# NAVIGATION AND FORMATION CONTROL EMPLOYING COMPLEMENTARY VIRTUAL LEADERS FOR COMPLEX MANEUVERS

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Abstract: Complex maneuvers of formations of car-like autonomous vehicles are investigated in this paper. The proposed algorithm provides a complete plan for the formation to solve desired tasks and actual control inputs for each robot to ensure collision-free trajectories with respect to neighboring robots as well as dynamic obstacles. The method is based on utilization of complementary virtual leaders whose control inputs are obtained in one merged optimization process using receding horizon control methodologies. The functionality of the system, which enables reverse driving and arbitrary rotations of formations of nonholonomic robots, is verified by simulations of multi-robot tasks and by hardware experiments.

## 1 INTRODUCTION

In this paper, we present a novel approach for a nonholonomic robots formation driving and a trajectory planning to reach a target region. The method allows arbitrary maneuvers, including reverse driving and turning of the formation in limited environment with static as well as dynamic obstacles. Such skills are crucial for the control of autonomous ploughs during cooperative airport snow shoveling (Saska et al., 2008; Hess et al., 2009) being our target applications.

Tasks for which autonomous vehicles in formation are used usually involve physical constraints imposed by the vehicles (mobility constraints) or physical obstacles (environment constraints) and constraints enforced by inter vehicle relations (shape of the formation). Thus, it is important for control methodologies to incorporate system's constraints into the controllers design while preserving overall system stability which makes the Receding Horizon Control (RHC) especially appealing. Receding horizon control (also known as model predictive control) is an optimization based control approach often used for stabilizing linear and nonlinear dynamic systems (*e.g.*, see (Alamir, 2006) and references reported therein). For a detailed survey of RHC methods we refer to (Mayne et al., 2000) and references reported therein. The works applying RHC for formation driving control are presented in (Dunbar and Murray, 2006) (Franco et al., 2008). These papers have utilized RHC for the formation forming and/or following predefined trajectory in a workspace without obstacles. We will apply the Receding Horizon Control for the virtual leaders trajectory planning to desired goal area and for the followers stabilization in the formation.

Our contribution is a general approach that accounts for a nonholonomic robots' formation stabilization and a trajectory planning (enforced by a stability constraint) to reach a target region, also considering the response to dynamic changes in the environment. We developed a new concept of RHC by combining both, the trajectory planning to the desired goal region and the immediate control of the formation, into one optimization process. Our method can continuously respond to changes in environment of the robots while the cohesion of the immediate control inputs with direction of the formation movement in future is kept. Furthermore, we extended the common leader-follower concept with the idea of two virtual leaders, one for the forward and one for the backward movement. Such an approach is necessary for the complicated maneuvers of the formation of car-like robots. The proposed method provides a complete plan together with decisions when and how often to switch between the guidance of the virtual leaders.

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# 2 PRELIMINARIES

Let $\psi_L(t) = \{x_L(t), y_L(t), \theta_L(t)\}$ denote the configuration of a virtual leader $R_L$ at time $t$,[1] and $\psi_i(t) = \{x_i(t), y_i(t), \theta_i(t)\}$, with $i \in \{1,\dots,n_r\}$, denote the configuration for each of the $n_r$ followers $R_i$ at time $t$. The Cartesian coordinates $(x_j(t), y_j(t))$, $j \in \{1,\dots,n_r,L\}$ for an arbitrary configuration $\psi_j(t)$ define the position $\bar{p}_j(t)$ of a robot $R_j$ and $\theta_j(t)$ denotes its heading. Let us assume that the environment of the robots contains a finite number $n_0$ of compact obstacles collected in a set of regions $\mathcal{O}_{obs}$. Let us also define a target region $S_F$ as a circle with radius $r_{S_F}$ and center $C_{S_F}$ such that $\mathcal{O}_{obs} \cap S_F = \emptyset$. Finally, we need to define a circular detection boundary with radius $r_s$ and a circular avoidance boundary with radius $r_a$, where $r_s > r_a$. Single robots should not respond to obstacles detected outside the region with radius $r_s$. On the contrary, distance between the robots and obstacles less than $r_a$ is considered as inadmissable. These concepts are adopted from the concept of avoidance control (Leitmann, 1980) to be used in this paper for the collision avoidance guaranties. Moreover, we need to extend these zones for the virtual leaders depending on size of the formation to ensure that the result of the leader trajectory planning is feasible also for the followers. We will denote the extended radiuses as $r_{s,L}$ and $r_{a,L}$.

