

# TREE-STRUCTURED TEMPORAL INFORMATION FOR FAST HISTOGRAM COMPUTATION

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Abstract: In this paper we present a new method for fast histogram computing. Based on the known tree-representation histogram of a region, also called reference histogram, we want to compute the one of another region. The idea consists in computing the spatial differences between these two regions and encode it to update the histogram. We never need to store complete histograms, except the reference image one (as a preprocessing step). We compare our approach with the well-known integral histogram, and obtain better results in terms of processing time while reducing the memory footprint. We show theoretically and with experimental results the superiority of our approach in many cases. Finally, we demonstrate the advantage of this method on a visual tracking application using a particle filter by improving its time computing.

## 1 INTRODUCTION

Histograms are often used in image processing for feature representation (colors, edges, *etc.*). The complex nature of images implies a large amount of information to be stored in histograms, requiring more and more computation time. Many approaches in computer vision require multiple retrievals of histograms for rectangular patches of an input image. Each one is developed for a specific application, such as for image retrieval (Halawani and Burkhardt, 2005), contrast enhancing (Caselles et al., 1999) or object recognition (Gevers, 2001). In such approaches, we dispose a reference histogram and try to find the region of the current image whose histogram is the most similar. The similarity is given by a measure that has to be computed between each target histogram and the reference one. This implies the computation of a lot of target histograms, that can be very time consuming, and may also need a lot of storage. The main goal is then to reduce the computation time, while using small data structures, requiring less memory.

In this article, we propose a new histogram computation by using a data structure only coding the pixel differences between two frames of a video sequence. This data structure is updated over time on pixel changes information and is used to define the histogram of the whole new image or a part of it. We never need to store the complete histogram and our representation is compact because it only contains

variation information between two frames. The main advantages of our approach are that it is not dependent on the histogram quantization (i.e. number of bins), it is fast to compute (comparing to other approaches) and compact. Section 2 reviews some of the previous works on fast histogram computation. Section 3 presents our method compared with the well-known integral histogram. Section 4 gives some theoretical considerations about time computation and size of storage needed. In Section 5, some experimental results show the benefit of our approach. In Section 6 we illustrate the capability of our method on a real application: object tracking using particle filtering. Finally, we give concluding remarks in Section 7.

## 2 PREVIOUS WORKS

An histogram is computed into a region by browsing all the pixels of this region. If lots of histograms have to be computed locally around a set of salient points, it can be advantageous to use the histogram of a nearby region and to update it to obtain the histogram of the current region, instead of computing all the histograms. This can be applied in cases of spatial filtering, but also in temporal filtering in video sequences, when trying to find displacements of objects between frames. A lot of works have been proposed to reduce the histogram computing time.

One of the first work on redundancy computation reduction was proposed by (Tang et al., 1979), in the context of image filtering. Considering the histogram  $H_R$  of a region  $R$ , the histogram of a region  $Q$  is computed by keeping the histogram of their intersection region, removing the pixels of  $R$  that do not belong to  $Q$ , and adding those from  $Q$  that do not belong to  $R$ . This approach is efficient only in cases of large intersection between regions. Recently this method has been improved by (Perreault and Hebert, 2007) in the context of median filtering. In (Sizintsev et al., 2008) the authors present the distributive histogram. They use the property for disjoint regions  $R$  and  $Q$  that  $H(R \cup Q) = H(R) + H(Q)$ . Their approach can then easily be adapted to non-rectangular regions that is not the case of previous approaches.

A fast way to compute histograms in terms of time computation is the integral histogram (Porikli, 2005) (IH), inspired from integral images (Viola and Jones, 2001), that is now used in many applications needing massive histogram computations by localized searches, especially in recent tracking algorithms (Adam et al., 2006; Wang et al., 2007). This approach is inspired from integral image and consists in computing the histogram of any region of an image using only four operations (two additions and two subtractions). IH is a cumulative function whose cells  $IH(r, c)$  contain the histogram of an image area containing its  $r$  first rows and  $c$  first columns. Then:

$$\begin{aligned} IH(r, c) &= I(r, c) + IH(r-1, c) \\ &+ IH(r, c-1) - IH(r-1, c-1) \end{aligned}$$

Once the integral histogram has been computed over all cells, we can derive any histogram of a sub-region only using four elementary operations, see (Porikli, 2005) for more details. For example, the histogram of a  $w \times h$  region  $R$  with pixel  $(r, c)$  as bottom right corner is given by:

$$\begin{aligned} H_R &= IH(r, c) - IH(r-h, c) \\ &- IH(r, c-w) + IH(r-h, c-w) \end{aligned}$$

The main drawback of integral histogram is the large amount of data needed to be stored. For an  $N \times M$  image, the size of the array  $IH$  needed is  $N \times M \times B$ , where  $B$  is the number of bins in the histogram. We can find a good comparative study of some of these previous exposed method in (Sizintsev et al., 2008). However, our approach is totally different than previous ones: we never need to encode histograms (except the reference one), but only the temporal differences between two images, and use them to determine new histograms. The size of the data structure, and the histogram computation time only depends on the variations between frames.

