# STRUCTURE FROM MOTION OF LONG VIDEO SEQUENCES

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Abstract: In this paper we introduce an approach for "Structure from Motion" from long video sequence. Our approach starts from an initialization of first several frames and adopts an incremental strategy to allow more frames to be added into the SFM system. The main contribution lies in that we introduce an update propagation to modify the entire SFM system to accommodate changes brought by the local bundle adjustment applied to newly added frames. With this step, our approach gains a significant accuracy improvement at a cost of relatively small extra computation overhead.

## **1 INTRODUCTION**

This paper presents a structure from motion approach for long video sequences with fixed intrinsic camera parameters. The term "Structure from motion" (SFM) (Hartley and Zisserman, 2004) refers to the process of automatically recovering camera parameters and reconstructing the 3D scene structure from recorded 2D images. It has been a central issue of computer vision for decades and there are many implemented systems (2d3, 2000; Digi-lab, 2005). Many applications of SFM can be found in different fields, such as imagebased rendering , virtual reality and stereo.

Most SFM algorithms are based on point correspondence form different views, from which the epipolar geometry, e.g., the fundamental matrix for two-views, can be extracted. A projective reconstruction of camera parameters can be calculated from the epipolar geometry, which is then upgraded to a metric one using the intrinsic camera parameter. Once cameras are recovered, 3D points can be constructed using the triangulation. Up to this stage, we are given a very coarse estimate, upon which a further optimization is required to minimize the reprojection error. For normal pinhole cameras, the most common optimization approach is the bundle adjustment (Triggs et al., 1999), which minimizes reprojection errors by simultaneously refining the 3D structure and cameras in a non-linear style. The bundle adjustment requires an initialization for following iterative adjustments. Currently, most bundle adjustment algorithms are implemented using the Levenberg-Marquardt method (Hartley and Zisserman, 2004; Lourakis and Argyros, 2009), which, compared to the traditional Gauss-Newton method, is faster and less sensitive to poor initial estimates. However, a good initialization is still of great importance, because it can provide a faster convergence for the iteration process, and more crucially, in some cases, it can even determine the success of the bundle adjustment.

For long sequences, the reconstruction usually starts from several reference frames, and an incremental mechanism is necessary to allow more views to be added into the current system. As the sequence length increases, the accumulated error may cause the initial estimate far away from the real one, and therefore, provide the bundle adjustment a poor basis. To solve this problem, one may suggest performing the bundle adjustment over all frames so far handled at each increment step. However, the bundle adjustment is computationally expensive. According to (Shum et al., 1999), the complexity of each iteration of bundle adjustment is  $O(m \cdot n^3)$ , where m is the number of 3D points and n is the number of frames. To improve the efficiency, it is desired to reduce the number of fames involved. For the incremental SFM system, the local bundle adjustment, which restricts the refinement to only several frames around the currently added one, is preferable (Zhang and Shan, 2003; Mouragnon et al., 2009; Zhang et al., 2007).

This paper aims to deliver a robust incremental

SFM approach, which supplies a good starting point for global bundle adjustments. In addition, such a incremental system itself is useful in many applications. Imagining the case that a global bundle adjustment is not affordable due to the efficiency requirement, an incremental SFM approach which can provide a result with almost the same precision is really needed.

The local bundle adjustment is adopted in our approach for incrementally expanding the current reconstruction. A problem arising is that the local bundle adjustment of each expansion step might change the existing reconstruction. Obviously these changes will bring extra errors to the previous reconstruction. And more importantly, with increasing sequence length, these relatively small changes can be accumulated to a sufficient amount that invalidate the previous reconstruction. In this case, the incremental algorithm is at risk of losing the basis for further expansion. To alleviate this problem, we adopts an update propagation method, which modify the entire reconstruction to cater for changes brought by the local bundle adjustment. Compared to the pure local bundle adjustment, our approach is clearly slower due to the extra expense of the update propagation, but gains a significant accuracy improvement.

In this paper, we assume the intrinsic parameter of the camera is known and fixed, i.e., we will not deal with the self-calibration problem.