## 2.1 Kinematic Model and Constraints

The kinematics for any robots $R_j$, $j \in \{1,\dots,n_r,L\}$, in the formation is described by the simple nonholonomic kinematic model: $\dot{x}_j(t) = v_j(t)\cos\theta_j(t)$, $\dot{y}_j(t) = v_j(t)\sin\theta_j(t)$, $\dot{\theta}_j(t) = K_j(t)v_j(t)$, where velocity $v_j(t)$ and curvature $K_j(t)$ represent control inputs $\bar{u}_j(t) = (v_j(t), K_j(t)) \in \mathbb{R}^2$.

Let us define a time interval $[t_0, t_{N+M}]$ containing a finite sequence with $N+M+1$ elements of nondecreasing times $\mathcal{T}(t_0, t_{N+M}) = \{t_0, t_1, \dots, t_{N-1}, t_N, \dots, t_{N+M-1}, t_{N+M}\}$, such that $t_0 < t_1 < \dots < t_{N-1} < t_N < \dots < t_{N+M-1} < t_{N+M}$.[2] Also, let us define a controller for a robot $R_j$ starting from a configuration $\psi_j(t_0)$ by $\mathcal{U}_j(t_0, t_{N+M}, \mathcal{T}(\cdot)) := \{\bar{u}_j(t_0; t_1 - t_0), \bar{u}_j(t_1; t_2 - t_1), \dots, \bar{u}_j(t_{N+M-1}; t_{N+M} - t_{N+M-1})\}$ which is characterized as a sequence of constant control actions. Each element $\bar{u}_j(t_k; t_{k+1} - t_k)$, $k \in \{0,\dots,N+M-1\}$, of the finite sequence $\mathcal{U}_j(t_0, t_{N+M}, \mathcal{T}(\cdot))$ will be held constant during the time interval $[t_k, t_{k+1})$ with length $t_{k+1} - t_k$ (not necessarily uniform).

By integrating the kinematic model in over a given interval $[t_0, t_{N+M}]$, and holding constant control inputs over each time interval $[t_k, t_{k+1})$ (from this point we may refer to $t_k$ using its index $k$), we can derive the following model for the *transition points* at which control inputs change:

$$
\begin{aligned}
x_j(k+1) &= \begin{cases} x_j(k) + \frac{1}{K_j(k+1)}\left[\sin\left(\theta_j(k) + \right.\right. \\ \quad K_j(k+1)v_j(k+1)\Delta t(k+1)\big) - \\ \quad \left.\sin\left(\theta_j(k)\right)\right], \text{if} K_j(k+1) \neq 0; \\ x_j(k) + v_j(k+1)\cos\left(\theta_j(k)\right)\Delta t(k+1), \\ \quad \text{if } K_j(k+1) = 0 \end{cases} \\
y_j(k+1) &= \begin{cases} y_j(k) - \frac{1}{K_j(k+1)}\left[\cos\left(\theta_j(k) + \right.\right. \\ \quad K_j(k+1)v_j(k+1)\Delta t(k+1)\big) - \\ \quad \left.\cos\left(\theta_j(k)\right)\right], \text{if} K_j(k+1) \neq 0; \\ y_j(k) + v_j(k+1)\sin\left(\theta_j(k)\right)\Delta t(k+1), \\ \quad \text{if } K_j(k+1) = 0 \end{cases} \\
\theta_j(k+1) &= \theta_j(k) + K_j(k+1)v_j(k+1)\Delta t(k+1),
\end{aligned}
\tag{1}
$$

where $x_j(k)$ and $y_j(k)$ are the rectangular coordinates and $\theta_j(k)$ the heading angle for the configuration $\psi_j(k)$ at the transition point with index $k$, $v_j(k+1)$ and $K_j(k+1)$ are control inputs at time index $k+1$, and $\Delta t(k+1)$ is the sampling time. This model and notation allows us to describe long trajectories exactly using a minimal amount of information such as: i) the initial configuration $\psi_j(t_0)$, ii) the sequence of switching times $\mathcal{T}(\cdot)$, and iii) the sequence of controls actions $\mathcal{U}_j(\cdot)$.