### 3 PROPOSED APPROACH: TEMPORAL HISTOGRAM

Assume that we have a reference histogram  $H$  (from the reference image  $I_1$ ), and want to compute histograms in a new image  $I_2$  only using  $H$  and temporal variations between  $I_1$  and  $I_2$ . Temporal variations are obtained with the image difference and encoded by a tree data structure with height  $h_T = 3$ . Nodes at the level  $h = 1$  correspond to the rows  $r_i$  of the image  $I_2$  where there is a difference with  $I_1$ , and nodes at the level  $h = 2$  correspond to the columns numbers  $c_j$ . Leaf nodes contain, for each pixel  $(r_i, c_j)$ , the difference between  $I_1$  and  $I_2$ , the initial bin it was belonging to in  $H$ , and the bin it will belong to in the histogram of the updated histogram. Figure 1 shows a basic example of the construction of this data structure. On the left, the image difference between  $I_1$  and  $I_2$  shows only four different pixels, situated in three different rows  $r_1, r_2$  and  $r_3$ , and four different columns  $c_1, c_2, c_3$  and  $c_4$ . For each pixel  $(r, c)$ , we also have to store its original and new bins, respectively  $b_o$  and  $b_f$ . Algorithm 1 summarizes this process.

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**Algorithm 1.** Temporal data structure construction.

