all of which are equally likely to be thrown and there exists a six sided trick die with one side being twice as likely to be thrown as the remaining sides. The regular die has more entropy than the trick die because all sides are equally likely to occur in the regular die. The trick die is less uncertain because one side is twice as likely to be thrown as each of the remaining five; thus, one can have more confidence in the resulting trick die.

Klir and Wierman (Klir and Wierman 1999) discuss measuring conflict from evidence on sets. The ER problem examined here does not involve the entire set but only one state assigned to one or more set values. Klir (Klir 2006) elaborates on Shannon's measure of entropy and identifies *conflict* as the basis for the entropy measured by Shannon. De Luca and Termini (Deluca 1972) extended Shannon's measure of entropy to fuzzy uncertainty in a fuzzy set while others also presented alternative measures, see Yager (Yager 1979), and Higashi and Klir (Higashi and Klir 1982). Pal and Bezdek (Pal and Bezdek 1994) provide a good summary of many of the approaches used to measure entropy associated with a fuzzy set. All the previous approaches examined for fuzzy uncertainty quantified the entropy involved in an entire fuzzy set, whereas the current study examines quantifying the entropy involved in AR where one state is described using several fuzzy sets.

Shannon's measure of conflict for probability (p) has the form

$$S(p) = -\sum_{x\in X} p(x)\log_2 p(x). \tag{5}$$

Klir and Wierman provide an extended measure of conflict to *bea* values on sets, that is, in Equation 5, x is replaced with A and p is replaced with m. For ER applications, the focus is on a specific *x* assigned to A_i. Thus, the conflict in an ER vector result $(\vec{R}_{ER})$ is:

$$C(\vec{R}_{ER}) = -\sum_{i=1}^{n} m_{A_i}(x)\log_2 m_{A_i}(x), \tag{6}$$

De Luca and Termini's (Deluca and Termini 1972) measure for the entropy of a fuzzy set is similar to Shannon's but conceptually different. Shannon measures the conflict due to random uncertainty while De Luca and Termini measure the conflict due to the fuzzy uncertainty associate with a membership function for a fuzzy set. As shown in Equation 7, Deluca and Termini proposed quantifying the conflict of a fuzzy set from its membership function and the complement of its membership function. Pal and Bezdek (Pal and Bezdek 1994) indicate that inclusion of the complement in Equation 7 is necessary to satisfy maximality.

$$D(\tilde{A}) = -\sum_{i=1}^{n} \mu_{\tilde{A}}(x_i)\log_2\mu_{\tilde{A}}(x_i) + \mu_{\bar{\tilde{A}}}(x_i)\log_2\mu_{\bar{\tilde{A}}}(x_i) \tag{7}$$

In the previous approaches involving fuzzy uncertainty, the entropy quantified involves all the possible states described by a particular fuzzy set (Pal and Bezdek 1994, Klir and Wierman 1999, Klir 2006); whereas, in this application the entropy quantified is associated with only one state described linguistically using various fuzzy sets.

The outcome resulting from the AR is expressed as a vector of membership values for *x* in $\tilde{A}_i$. In an AR model the conflict is not among one fuzzy set but several, that is, there is conflict among all the fuzzy set alternatives having a non-zero degree of membership in the resulting vector. There exists a fundamental difference between the application for the previous approaches and the application of the current study. However, Equation 7 can be modified so that it is applicable to account for the conflict involved in imprecisely describing a specific state *x* with the various fuzzy sets $\tilde{R}_i$ in the resulting vector $\vec{R}$. The proposed equation, applicable to an AR result, is presented in Equation 8. Note, the major difference between Equation 7 and 8 is that Equation 8 involves one state *x* potentially described using *n* fuzzy sets, $\tilde{A}_{i,\cdots,n}$; whereas, Equation 7 involves one fuzzy set describing *n* different states, $x_{i,\cdots,n}$.

$$C(\vec{R}) = -\sum_{i=1}^{n} \mu_{\tilde{A}_i}(x)\log_2\mu_{\tilde{A}_i}(x) + \mu_{\bar{\tilde{A}}_i}(x)\log_2\mu_{\bar{\tilde{A}}_i}(x) \tag{8}$$

Where $\vec{R}$ is the vector consisting of the degree of membership for each fuzzy set in the AR result for one scenario, and C is the conflict, $\mu_{\tilde{A}_i}(x)$ is the degree of membership of state *x* in the fuzzy set $\tilde{A}_i$.