In real applications, the control inputs are limited by vehicle mechanical capabilities (i.e., chassis and engine). This constraints can be taken into account for each robot $R_j$, $j \in \{1,\dots,n_r,L\}$, limiting their control inputs by the following inequalities: $v_{min,j} \leq v_j(k) \leq v_{max,j}$ and $|K_j(k)| \leq K_{max,j}$, where $v_{max,j}$ is the maximal forward velocity of the $j$-th vehicle, $v_{min,j}$ is the limit on the backward velocity and $K_{max,j}$ is the maximal control curvature. These values can be different for each robot $R_j$.

## 2.2 Formation Driving using RHC

The presented approach relies on the well known leader-follower method frequently used in applications of car like robots (Barfoot and Clark, 2004). In the method, the followers $R_i$, $i \in \{1,\dots,n_r\}$ track the leader's trajectory which is distributed within the group. The followers are maintained in relative distance to the leader in curvilinear coordinates with two axes $p$ and $q$, where $p$ traces the leader's trajectory and $q$ is perpendicular to $p$ as is demonstrated in Figure 1. The positive direction of $p$ is defined from $R_L$ back to the origin of the movement and the positive

[1] Index $L$ denotes the virtual leader which has assigned the leadership at the moment while indexes $L_1$ and $L_2$ distinguish between particular virtual leaders only if it is necessary.

[2] The meaning of constants $N$ and $M$ will be explained in Section 3.

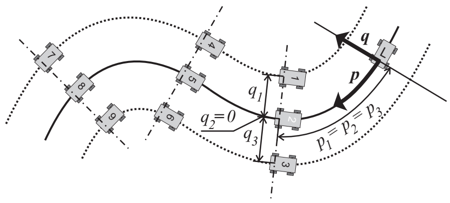



Figure 1: Formation described by curvilinear coordinates $p$ and $q$.

direction of $q$ is defined in the left half plane in direction of the forward movement.

The shape of the formation is then uniquely determined by the parameters $p_i(t)$ and $q_i(t)$, defined for each follower $i$, which can vary during the mission. To convert the state of the followers in curvilinear coordinates to the state in rectangular coordinates, the simple equations in (Barfoot and Clark, 2004) can be applied.

The main idea of the receding horizon control is to solve a moving finite horizon optimal control problem for a system starting from current states or configuration $\psi(t_0)$ over the time interval $[t_0, t_f]$ under a set of constraints on the system states and control inputs. In this framework, the length $t_f - t_0$ of the time interval $[t_0, t_f]$ is known as the control horizon. After a solution from the optimization problem is obtained on a control horizon, a portion of the computed control actions is applied on the interval $[t_0, \delta t_n + t_0]$, known as the receding step, where $\delta t_n := \Delta t n$.[3] This process is then repeated on the interval $[t_0 + \delta t_n, t_f + \delta t_n]$ as the finite horizon moves by *time steps* defined by the sampling time $\delta t_n$, yielding a state feedback control scheme strategy. Advantages of the receding time horizon control scheme become evident in terms of adaptation to unknown events and change of strategy depending on new goals or new events such as appearing obstacles in the environment.

# 3 METHOD DESCRIPTION

In this section, a concept of complementary virtual leaders, which enables backward driving of formations, will be proposed. The basic idea of the classical leader-follower concept (see Figure 1) is based on the fact that the followers continue with forward movement until the place where the leader changed the polarity of its velocity. Such a behavior could cause collisions or unacceptable disordered motion leading to a breakage of the shape of formation. Natural conception of the formation movement supposes that the entire group keeps compact shape and so all members should change the polarity of their velocity in the same moment.

Such an approach requires an extension of the standard method employing one leader to an approach with two virtual leaders, one for the forward movement and one for the backward movement. Their leading role is switched always when the sign of the leader's velocity is changed. The suspended virtual leader becomes temporarily a virtual follower. The virtual follower traces the virtual leader similarly as the other followers to be able to undertake its leading duties at time of the next switching. Therefore, all robots in our system will be considered as followers and there is no physical leader. The virtual leaders will be positioned at the axis of the formation, one in front of the formation and one behind the formation.