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T ← {}
Compute the image difference D = I1 - I2
for all D(r, c) ≠ 0 do
    bo ← bin of I1(r, c); bf ← bin of I2(r, c)
    if node r does not exist in T then
        Create branch r - c - (bobf) in T
    else
        Add branch c - (bobf) to node r in T
    end if
end for

```

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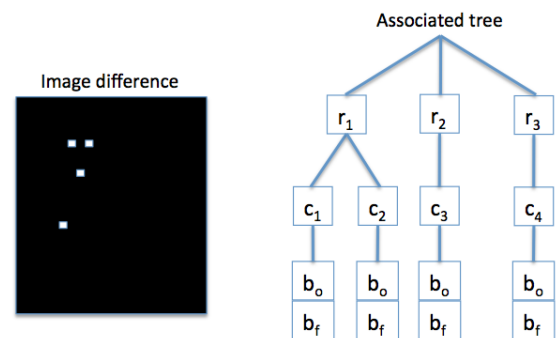


Figure 1: Construction of the data structure associated with the image reference  $I_d$ , on the left. For each non-zero value pixel of  $I_d$ , we store its new row number  $r_i$ , column number  $c_j$  and original and final bins, respectively  $b_o$  and  $b_f$ .

Once we have the reference histogram  $H_R$  and the difference tree  $T$ , we can derive any histogram of a re-

gion  $R$  in  $I_2$ , as described in Algorithm 2. We then just need to browse the data structure to determine if some pixels have changed in this region between the two images. For each changed pixel, we have to modify  $H_R$  by removing one from the bin  $b_o$  and adding one to the bin  $b_f$ . This is a very simple but efficient way to compute histograms because we just perform the necessary operations (where change has been detected). In the next Section, we give some theoretical comparative results between our approach and integral histogram.

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**Algorithm 2.** Histogram computation of a region  $R$ .

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Extract the sub-tree  $T_R$  from  $T$ , containing changing pixels in  $R$  between the two frames  
**for all** node branch  $r - c - b_o - b_f$  in  $T_R$  **do**  
      $H_R(b_o) \leftarrow H_R(b_o) - 1$   
      $H_R(b_f) \leftarrow H_R(b_f) + 1$   
**end for**

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## 4 THEORETICAL STUDY: MEMORY AND COMPUTATION COST

IH is, in our opinion, the best in the sense that it requires low computation time and is flexible enough to adapt to many applications therefore it is the one we have chosen for comparison with our approach. In this section, we compare our approach with IH in terms of number of operations necessary to compute histograms, and size of storage needed for the data structures. We are considering an image  $I$  of size  $N \times M$ , and  $B$  is the number of bins in the histograms. The histogram of the reference image has to be computed as a preliminary step for both approaches: we then do not consider this common step. We also do not consider the allocation operations for the two data structures (an array for IH and a tree for TH), but this is clear that the tree needs less allocation operations than an array, for a fixed number of pixels, because it only stores 4 values per changing pixel, whereas IH stores a whole histogram per pixel. The determination of the bin of a current pixel requires one division and one floor: we call  $f_b$  this operation, and  $a$  an addition (or a subtraction). Both methods require two steps:

1. the data structure construction, then
2. the data access for the computation a a new histogram.

We first consider and compare independently both steps.

### 4.1 Construction of Data Structures

For IH, we need to browse all pixels  $I(r, c)$  of the image, determine its bin value, and compute the integral histogram using four operations  $a$  (see Section 2), for each bin of the histogram. This part then needs a total number of operations of:

$$(n_d)_{\text{IH}} = (4a + f_b)NMB$$

This number of operations is a constant.

For TH, we first need to find non-zero values in the image difference  $D$  ( $NMa$  operations). By scanning  $D$  in the lexicographic order, we then create a branch in the tree data structure for each non-zero value: let's be  $s$  the total number of non-zero value pixels ( $s \leq (N \times M)$ ). For each of the  $s$  changing pixel, we have to determine its new bin. The number of operations needed for the construction of the tree is then:

$$(n_d)_{\text{TH}} = sf_b + NMa$$

Thus, to compare with IH, we have to consider two special cases:

- In the best case, all the pixels in the image difference are zero-valued pixels: we need  $(n_d)_{\text{TH}} = NMa$  operations to construct  $T$ .
- In the worst case, all the pixel values of the image difference are different from zero, the construction of  $T$  can be done using a total number of operations of:

$$(n_d)_{\text{TH}} = NMf_b + NMa = NM(a + f_b)$$

Even in the worst case (all pixels have changed), the number of operations necessary for the construction of  $T$  is less than the one necessary for the integral histogram construction. It should also be noticed that  $(n_d)_{\text{TH}}$  does not depend on the number  $B$  of bins of the histogram because we do not encode histogram (and so do not need to browse all bins), only temporal changes between images.