The rest of this paper is organized as: Section 2 introduces some math notations used in this paper and presents an overview of our approach. Section 3 describes how the reconstruction is initialized from reference frames. Section 4 introduce our incremental method for reconstruction expansion. Section 5 presents the results. Section 6 concludes this paper.

#### 2 THE FRAMEWORK

We first introduce the math notation used throughout this paper. Suppose we are given a sequence with *n* frames:  $\{I_i\}_0^{n-1}$ . For each  $I_i$ , the corresponding camera is modeled by the intrinsic parameter *K*, which is a 3 × 3 up-triangle matrix, (we assume *K* remains unchanged across the frame), and the extrinsic parameter  $[R_i | t_i]$ , where  $R_i$  is a 3 × 3 orthonormal rotation matrix and  $t_i = (tx_i, ty_i, tz_i)$  is a translation vector. The projection matrix of such a camera is that:  $P = K \cdot [R_i | t_i]$ .

To reduce the parameter number, the rotation of each camera can be described by its Euler angle:  $\omega_i = (\alpha_i, \beta_i, \gamma_i)$ . In this case, the camera  $C_i$  is parameterized by a 6-vector:  $C_i = (\alpha_i, \beta_i, \gamma_i, tx_i, ty_i, tz_i)$ . The projection of a 3D points  $X_j = [x_i, y_i, z_i]^T$  onto a 2D

image by  $C_i$  can be expressed by a non-linear function  $\Theta$  such that:  $\Theta(C_i, X_i)$ .

For a sequence to be reconstructed, sparse points are matched consecutively, i.e.,  $I_1$  against  $I_0$ ,  $I_2$ against  $I_1$  and etc. The point matching is based on certain feature tracking techniques, (we have tried bother SIFT (Lowe, 2004) and KLT points (Shi and Tomasi, 1994)). The Random Sample Consensus (RANSAC) algorithm is used to fit the fundamental matrix that encapsulates the epipolar constraint. Therefore, the track of a point can be lost in a given frame for two reasons: there is no matched point or the match does not conform to the epipolar constraint. For each track, a 3D point can be constructed if corresponding camera parameters are known.

If the camera moves slowly, we resample the sequence to select some keyframes that have wider base-lines. In addition, since information contained in a long track is more reliable than a short one. We only consider tracks that are not shorter than certain minimal track length (T). T is usually set to be 3 keyframes.

The reconstruction process is initialized from first several frames. Then more frames are added incrementally to expand the current reconstruction. Each expansion is accomplished by two steps: 1. Firstly, the local bundle adjustment is performed over several neighbouring frames around the newly added frames. 2. Then, the update brought by the local bundle adjustment is propagated to the entire reconstruction. After all frames are processed, if the remaining reprojection error is still large, an optional global bundle adjustment is applied to refine the result. Finally, Cameras of non-keyframes are calculated based on the reconstructed 3D points, if the sequence has been resampled.

## 3 RECONSTRUCTION INITIALIZATION

The first  $(0^{th})$  frame is always selected as the reference frame, such that  $R_0 = I$  and  $t_0 = 0$ . We select the first s ( $s \ge 3$ ) frames to initialize the reconstruction. The length of s is based on how many tracks can be seen from both the  $0^{th}$  and the  $i^{th}$  (0 < i < s) frame. The minimal number of such tracks is set to 30. For each of non-reference initialization frames, its fundamental matrix against the reference frame  $F_i$  is discovered based on inlier matches. With the intrinsic matrix K, the epipolar constraint can be upgraded from the fundamental matrix to an essential matrix  $E_i$ :  $E_i = K^T \cdot F_i \cdot K$ .

Then, the essential matrix is decomposed into an

orthonormal matrix corresponding to the rotation matrix and a skew-symmetric matrix corresponding to the translation vector:

$$E_i = [t_i]_{\times} R_i \tag{1}$$

The decomposition is implemented using the singlular value decomposition (SVD), which returns two twisted configurations of  $R_i$  and two reflected configurations of  $t_i$ , therefore in total four combinations. To remove the ambiguity, these four configurations are tested with a single reconstructed 3D point to make sure that such a point is in front of both the  $0^{th}$  and  $i^{th}$  camera (Hartley and Zisserman, 2004). Once the rotation matrix is computed, its corresponding Euler angle  $\omega_i$  can be extracted.