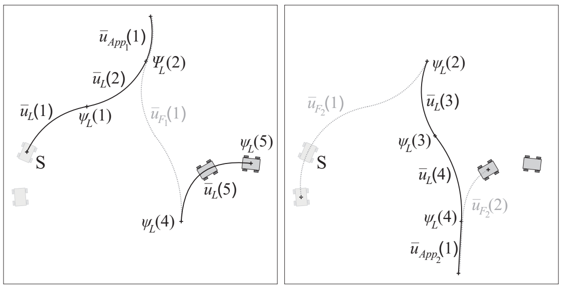
In the presented approach, we propose to solve collision free trajectory planning and optimal control together for both virtual leaders in one optimization step. This ensures integrity of the solution for the separated plants. Beyond this, we extend the standard RHC method with one control horizon into an approach utilizing two finite time intervals $T_N$ and $T_M$. The first time interval $T_N$ should provide immediate control inputs for the formation regarding the local environment. By applying this portion of the control sequence, the group is be able to respond to changes in workspace that can be dynamic or newly detected static obstacles. The difference $\Delta t(k+1) = t_{k+1} - t_k$ is kept constant (later denoted only $\Delta t$) in this time interval and should satisfy the requirements of the classical receding horizon control scheme.

The second interval $T_M$ takes into account information about the global characteristics of the environment to navigate the formation to the goal and to automatically compose the entire maneuver containing usually multiple switching between the virtual leaders. Here, we should highlight that also the number of switchings between the virtual leaders is designed automatically via the optimization process. The transition points in this part can be distributed irregularly to effectively cover the environment. During the optimization process, more points should be automatically allocated in the regions where a complicated maneuver of the formation is needed. This is enabled due to the varying values of $\Delta t(k+1) = t_{k+1} - t_k$ that will be for the compact description collected into the vector $\mathcal{T}^{\Delta}_{L,M} = \{\Delta t(N+1), \ldots, \Delta t(N+M)\}$.

To define the trajectory planning problem with two time intervals in a compact form we need to gather states $\psi_L(k), k \in \{1, \ldots, N\}$ and $\psi_L(k), k \in$

[3] Number of applied constant control inputs $n$ is chosen according to computational demands as was explained in (Saska et al., 2009).