### 4.2 Histogram Computations

For both approaches, we consider the problem of computing the histogram  $H_R$  of any region  $R = [R_h \times R_w]$  of a new image  $I_2$  knowing histograms in  $I_1$ . For both approaches, we consider the data access or extraction as a negligible constant respectively  $c_{\text{IH}}$  and  $c_{\text{TH}}$  (experimental results in Section 5 show that this is not a strong assumption).

For IH we just need two additions and two subtractions between values stored in the data structure, for each bin of the histogram (see Section 2). Then, to

compute any histogram, we need a constant number of operations:

$$(n_c)_{\text{IH}} = 4aB$$

This is a bit more complicated for TH. We first need to extract the region  $R$  from  $T$ , if it does exist (we have its histogram, see the introduction of this section). Then, for each of the  $s_R$  differences ( $s_R \leq s$  if  $R$  is a subregion of  $I$ , otherwise  $s_R = s$ ), we have to remove one from the bin  $b_o$  and add one to the bin  $b_f$ . The computation of a new histogram  $H_R$  is done by a total number of operations of:

$$(n_c)_{\text{TH}} = 2as_R$$

We consider the following special cases :

- In the best case, there is no difference between the two considered regions: we need  $(n_c)_{\text{TH}} = 0$  operation.
- In the worst case, all the pixel values have changed between the two regions, we can compute the histogram using a number of operations of:

$$(n_c)_{\text{TH}} = 2a|R|$$

where  $|R| = R_h \times R_w$  is the number of pixels in  $R$ . If  $R = I_2$ , then  $(n_c)_{\text{TH}} = 2aN M$ .

The efficiency of our approach for the new histogram computation depends on the size of  $R$  and on the number of changing pixels between  $I_1$  and  $I_2$ . In the general case, we have:

$$(n_c)_{\text{TH}} < (n_c)_{\text{IH}} \quad \text{if} \quad 2as_R < 4aB \Leftrightarrow s_R < 2B$$

We then conclude that TH is better as long as the number of changing pixels is less than twice the number of bins on the histogram.

### 4.3 Total Computation

The total histogram computation time of a region in the new image  $I_2$  is fixed for IH:

$$(n_t)_{\text{IH}} = (4a + f_b)NMB + 4aB$$

For TH, it depends on two major factors: (i) the number  $s$  of changing pixels between  $I_1$  and  $I_2$  and the number  $s_R$  of changing pixels between  $(R)_{I_1}$  and  $(R)_{I_2}$ . We need a total number of operation:

$$(n_t)_{\text{TH}} = sf_b + NMa + 2as_R$$

As previously, we can distinguish two cases:

- In the best case, there is no differences between the considered regions and then  $T$  is empty, we need:  $(n_t)_{\text{TH}} = NMa$  operations.

- In the worst case, all the pixel are different between  $I_1$  and  $I_2$ , so they are between  $(R)_{I_1}$  and  $(R)_{I_2}$ , and we then need at total number of operations:

$$(n_t)_{\text{TH}} = NM(a + f_b) + 2a|R|$$

$$\text{If } R = I_2, \quad (n_t)_{\text{TH}} = NM(a + f_b) + 2aN M = NM(3a + f_b).$$

We can compare the worst case with the fixed number  $(n_t)_{\text{IH}}$ . IH depends on the size  $N \times M$  of the image and on the number  $B$  of bins of the histogram. TH depends on the size of the image (conditioning the potential number of changing pixels  $s$ ) but also on the region on which we compute the histogram. But the data structure construction step requires less operations for TH (see Section 4.1). An histogram computation will require a number of operation depending on the number of changing pixels between  $I_1$  and  $I_2$  (see Section 4.2)

### 4.4 Storage

We now compare the quantity of information necessary for both approaches.

For IH, we need a constant-size array, containing a total number of cells of:

$$(c)_{\text{IH}} = NMB$$

We need one  $B$ -size array for each pixel  $(r, c)$ , corresponding to the histogram of the region from rows 1 to  $N$  and columns 1 to  $M$ .

For TH we use a tree  $T$  as data structure whose size depends on the number  $s$  of changing pixels between images  $I_1$  and  $I_2$ . If we call  $n_r$  the number of rows in  $I_2$  containing changing pixels, the number of nodes of  $T$  is:

$$(c)_{\text{TH}} = n_r + 3s$$

i.e.  $n_r$  for the rows, and 3 nodes for each changing pixel. We can distinguish two cases :

- In the best case, there is no difference between regions,  $T$  is empty:  $(c)_{\text{TH}} = 0$ .
- In the worst case, all the pixels are different, and the size if the required data structure  $T$  is:

$$(c)_{\text{TH}} = N + 3NM$$

Then, in the worst case (i.e. all the pixels have changed between the two images or regions, that can rarely happen):