Another type of entropy, known as non-specificity, reflects the ambiguity in specifying the exact solution (Klir 2006). Hartley (Hartley) first proposed measuring the lack of specificity which is simply related to the number of alternatives present. Klir simply defines the Hartley measure of uncertainty as:

$$\mathrm{H}(f_E) = log_2|E|, \tag{9}$$

where f_E is any function of the subset *E*. Klir discusses the Hartley measure as it applies to probability distribution functions and membership functions which are not discussed here and the reader is referred to (Klir 2006, Klir and Wierman 1999). In this paper, the measure of non-specificity is considered as a means to determine the lack of specificity in the resulting AR or ER vector using the number of non-zero alternatives in the vector. By considering that f_E instead represents a vector result and *E* represents *R,* the number of nonzero values in the resulting vector, the non-specificity of the resulting vector is determined. The measure for non-specificity in an AR or an ER result is thus quantified using Equation 10:

$$\mathrm{N}(\vec{R}) = log_2|R|, \tag{10}$$

Where *R* is the number of linguistic sets in the resulting AR or ER vector having a non-zero degree of membership or *bea,* respectively.

Random uncertainty may be present in available information elicited from SMEs but it is at an epistemic level and captured in the linguistic values provided by the SMEs. As a result the conflict due to random uncertainty is captured by Equation 6 for ER or Equation 8 for AR. Conflict is determined differently in AR and ER applications due the restrictions of Equation 2 on the degree of membership and the restrictions of Equations 3 and 4 on the *bea*. Equations 6, 8 and 10 have units of bits of information from the use of the logarithm base 2 (Klir 2006). A simple determination of maximum confidence can be made from minimum information uncertainty among competing alternatives.

3.1 Entropy in AR and ER Results

The quantification of conflict and non-specificity in AR and ER results are demonstrated here using the examples provided in Section 2. Using Equation 6 the conflict involved in the ER result F1(effectiveness of physical inventory):[0, 0.1, 0.9, 0], is calculated as.

$$C = -[0.1\log_2(0.1) + 0.9\log_2(0.9)] = 0.469$$

The non-specificity involved in the ER result is calculated using Equation 10.

$$\mathrm{N}(\vec{R}) = log_2|2| = 1$$

Using Equation 8 the conflict involved in the AR result S1(risk): [0, 0, 0.57, 0, 0], is calculated as follows. Recall that the membership of the complement is determined from Equation 2.

$$C(\vec{R}) = -[(0.57\log_2 0.57) + 0.43\log_2 0.43 = 0.9858$$

The non-specificity involved in the AR result is calculated using Equation 10.

$$\mathrm{N}(\vec{R}) = log_2|1| = 0$$

In addition to the ER and AR example provided previously two additional ER results and AR results are provided. The ER and AR results and their quantities of information uncertainty are presented in Tables 4 and 5, respectively.

Table 4: ER entropy results for Effectiveness of Physical Inventory example.

ER result	Conflict	Non-specificity
F1[0, 0.1, 0.9, 0]	0.469	1
F2[0, 0.2, 0.8, 0]	0.722	1
F3[0, 0.15, 0.75, 0.1]	1.054	1.585

Table 5: AR Entropy results for Economic Risk example.

AR result	*Conflict*	*Non-specificity*
S1[0, 0, 0.57, 0, 0]	0.9858	0
S2[0, 0.3, 0.7, 0.2, 0]	2.883	1.585
S3[0, 0.2, 0.6, 0.2, 0.1]	2.484	2.000

The results demonstrate the utility of quantifying information uncertainty to compare the results. In Table 4, the effectiveness of physical inventory, F1, F2 and F3 all result in a linguistic value as “mostly moderate”. There is an observable difference in each resulting vector; however, a realistic comparison is not possible without a useful metric. Entropy measures, specifically conflict, provide a recognizable and comparable difference with all three ER results. In the case of the AR results, Table 5, there is also a recognizable difference in the conflict and the non-specificity. The non-specificity reflects a difference that can also be discerned visually, i.e. the greater number of non-zero alternatives the greater the non-specificity. Alternatively, measuring the conflict provides comparative information that is not as easily discerned visually.

Tables 4 and 5 illustrate the quantification of the conflict and non-specificity using simple AR and ER models. Based on information uncertainty, the alternative with the least information uncertainty is also the alternative with the most confidence. Therefore, F1 and S1 are the alternatives providing the most confidence.

4 CONCLUSIONS

ER and AR results for SRAMA have quantifiable amounts of information uncertainty and this study extends information theory to AR and ER SRAMA models. Straight-forward extensions of previous approaches are presented in this paper and used to quantify the information uncertainty in AR results. The information uncertainty measurements of conflict and non-specificity associated with AR and ER results are illustrated and used to compare the results to one another. Maximum confidence is simply based on minimum measured information uncertainty in each result. Through ongoing research, the results can be further extended through the development of a metric comparing measured confidence to the maximal potential value of confidence determined from a combined measure of information uncertainty. Moreover, future work will involve comparisons of the results obtained using the proposed metrics to rank the results to those obtained from a SME ranking of the results.

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