$\{N+1,\ldots,N+M\}$ into vectors $\Psi_{L,N} \in \mathbb{R}^{3N}$ and $\Psi_{L,M} \in \mathbb{R}^{3M}$. Similarly the control inputs $\bar{u}_L(k), k \in \{1,\ldots,N\}$ and $\bar{u}_L(k), k \in \{N+1,\ldots,N+M\}$ can be gathered into vectors $\mathcal{U}_{L,N} \in \mathbb{R}^{2N}$ and $\mathcal{U}_{L,M} \in \mathbb{R}^{2M}$. In the case of the direction of movement alternation, the formation has to travel into the distance $\max_{i=\{1\ldots n_r\}} p_i$ to place the second virtual leader on the position of the first one. Such a movement creates an *appendix* in the entire trajectory that is followed. The minimal length of this additional trajectory is defined by the size of the formation and only the curvature of the leading robot is not uniquely determined. Therefore, we have to define a vector $\mathcal{K}_{App}$ collecting variables $K_{App}(j), \forall j \in \{1,\ldots,N+M-1\}$ applied for the movement of the virtual leaders in the appendixes. $N+M-1$ is maximal possible number of switchings. Usually the number of switchings is significantly lower and the unused values of $K_{App}(\cdot)$ will not be considered in the evaluation of the optimization vector. All variables describing the complete trajectory from the actual position of the first virtual leader until target region can be then collected into the unique optimization vector $\Omega_L = [\Psi_{L,N}, \mathcal{U}_{L,N}, \Psi_{L,M}, \mathcal{U}_{L,M}, \mathcal{T}^{\Delta}_{L,M}, \mathcal{K}_{App}] \in \mathbb{R}^{6N+7M-1}$.u The vector $\Omega_L$ contains necessary information for both leaders, but we have to decide, which part of the vector will be assigned to which leader. Firstly, let us define an ordered set of indexes of samples where the polarity of velocity is changed as $I_{sw} := \{i : v_L(i)v_L(i+1) < 0\}, \forall i \in \{1,\ldots,N+M-1\}$, where the values of $v_L(\cdot)$ can be extracted from the vector $\mathcal{U}_{L,MN} = [\mathcal{U}_{L,N}, \mathcal{U}_{L,M}] \in \mathbb{R}^{2(N+M)}$. Now, we can propose to collect the control inputs used for the first virtual leader as $\mathcal{U}_{L_1,MN} = [\bar{u}_L(1), \bar{u}_L(2), \ldots, \bar{u}_L(I_{sw}(1)), \bar{u}_{App_1}(1), \bar{u}_{F_1}(1), \bar{u}_L(I_{sw}(2)+1), \bar{u}_L(I_{sw}(2)+2), \ldots, \bar{u}_L(I_{sw}(3)), \bar{u}_{App_1}(2), \bar{u}_{F_1}(2), \ldots, \bar{u}_L(I_{sw}(2n_{sw_1}-2)+1), \bar{u}_L(I_{sw}(2n_{sw_1}-2)+2), \ldots, \bar{u}_L(I_{sw}(2n_{sw_1}-1)), \bar{u}_{App_1}(n_{sw_2}), \bar{u}_{F_1}(n_{sw_2})]$, if $n_{sw_1} = n_{sw_2}$ and $\mathcal{U}_{L_1,MN} = [\bar{u}_L(1), \bar{u}_L(2), \ldots, \bar{u}_L(I_{sw}(1)), \bar{u}_{App_1}(1), \bar{u}_{F_1}(1), \bar{u}_L(I_{sw}(2)+1), \bar{u}_L(I_{sw}(2)+2), \ldots, \bar{u}_L(I_{sw}(3)), \bar{u}_{App_1}(2), \bar{u}_{F_1}(2), \ldots, \bar{u}_{App_1}(n_{sw_2}), \bar{u}_{F_1}(n_{sw_2}), \bar{u}_L(I_{sw}(2n_{sw_1}-2)+1), \bar{u}_L(I_{sw}(2n_{sw_1}-2)+2), \ldots, \bar{u}_L(N+M)]$, if $n_{sw_1} \neq n_{sw_2}$. The control inputs for the second virtual leader as $\mathcal{U}_{L_2,MN} = [\bar{u}_{F_2}(1), \bar{u}_L(I_{sw}(1)+1), \bar{u}_L(I_{sw}(1)+2), \ldots, \bar{u}_L(I_{sw}(2)), \bar{u}_{App_2}(1), \bar{u}_{F_2}(2), \bar{u}_L(I_{sw}(3)+1), \bar{u}_L(I_{sw}(3)+2), \ldots, \bar{u}_L(I_{sw}(4)), \bar{u}_{App_2}(2), \bar{u}_{F_2}(3), \ldots, \bar{u}_{App_2}(n_{sw_1}-1), \bar{u}_{F_2}(n_{sw_1}), \bar{u}_L(I_{sw}(2n_{sw_2}-1)+1), \bar{u}_L(I_{sw}(2n_{sw_2}-1)+2), \ldots, \bar{u}_L(N+M)]$, if $n_{sw_1} = n_{sw_2}$ and $\mathcal{U}_{L_2,MN} = [\bar{u}_{F_2}(1), \bar{u}_L(I_{sw}(1)+1), \bar{u}_L(I_{sw}(1)+2), \ldots, \bar{u}_L(I_{sw}(2)), \bar{u}_{App_2}(1), \bar{u}_{F_2}(2), \bar{u}_L(I_{sw}(3)+1), \bar{u}_L(I_{sw}(3)+2), \ldots, \bar{u}_L(I_{sw}(4)), \bar{u}_{App_2}(2), \bar{u}_{F_2}(3), \ldots, \bar{u}_L(I_{sw}(2n_{sw_2}-1)+1), \bar{u}_L(I_{sw}(2n_{sw_2}-1)+2), \ldots, \bar{u}_L(I_{sw}(2n_{sw_2})), \bar{u}_{App_2}(n_{sw_1}-1), \bar{u}_{F_2}(n_{sw_1})]$, if




(a) Trajectory of the first virtual leader.

(b) Trajectory of the second virtual leader.

Figure 2: Trajectories with denoted control inputs and states of the two virtual leaders. The solid black curves denote trajectories where the virtual leader is leading the formation while the gray curves denote trajectories where the virtual leader is the virtual follower. The initial positions of the leaders are depicted by gray shadows.