$$(c)_{\text{IH}} < (c)_{\text{TH}} \quad \text{if} \quad NMB < N(1 + 3M) \Leftrightarrow B \leq 3$$

In the most common case,  $(c)_{\text{IH}} < (c)_{\text{TH}}$  if  $NMB < n_r + 3s$ . It is more than probable that the number of



changing pixels between the two images is less than the total number of pixels. At most, if all these changing pixels are located on different rows (negative scenario), we have  $n_r + 3s = s + 3s = 4s$ , so:

$$(c)_{\text{IH}} < (c)_{\text{TH}} \quad \text{if} \quad NMB < 4s \Leftrightarrow s > \frac{NMB}{4}$$

Globally our histogram computation needs then less storage.

The theoretical considerations about the number of operations and storage needed for both approaches developed in this section will next be verified with a number of experimental results in the next Section.

## 5 EXPERIMENTAL RESULTS

In this section, we systematically compare integral histogram (IH) with the proposed temporal histogram (TH), since no method has been proved to be more interesting than IH in terms of both computation time and storage: it would not be relevant to perform comparisons with other methods based on this criteria. In the next subsections, we call computation of a histogram the two-steps process needed for both approaches: data structure construction and histogram computation. All computation times reported in this section correspond to the mean value over 100 different tests.

### 5.1 Computation Time

In this section we propose to compare the computation times of integral and temporal histograms. In Section 4, we have highlighted some parameters that we directly involved, such as the number  $B$  of bins of the histograms, the size  $N \times M$  of the images, the number  $s$  of changing pixels in the whole image, and the number  $s_R$  of changing pixels in the considered region for histogram computation.

#### 5.1.1 Video Sequences

Tests on different complete video sequences have been performed. In this section we only present those made on sequences “Walking” (15 frames of size  $275 \times 320$ ), “Tennis” (89 frames of size  $240 \times 342$ ) and “Parking” (231 frames of size  $576 \times 768$ ), see examples of frames in Figure 2. Some frames of these sequences are shown in Figure 3. In these tests, we are interested in the total computation time (along all the sequence) needed for the computation of the histogram of randomly chosen regions of size  $10 \times 10$  in each  $I_t$  ( $t > 1$ ) depending on the number of bins.

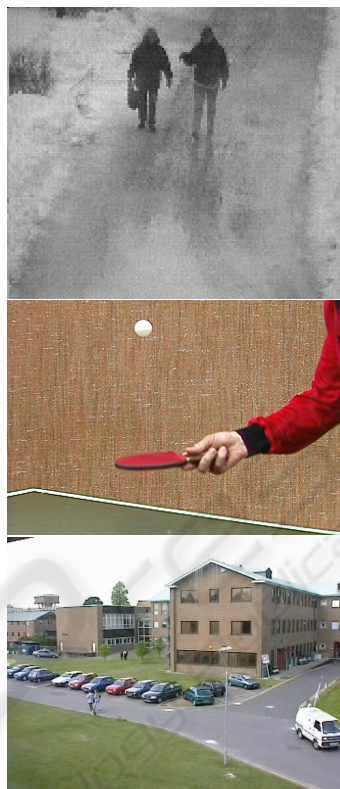


Figure 2: A frame from, from top to bottom: “Walking”, “Tennis” and “Parking” sequences.

We can see in Figure 3 that the computation times obtained with our approach are lower for each one of these sequences. This is in part due to the fact that the computation of the array of the integral histogram takes a lot of time (and is performed at each frame), even if the histogram time computation (just requiring four operations) is small. This also shows that our approach is relatively stable with respect to the increasing number of bins  $B$ , contrary to integral histogram, whose computation time increases with  $B$  (drastically for  $B = 256$ ). This is due to the fact that, contrary to the IH, the number  $B$  of bins does not affect a lot the computation time (see Section 4.3) of TH is there are only few changing between two frames. As there is no “best” number of bins, and different bin numbers can reveal different features of the data: it is difficult to determine an optimal number of bins, without making strong assumptions about the shape of the distributions. With our approach, it is not necessary to make such assumptions.

#### 5.1.2 Image and Size Variations

The performance of our approach principally depends on the number  $s$  of changing pixels. The larger  $s$ , the more consuming the method is. We then have tested

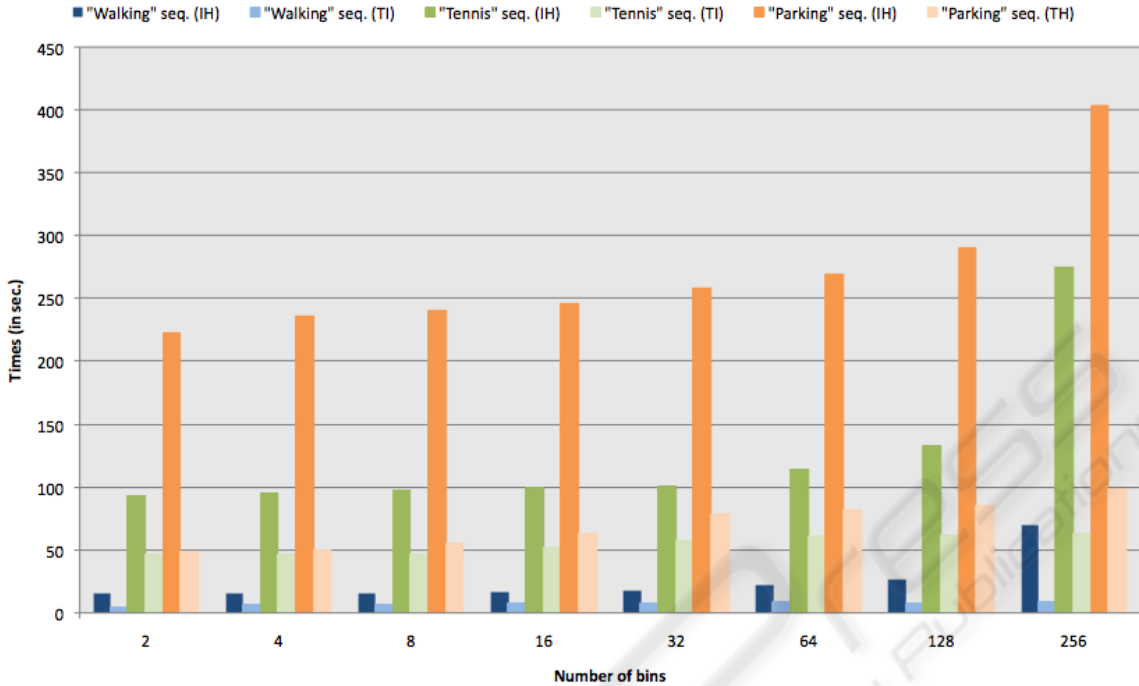


Figure 3: Tests on different video sequences (left column for an example of frame of, from top to bottom “Walking”, “Tennis” and “Parking” sequences). Bar diagram of an histogram computation time for both approaches, for different sequences and increasing number of bins: “Walking” in blue, “Tennis” in green and “Parking” in orange (IH is represented as plain color, TH as a transparency color).

the computation time as a function of  $s$  and compared results with those obtained by the IH method.

In the first test, on the “Walking” video sequence,  $I_1$  is used as reference image and we evaluated the histogram computation time for a region in the next frame. Tests have been performed in frames  $I_1$ ,  $I_2$  and  $I_{10}$ , in which respectively 0%, 25% and 40% of the pixels vary from the first frame. Results are shown in Figure 4, for different values of  $B$ . We can see that the computation time of our approach increases as the number of changing pixels increases, as highlighted in Section 4, but stay below the one obtained with IH.