Since translation vectors achieved from the above process are normal vectors that only parameterizes directions, relative scales have to be discovered. We define the translation between the  $0^{th}$  and  $1^{st}$  camera as a reference scale. The following scale of translation can be determined using the 3D points triangulated from the  $0^{th}$  and  $1^{st}$  camera. Suppose that the  $x_{ij}$  is the observed 2D point of the projection from  $j^{th}$  3D point onto the  $i^{th}$  frame, the problem of determining the scale l is formulated into a linear Least Squares estimation:  $A \cdot l = B$ . Each visible  $X_j$  provides two linear functions:

$$\underbrace{\left[\begin{array}{c} \left((K\cdot t_{i})^{1\top}-u_{ij}\cdot(K\cdot t_{i})^{3\top}\right)\\ \left((K\cdot t_{i})^{1\top}-v_{ij}\cdot(K\cdot t_{i})^{2\top}\right)\end{array}\right]}_{A_{j}} \cdot l = \underbrace{\left[\begin{array}{c} u_{ij}\cdot(K\cdot R_{i}\cdot X_{j})^{3\top}-(K\cdot R_{i}\cdot X_{j})^{1\top}\\ v_{ij}\cdot(K\cdot R_{i}\cdot X_{j})^{3\top}-(K\cdot R_{i}\cdot X_{j})^{2\top}\end{array}\right]}_{B_{j}}_{B_{j}}$$
(2)

where  $()^{i\top}$  denotes the *i*<sup>th</sup> row of a matrix. The linear system is solved by SVD.

After all the initialization frames are processed, tracks falling into this segment are selected to construct the corresponding 3D points using the linear DLT triangulation (Hartley and Zisserman, 2004). Then the bundle adjustment is applied to guarantee an accurate initialization. Suppose that the Euclidean distance between the observed image and the reprojected point from the estimated  $X_j$  and  $C_i$  is denoted by  $d(x_{ij}, \Theta(C_i, X_j))$ . Assuming that, so far, we have been given *s* frames and  $m_c$  3D points (or tracks). The goal of the bundle adjustment is to find an optimal estimation that has the maximal likelihood given the observed data, in other words, minimizes the sum of distances between the observed 2D points and the 2D points predicted by the estimation, specifically:

$$\min_{\{C_i\}_1^{s-1}, \{X_j\}_0^{m_c-1}} \sum_{j=0}^{m_c-1} \sum_{i=1}^{s-1} d(x_{ij}, \Theta(C_i, X_j))^2$$
(3)

There are in total  $6 * (s - 1) + 3 * m_c$  parameters involved in the minimization, as we assume the ref-

erence frame is fixed. It is usually not reliable to initialize from only first several frames. In practice, we make use of more frames s', which can be chosen based on the total length of the sequence, say  $\frac{n}{5}$ . The reconstruction is extended from *s* to *s'* using the expansion strategy presented in the next section.

## 4 RECONSTRUCTION EXPANSION

Suppose that frames from  $I_0$  to  $I_{i-1}$  has been processed, so that the corresponding cameras  $C_0$  to  $C_{i-1}$  are recovered and a set of 3D points are reconstructed. The new frame  $I_i$  is added into the current SFM system. We initialize the camera  $C_i$  using the previously reconstructed 3D points that are visible in  $I_i$ . Suppose that there are  $m_v$  such 3D points, the cost function for calculating  $C_i$  is that:

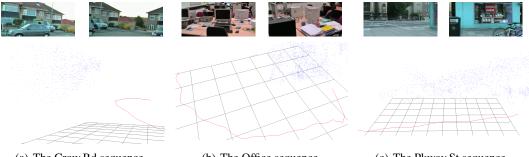
$$\min_{C_i} \sum_{j=0}^{m_v - 1} d(x_{ij}, \Theta(C_i, X_j))^2$$
(4)