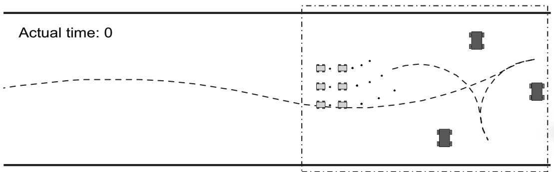
$n_{sw_1} \neq n_{sw_2}$.
The value $n_{sw_1}$ (resp. $n_{sw_2}$) is number of time intervals in which the first (resp. the second) virtual leader has the leadership. Control inputs $\bar{u}_{F_1}(j), \forall j \in \{1,\ldots,n_{sw_2}\}$ (resp. $\bar{u}_{F_2}(j), \forall j \in \{1,\ldots,n_{sw_1}\}$) are obtained by the formation driving method when the first (resp. the second) virtual leader is the virtual follower. The control inputs $\bar{u}_{App_1}(j), \forall j \in \{1,\ldots,n_{sw_1}\}$ (resp. $\bar{u}_{App_2}(j), \forall j \in \{1,\ldots,n_{sw_2}-1\}$) are applied for the formation driving in the appendixes and are obtained using $\mathcal{K}_{App}$ and $v_{max,L}$.

The vectors $\mathcal{T}^{\Delta}_{L_1,MN}$ (resp. $\mathcal{T}^{\Delta}_{L_2,MN}$) will be compiled similar as the vector $\mathcal{U}_{L_1,MN}$ (resp. $\mathcal{U}_{L_2,MN}$), because the variables $\bar{u}(\cdot)$ and $\Delta t(\cdot)$ are inseparably joined. An example illustrating this approach is presented in Figure 2 where the parameters and outputs of the optimization are: $N = 0$, $M = 5$, $I_{sw} \in \{2,4\}$, $\mathcal{U}_{L_1} = [\bar{u}_L(1), \bar{u}_L(2), \bar{u}_{App_1}(1), \bar{u}_{F_1}(1), \bar{u}_L(5)]$, $\mathcal{U}_{L_2} = [\bar{u}_{F_2}(1), \bar{u}_L(3), \bar{u}_L(4), \bar{u}_{App_2}(1), \bar{u}_{F_2}(2)]$.

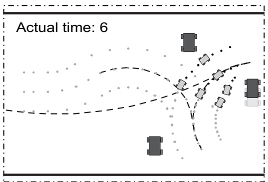
The trajectory planning and the static as well as dynamic obstacle avoidance problem for both virtual leaders can be transformed to the minimization of single cost function subject to sets of equality and inequality constraints. Accordingly the leader-follower concept, the trajectory computed as the result of this minimization will be used as an input of the trajectory tracking for the followers. Details of the description of both, the leader trajectory planning and the trajectory tracking for followers, can be found in our previous publication in (Saska et al., 2009), where a method with one leader enabling only forward movement is presented.

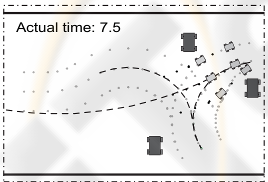
# 4 EXPERIMENTAL RESULTS

The results presented in this section have been obtained using the introduced algorithm with parameters: $n = 2$, $N = 4$, $M = 8$, and $\Delta t = 0.25s$. In the scenario of a road with obstacles in Figure 3 a formation of 6 robots is aimed to reach the target region in minimum time. In accordance with real application, the maximum forward and backward velocities of the virtual leader of the formation were set unsymmetrically as $v_{min,L} = -v_{max,L}/2$. Putting the final region sufficiently far behind the formation, the solution of the *formation to target zone problem* is to turn the formation and then continue forward to reach the desired area. Such a maneuver, which contains two switching between the virtual leaders, is denoted by dashed curves in Figure 3(a). The solution of the task keeps the formation outside the obstacles during the whole maneuver. Beyond this, the ability to avoid dynamic obstacles with collision course detected by rear sensors of the robots during the reverse driving is demonstrated in the simulation (see snapshots in Figure 3(b) and in Figure 3(c)). The complete history of the turning can be seen in Figure 3(d).