For the second test we have generated synthetic  $N \times N$  images for different values of  $N$  and compared times for the computation of the histogram with  $B = 16$  bins of this whole image (no pixel variation) for both approaches. Comparative results (in seconds) are reported in Table 1. The increase of  $N$  does not influence a lot our approach, much while drastically decreasing IH performance. As no pixel have changed between the two considered frames, the small time computation increasing for TH is just due to the pixel scanning of the new image that takes more time for a large image than a smaller one: this explains why the time computation for TH is equal to 0.0064 seconds for  $N = 256$  and to 0.4 for  $N = 2048$ .

Table 1: Time computation (in sec.) of an histogram with  $B = 16$  bins depending on the size of the image.

$N$	256	512	1024	2048
IH	0.8	3.23	12.9	52.1
TH	0.0064	0.02	0.09	0.4

The third test consists in considering a  $1024 \times 1024$  synthetic image and simulating a number  $s$  of changing pixels, then computing the histogram with  $B = 16$  bins of this new image. We have compared the computation times between both approaches: results are reported in Table 2. The computation time with our approach stays below IH’s one until  $s = 10^6$ . This is not surprising, because our approach depends on the number of changing pixels between images. Anyway,  $s$  should has to have a large value before increasing drastically our computation time.

Table 2: Time computation (in sec.) of a histogram with  $B = 16$  bins of a  $1024 \times 1024$  synthetic image after having  $s$  changed pixels.

$s$	$10^2$	$10^3$	$10^4$	$10^5$	$10^6$
% changing pixels	0.01	0.1	1	10	100
IH	12.9	13	12.9	13.1	12.9
TH	0.14	0.24	0.47	2.49	27.6

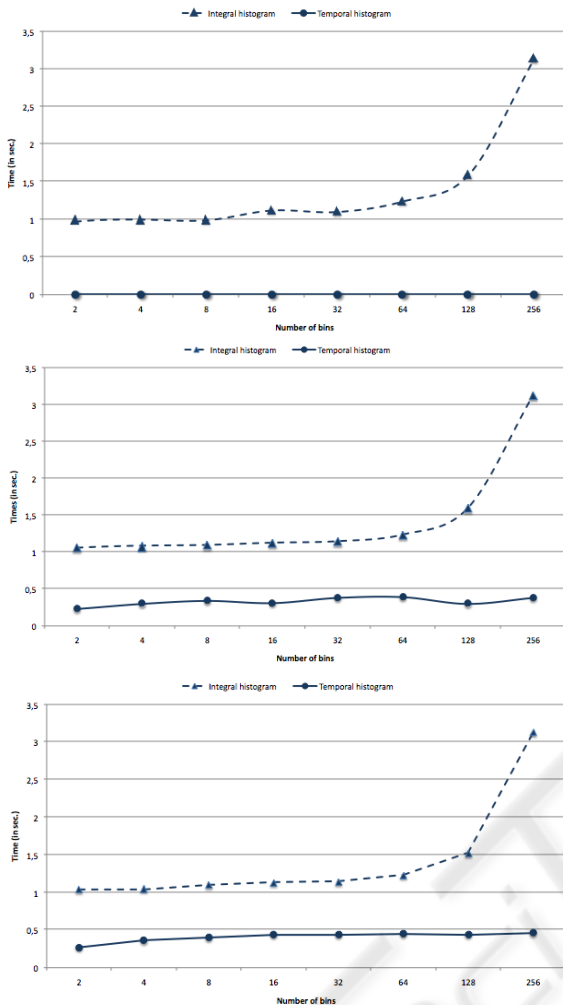


Figure 4: Comparison of integral histogram (IH) and temporal histogram (TH) depending on the number of changing pixels between frames (“Walking” sequence,  $275 \times 320$  frames). From top to bottom: 0% (between  $I_1$  and  $I_1$ ), 25% (between  $I_1$  and  $I_2$ ) and 40% (between  $I_1$  and  $I_{10}$ ) changing pixels.

### 5.1.3 Number of Histogram Computations

The most time consuming part of IH is the construction of the array. However this array allows computing very quickly any histogram (or set of histograms) using only four operations per histogram. In this subsection, we have launched a massive number of histogram computations and compared both approaches in terms of computation time. The idea is to simulate the computation of target histograms in a search window around a precise position, such as in spatial filtering or temporal filtering (particle filtering for example). Test have been made on the “Walking” sequence. We have chosen to present the results for different values of  $B$ . Results are shown in Figure 5.

Once again the performance depends on the quantization of the histograms. For a strong quantization ( $B = 2$  or  $4$ ), IH and TH become equivalent for 1000 computations of histograms. For  $B = 8, 16, 32$ , methods are equivalent for 5000 computations. For  $B = 128$ , 10000 computations are needed, and 25000 for  $B = 256$ . This interesting result shows that we can keep good results (compared to IH) with no need for a strong quantization of the histogram: TH does not need to approximate histograms to provide good computation times, which is a real advantage for histogram based search applications. We can however see one limitation of the proposed approach when dealing with too much histograms. In Section 4.2 we mentioned that TH is better than IH as long as the number of changing pixels is less than twice the number of bins on the histogram. It is clear that the performances of TH then depends on the number of changing pixels between the two considered regions on which we compute histograms, and that is the reason why our computation times increase with the number of computed histograms considered. We can notice that for a small histogram quantization, we give very good results. Our TH is the suitable for visual tracking applications where it is better not to quantify histograms too much.