For each expansion step, we add into the SFM system *h* new frames. *h* varies according to how many 3D points can be seen in the newly added frame. The minimal visible 3D points are usually set to be  $m_v = 30$ . For the local bundle adjustment, an overlapping number *o* is chosen to set how many previously recovered cameras are involved in the current refinement, i.e., the local bundle adjustment is performed over frames  $\{I_{i-o}, ..., I_{i-1}, ..., I_{i+h-1}\}$ . In order to maintain information from previous reconstruction, the fixation number *fix* is set to prescribe how many frames starting from  $I_{i-o}$  should keep unchanged in the current local bundle adjustment ( $2 \le fix \le o$ ).

3D points that have been refined by the current local bundle adjustment can be divided into two groups: those have been previously reconstructed and can be seen from frames  $\{I_0, ..., I_{i-o-1}\}$ , which is denoted by  $\mathcal{V}$ , and those cannot. For the former, changes brought by the local bundle adjustment have to be propagated. For each frame falling in  $\{I_0, ..., I_{i-o-1}\}$ , we check if there is a 3D point from  $\mathcal{V}$  is visible in that frame. If so, we refine the corresponding camera *C* using 4. The minimization process takes the current parameter as the initial guess. An illustration of this process is presented in Figure 1.

After cameras of all the affected frames haven been updated, a further reconstruction of all 3D points that have been involved so far is introduced, such that:

$$\min_{\{X_j\}_0^{m-1}} \sum_{j=0}^{m-1} \sum_{i=1}^{i+h-1} d(x_{ij}, \Theta(C_i, X_j))^2$$
(5)



(a) The Crow.Rd sequence

(b) The Office sequence

(c) The Pkway.St sequence

Figure 2: Example frames of testing sequence and visualizations of reconstructed 3D points and cameras.

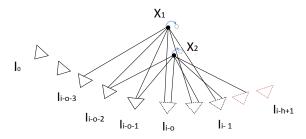


Figure 1: Illustration of the local bundle adjustment and update propagation: Newly added cameras are denoted by red dashed lines and camera involved in current local bundle adjustment are denoted by dashed line. Suppose that the previously constructed 3D points  $X_1$  and  $X_2$  are updated due to the current local bundle adjustment,  $X_1$  is visible in  $I_{i-o-3}$ ,  $I_{i-o-2}$  and  $I_{i-o-1}$ ,  $X_2$  is visible in  $I_{i-o-2}$  and  $I_{i-o-1}$ . Therefore, these cameras have to be updated.

(Steedly and Essa, 2001) proposes an approach in a similar style to incorporate updates brought by new frames. However, we are more concerned with the local information propagation, rather than achieving a global minimum for each expansion step. As mentioned above, after cameras of affected frames are solved, we reconstruct 3D points assuming that all camera parameters are optimized. Compared to the section-resection framework that alternates between the process of camera parameters estimation and 3D reconstruction (Mahamud et al., 2001), this is conceptually equivalent to forcing such process to stop at that moment, otherwise our update propagation will become another global bundle adjustment. Indeed, we will show in Section 5 that this will not incur a large precision lost.

## **5 EXPERIMENTS**

Our SFM system is implemented using the C++ programming language on the WindowsXP platform. The experiment is conducted on a desktop PC with In-

Table 1: Statistics of test sequences.

	Crow.Rd	Pkway.St	Office	
Frames N	810	227	672	
Keyframes N	73	75	112	
Tracks (3D pts) N	1712	2020	2784	
Projection N	12496	16135	22441	
Overlapping (o) N	5	8	5	
Fixation (fix) N	5	6	5	
Initialization (s') N	15	15	22	

tel Pentium 2.40 GHz CPU and 1.50GB RAM memory.

Three video sequences are used as our testing data. The Crowther Road sequence is captured around a corner of a living neighborhood of Bristol with a hand-held camera. The Parkway street sequence captures a business street of Bristol with a camera mounted on a moving vehicle and The Office sequence films scenes of our office with a hand-held camera. All these three sequences are not closed. Statistics of these three sequence are presented in Table 1. Reconstructions are visualized (using our 3D visualizer) in Figure 2 together with some example input frames.