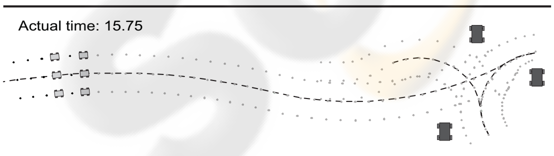

(a) Initial position of the formation with complete plan for both virtual leaders denoted by the dash curve.




(b) Detection of movement of one obstacle and consequential reaction of the followers.

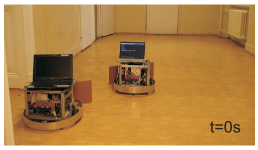
(c) The followers successfully passed by the obstacle and they are going back to their position in the formation.




(d) Accomplished task with denoted trajectories of the robots.

Figure 3: Snapshots of turning 180 degrees in the environment with static as well as dynamic obstacles.

The presented simulations have shown only simple experiments to demonstrate a basic behavior of the presented approach. More complicated scenarios proving the ability to avoid a damaged follower crossing trajectories of neighbors, to utilize different shapes of formations or to deal with local minima in a complex environment cannot be offered due to space limitation of this paper, but can be found in (Saska et al., 2009; Saska et al., 2007).





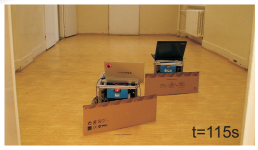
(a) Two indoor robots in positions appropriate for cooperative shoveling materials to the side.

(b) Robots have changed formation to a shape appropriate for turning.



(c) The second "plough" is overtaking the leadership.

(d) The leading duties are going back to the first robot.

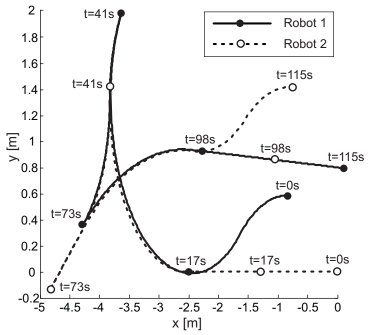

(e) The formation has finished the turning maneuver.

(f) Robots are again ready for shoveling.

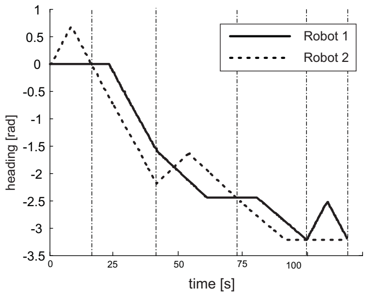
Figure 4: Formation turning at the end of a blind corridor.

The presented hardware experiment is a part of airport snow shoveling project using formations of autonomous ploughs, which has been reported in (Saska et al., 2008; Hess et al., 2009). Here, we applied the approach presented in this paper for turning the formation of snow ploughs at the end of cleaning runway. The experiment has been performed on G2Bot robotic platform equipped with SICK laser range finder (not utilized in the experiment) and odometry.

In a simplified version of the experiments (see snapshots in Figure 4, data from odometry in Figure 5 and a movie of the experiment in (MOVIE, 2010)), two autonomous indoor robots are facing a blind corridor with an aim to turn at the end. The map of the environment and positions of the vehicles are known before the mission. During the experiment, positions are updated using a dead reckoning because an exter-




(a)




(b)

Figure 5: Data captured from odometry of both ploughs with depicted times of snapshots in Figure 4.

nal positioning system is not necessary in such a short term experiment. The actual position of robots and their plans are shared via wireless communication. The plan of robots is as follows: 1) build a compact formation appropriate for turning, 2) turn 180 degrees and 3) return back to the shoveling formation. Such an approach is necessary in a case of larger formations covering the whole runway and it can also better clarify the method employing two virtual leaders. The transitions between the different formations as well as the turning maneuver are computed automatically using the methods presented in this paper. Only the positions of the vehicles within the formations (safety distances, required overlapping of shovels etc.) are given by experts.

# 5 CONCLUSIONS

In this paper, we have presented an RHC approach to formation driving of car-like robots reaching a target area in environments with obstacles. The proposed approach is novel in the sense of utilized pair of complementary virtual leaders necessary for guidance of the formation during backward driving. This allows us arbitrary maneuvers of formations including the U-turn in bordered environment, which is necessary for complete autonomous airport snow shoveling system. The applicability of the presented approach has been verified with simulations and hardware experiment.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