## 5.2 Storage

In this section we show that our approach does not need to store a lot of information, contrary to integral histogram. Table 3 reports the size (number of elements) required for the two data structures used for the test shown in Figure 4, middle row (between two consecutive frames of the sequence). The number of elements necessary for IH increases with the number of bins, according to the results of Section 4.4, in which we found  $(c)_{IH} = NMB$ . For TH, it depends on the number of changing pixels between the two frames,  $(c)_{TH} = n_r + 3s$ . A pixel is said to be changing between the two images if he changes its bins in the histogram. This notion then strongly depends on the histogram quantization: the more histogram is quantified, the less a pixel changes its bins between two images. That is the reason why the number of elements of our data structure indirectly depends on  $B$ . Note that the number of elements needed for IH does not change if images are really different, which is not the case for TH. We report in Table 4 the size of these data structures depending on the number  $s$  of changing pixels between two images generated as random  $1024 \times 1024$  matrices (same experiments as in Section 5.1.2). We fix  $B = 16$ . For this case, the number of elements necessary for integral histogram

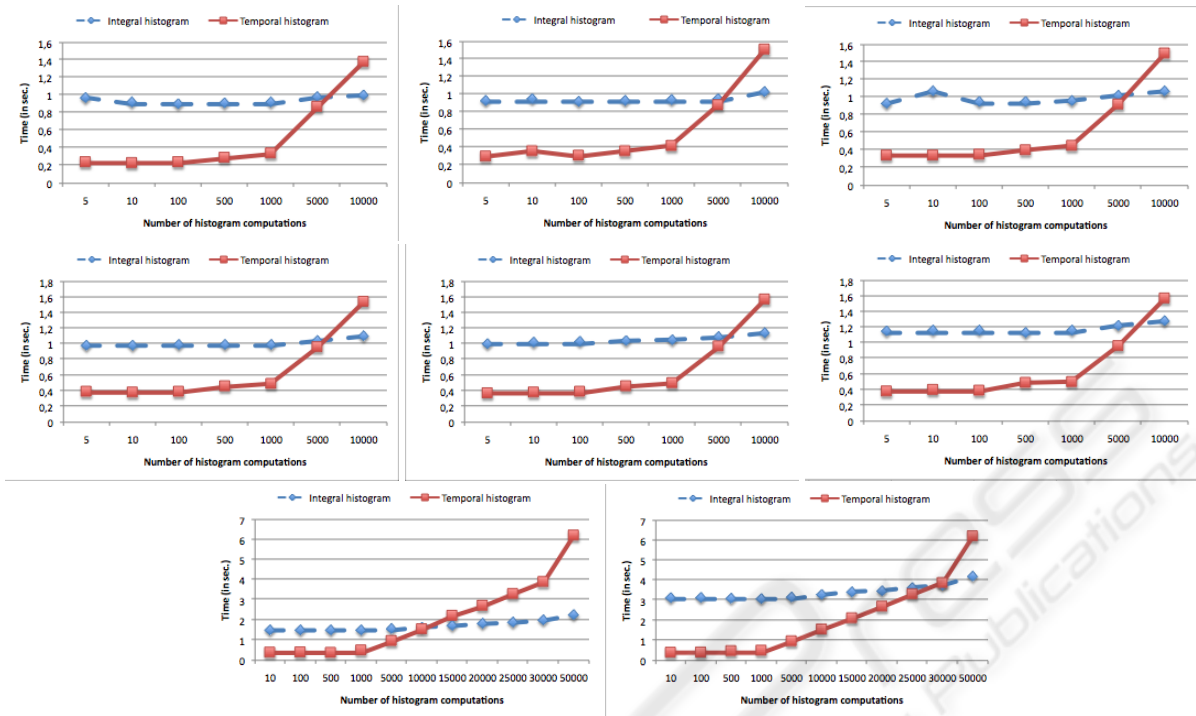


Figure 5: Comparison of IH (dotted lines) and TH (plain lines) results for a massive number of histogram computations, for different values of  $B$ , from top to bottom, from left to right, 2, 4, 8, 16, 32, 64, 128 and 256.

Table 3: Size (number of elements) of data structures required for both approaches, depending on  $B$ , the test corresponds to the one of the middle row of Figure 4.

$B$	IH	TH
2	$1.76 \times 10^5$	$9.1 \times 10^3$
4	$3.52 \times 10^5$	$1.78 \times 10^4$
8	$7.04 \times 10^5$	$2.93 \times 10^4$
16	$1.4 \times 10^6$	$4.5 \times 10^4$
32	$2.81 \times 10^6$	$4.5 \times 10^4$
64	$5.63 \times 10^6$	$4.5 \times 10^4$
128	$1.12 \times 10^7$	$4.53 \times 10^4$
256	$2.25 \times 10^7$	$4.53 \times 10^4$

Table 4: Size (number of elements) of data structures required for both approaches, depending on the number  $s$  of changing pixels between two images generated as random  $1024 \times 1024$  matrix, for  $B = 16$ . Percentage (%) of changing pixel are reported on the second line of this table.

$s$	$10^2$	$10^3$	$10^4$	$10^5$	$10^6$
%	0.01	0.1	1	10	100
IH	$1.6 \times 10^7$	$1.6 \times 10^7$	$1.6 \times 10^7$	$1.6 \times 10^7$	$1.6 \times 10^7$
TH	$1.3 \times 10^3$	$3.8 \times 10^3$	$2.9 \times 10^4$	$2.8 \times 10^5$	$2.8 \times 10^6$

is fixed such that  $(c)_{\text{IH}} = NMB = 1024 \times 1024 \times 16 = 1.6 \times 10^7$ . Even considering  $10^6$  changing pixels (i.e. 100% of the initial image) in the region the histogram is computed, the number of elements needed to store it is always below the one IH needs.

To our opinion, TH is a good alternative to his-

ogram computation in a lot of cases because it gives a compact description of temporal change and good computation time results for histogram computation. Moreover, we never need to store histograms (except for the reference image), that is a real advantage when working on video sequences (for these cases, the reference image is the first of the sequence, and the histogram computation can be seen as a preprocessing step).

## 6 INTEGRATION INTO PARTICLE FILTER: FAST PARTICLE WEIGHT COMPUTATION

A good tracker should be able to predict in which area of a new frame the object is. Among all the methods, one can cite probabilistic trackers. In such approaches, an object is characterized by a state sequence  $\{x_k\}_{k=1, \dots, n}$  whose evolution is specified by a dynamic equation

$$x_k = f_k(x_{k-1}, v_k)$$

The goal of tracking is to estimate  $x_k$  given a set of observations. The observations  $\{y_k\}_{k=1, \dots, m}$ , with  $m < n$ ,



are related to the states by

$$y_k = h_k(x_k, n_k)$$

Usually,  $f_k$  and  $h_k$  are vector-valued, nonlinear and time-varying transition functions, and  $v_k$  and  $n_k$  are white Gaussian noise sequences, independent and identically distributed. Tracking methods based on particle filters (Gordon et al., 1993; Isard and Blake, 1998) can be applied under very weak hypotheses and consist of two main steps:

1. a prediction of the object states in the scene (using previous information), that consists in propagating particles according to a proposal function (see (Chen, 2003)) ;
2. a correction of this prediction (using an available observation of the scene), that consists in weighting propagated particles according to a likelihood function.