To show the strength of our incremental approach (local bundle adjustment plus update propagation), we compare our result with those of pure local bundle adjustment and global bundle adjustment. For simplicity, these three approaches are denoted as LBA+UP, LBA, and GBA respectively. For comparison, we consider two facts, the accuracy and the computation cost, which are used to demonstrate that with a relatively small expense, our LBA+UP approach is much more robust than the LBA approach, and sometimes it is even comparable with the GBA approach. In addition, for showing that our LBA+UP is able to provide a more solid basis for a final GBA, we also compare results of two types of GBA, one takes the traditional LBA result as the initial estimate, and the

	RMSE (pixel)			Time (Second)			RMSE of GBA based on LBA
	LBA	LBA+UP	GBA	LBA	LBA+UP	GBA	Daseu on LDA
Crow.Rd	2.11884	0.906058	0.794769	124.772	170.519	400.033	1.62013
Pkway.St	3.36435	0.898319	0.568826	176.262	247.37	355.945	1.36299
Office	1.86079	0.745086	0.663738	133.615	238.804	591.199	0.671995

Table 2: Comparison results of different approaches.

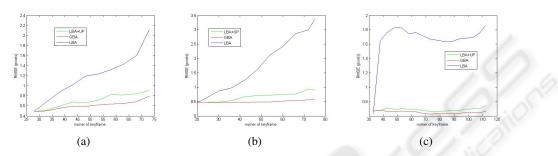


Figure 3: Visualization of how accuracy varies with the increasing keyframe number. From left to right: the Crow.Rd, the Pkway St and Office.

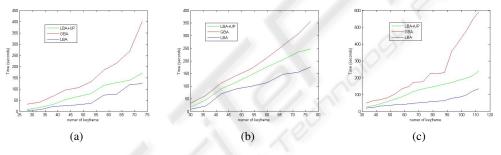


Figure 4: Visualization of how computation cost varies with the increasing keyframe number. From left to right: the Crow.Rd, the Pkway St and Office.

other takes our LBA+UP result as the initial guess. For each sequence, experiments are carried out with the same parameters, such as the overlapping segment length (o), the fixation number fix, the initialization length s', and other arguments like stopping conditions of the bundle adjustment.

Since there is no ground truth available, the reprojection error is used to measure the accuracy. The reproejction error is expressed by the Residual Mean Squared Error (in pixels):  $RMSE = \frac{\sum d(x_{ij},\Theta(C_i,X_j))}{N_{proj}}$ , where  $N_{proj}$  is the total number of projection. The computation cost is measured by the time (in seconds). We only record the time related to solving cameras and 3D points, as the efficiency of feature tracking is not considered in this paper.

Table 2 presents comparison results, from which it can be seen that: for long sequences, the performance of pure LBA is not reliable, while the accuracy of our LBA+UP approach is quite close to the GBA (here it takes the result of LBA+UP as the initial guess). Although our LAB+UP would bring some extra overhead, compared to the GBA, it still saves a lot of time. Considering the high precision improvement, we would say that it is worth of accepting such relatively small efficiency lost.

In addition, we also present the RMSE of GBA that takes the LBA as its initial guess, (the last column of Table 2). Compared to that based on the LBA+UP (the third column), we can see that our approach provides a much better starting point for the GBA.

Figure 3 and 4 visualize how those comparison statistics changes with the increase of sequence length. It is can be observed that with the sequence extended, the accuracy of LBA drops rapidly and the computation cost of GBA grows steadily, while our LBA+UP approach achieves a best tradeoff between them.

#### 6 CONCLUDING REMARKS

In this paper we present a new incremental SFM approach with fixed intrinsic camera parameter. Apart from the local bundle adjustment carried out for each expansion step, we introduce an update propagation step which modify the entire current reconstruction system to cater for changes brought by the local adjustment. Experiments on real data shows our approach works much better than those with merely local bundle adjustment, in that it is more accurate itself and provides a better initial guess for the global bundle adjustment.

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