Joint Probability Data Association Filter (JPDAF) (Vermaak et al., 2004) provides an optimal data solution in the Bayesian framework filter and uses a weighted sum of all measurements near the predicted state, each weight corresponding to the posteriori probability for a measurement to come from an object. Between two observations, the set of particles evolves according to an underlying Markov chain, following a specific transition function. Given a new observation, each particle is assigned a weight proportional to its likelihood of belonging to a tracked object. New particles are randomly sampled to favor particles with higher likelihood. A classical approach consists in integrating the color distributions given by histograms into particle filtering (Pérez et al., 2002), by assigning a region (e.g. validation region) around each particle and measuring the distance between the distribution of pixels in this region and the one in the area surrounding the object detected in a previous frame. This context is ideal to test and compare our approach in a specific framework.

For this test we measure the total computation time of processing particle filtering in the first 60 frames of the ‘‘Rugby’’ sequence ( $240 \times 320$  frames, see a frame in Figure 6): we are just interested on the  $B = 16$  bin histogram computation time around each particle locations (that is the point of our paper). In the first frame of the sequence, the validation region (fixed size  $30 \times 40$  pixels) containing the object to track (one rugby player) is manually detected. JPDAF is then used along the sequence to automatically track the object using  $N_p$  particles. Then, the total computation times needed for each method is detailed below:

- For IH: one integral histogram  $H_i$  in each frame  $i = 1, \dots, t$  of the sequence, then  $N_p$  histogram

(one for each particle) computation using four operations on  $H_i$ .

- For TH: one integral histogram  $H$  (only in the first frame), one tree  $T_i$  construction in each frame  $i = 1, \dots, t$  of the sequence, then, a histogram update for each particle using  $H$  and  $T_i$ .

Computation times are reported in Table 5 for different values of  $N_p$ . Computation times are lower with our approach until  $N_p = 5000$  (tests have shown that for  $N_p = 8500$ , computation times are the same for both methods). Note that, in practice, we do not need so much particles in a classical problem. Our approach permits real-time particle filter based tracking for a reasonable number of particles, which is a real advantage. Note that the purpose of this test was not to deal with tracking performances (that is the reason why we do not give any results about quality results): we just want to show that integrating TH into particle filter correction step instead of IH can accelerate the process. Moreover, we have shown than for similar computation times, we can use more particles into the frameworks integrating TH. As it is well-known (Gordon et al., 1993) that the particle filter converges with a high number of particles  $N_p$ , we can argue that integrating TI into a particle algorithm improves visual tracking quality. Note that we have obtained same kinds of results on different video sequences (people tracking on ‘‘Parking’’ sequence and ball tracking on ‘‘Tennis’’ sequence’’)

Table 5: Total computation times (in sec.) for all histograms ( $B=16$ ) in the particle filter framework, depending on the number  $N_p$  of particles.

$N_p$	50	100	1000	5000	10000
IH	47.75	47.4	47.5	50.48	53.59
TH	23.5	23.6	28.31	40.4	76.24

## 7 CONCLUSIONS

We have presented in this paper a new method for fast histogram computation, called temporal histogram (TH). The principle consists in never encoding histograms, but rather temporal changes between frames, in order to update a first preprocessed histogram. This technique presents two main advantages: we do not need a large amount of information to store whole histograms and it is less time consuming for histogram computation. We have shown by theoretical and experimental results that our approach outperforms the well-known integral histogram in terms of total computation time and quantity of information to store. Moreover, the introduction of TH into the particle filtering framework has shown its usefulness for real-

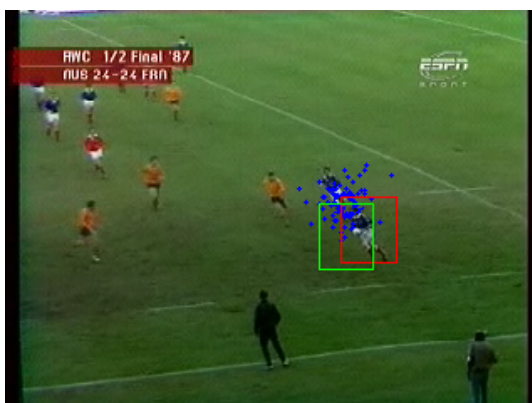


Figure 6: Example frame of the “Rugby” sequence: red rectangle is the validation region, green one the target region associated to one particle. Blue crosses symbolize particle positions in frame around which we compute histograms.

time applications in most common cases. Integral histogram requires the computation of the accumulator array in each new image which takes a lot of time (rarely taken into account in classical approaches). TH computes histogram only if necessary (i.e. some changes between images have been detected). Future works will concern the generalization of this reasoning on different distance computation between histograms, that requires to work directly on histogram bins (Bhattacharyya (Bhattacharyya, 1943),  $L_1$  norm, euclidean distance *etc.*): the update would be done on this distance, not on the histogram.

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