# A SOFTWARE SYSTEM FOR DATA INTEGRATION AND DECISION SUPPORT FOR EVALUATION OF AIR POLLUTION HEALTH IMPACT

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Keywords: Data Integration, Forecast, Health-environment Correlation.

Abstract: In this paper we present a software system for decision support (DSS – Decision Support System) aimed at forecasting high demand of admission on health care structures due to environmental pollution. The algorithmic kernel of the system is based on machine learning, the software architecture is such that both persistent and sensor data are integrated through a data integration infrastructure. Given the actual concentration of different pollutants, measured by a network of sensors, the DSS allows forecasting the demand of hospital admissions for acute diseases in the next 1 to 6 days. We tested our system on cardiovascular and respiratory diseases in the area of Milan.

## **1 INTRODUCTION**

The evaluation of the quality of the environment is important for health impact assessment, which in turn is crucial for taking actions for protecting the public health. Many epidemiological studies have shown that acute or chronic health effects can be associated with high concentrations of environment pollutants. This paper stems from the research project LENVIS (Localised ENVironmental and health Information Services for all) aimed at designing a Decision Support System (DSS) based on statistical learning. The main aim of the project is to leverage on-line environmental monitoring into alerts during episodic pollution events, in order to inform people of the current environmental situation, give warnings to health care providers about peaks in requests of hospitalization, support public authorities in deciding which actions have to be carried out (e.g. prevent risks by reducing excess pollution and minimizing exposure, prepare hospitals to peaks of emergency admissions).

A number of techniques have been developed for the prediction of pollutant concentrations (Gokhale and Raokhande 2008) (Perez and Salini 2008) or health indicators (Dong et al., 2009). The most interesting and closest to our approach in its objectives is (Wall and Li 2009), in which monthly counts of medical visits for persons identified to have alcoholism problems are modeled using twostate hidden Markov models.

As far as the health data modelling is concerned, typical studies rely on epidemiological approaches (Lanki et al. 2006) (Zanobetti et al. 2009) and builds on existing modelling technologies like GAM (Generalized Additive Modelling) (Chan et al. 2009), regressions and statistics (Bellini et al. 2007). We develop an original approach linking directly environmental and hospitalization data. A future evolution and exploitation of our system could include any model in the same way defined by the architecture, among the models which are actually integrated. Another reason for our approach to be driven by real hospitalization data is that the complex relationship between health and the environment limits the availability of causative evidence and that a data driven model is both simpler and of a more general application.

As far as the data integration task is concerned, we develop a data integration infrastructure suitable for the online access to multiple, distributed and heterogeneous data sources, which is unique in the state of the art of health modelling systems.

Essentially, forecasting approaches can be grouped in empirical models, fuzzy logic based systems, data driven statistical models and model-

Toscani D., Bargna F., Quarenghi L., Archetti F. and Giordani I. (2010). A SOFTWARE SYSTEM FOR DATA INTEGRATION AND DECISION SUPPORT FOR EVALUATION OF AIR POLLUTION HEALTH IMPACT. In Proceedings of the 12th International Conference on Enterprise Information Systems - Artificial Intelligence and Decision Support Systems, pages 284-290 Copyright © SciTePress driven statistical learning methods. These last methods, in particular State space models and Bayesian Networks, appears as the most interesting, since the "learning from data" and the probabilistic links among quantities make them able to exploit patterns and relations that cannot be explicitly handled by fuzzy logic systems.

Statistical analysis of time series, like *Multiple Linear Regression* (MLR), is based on local information and assumes stationarity, or weak stationarity, of the data. Analyzing health data, asymmetries between growth and decline periods are underlined and it is often difficult to distinguish between long-term trends and noise. As result, a forecasting model is valid only for a short period. For that reasons it's particularly interesting a class of probabilistic state space models: the *Autoregressive HMM* (*AHMM*) (Messina and Toscani 2008). This is a class of the *Hidden Markov Model* (*HMM*), a set of models widely applied in several fields (Dong et al. 2006) which assumes that an observation depends on the current state and also on the previous *p* values.

The two main contributions of this paper are: (1) a data access and integration component in order to have a "stream reasoning capability" for DSS; (2) a new algorithm for forecasting the numbers of hospitalizations.

# **2** SYSTEM ARCHITECTURE

The objective of our system is to provide a tool to evaluate the impact of current pollution on the health status of the population, measured by the forecast of number of hospitalization requests due to environmental pollution, to deliver information to e.g. public authorities and hospital managers. Queries to the system can be formulated in the structured format defined by the Data Access Component (DAC), which represents the core of the system. Results are produced as streams of structured objects (DataObject) and provided through both Lenvis Data Services (i.e. web services defined for the exchange of data inside the Lenvis platform) and Presentation and User Interaction (e.g. the Service Oriented Business Intelligence (SOBI) functionalities for user interaction, query submission and graphical results presentation). Two principal use cases can be defined: the Calculation of health indicators, like statistical and descriptive indicators of the impact of pollution on the number of hospitalizations, and Forecast of health impact, i.e. given the actual concentration of pollutants and their forecast we produce a forecast of the number of hospitalizations for the next 1 to 6 days. In this paper we address only this second case.

Figure 1 depicts the main components of the system, grouped in four layers; the Data Layer contains the external data sources, from which are extracted the data to reply to queries, i.e. data stored in data bases or coming from Lenvis services. Integration Layer, represented by the DAC, performs the operations of query interpretation, data sources querying and orchestration, data integration and preparation of queries replies. The third level, the Analysis Layer, includes the computational components, which produce the online data to reply to complex queries (e.g. forecast). Finally, the Application Layer is the level at which users can submit queries and obtain the results. It includes the interactions with the users through Lenvis Data Services (web services) and the Presentation and User Interaction, through the Service Oriented Business Intelligence tools.

The system is implemented in Java and part of the computational engine in Matlab. External applications and user interfaces control the DSS and collect the outputs through Java web services and Java API (Application Programming Interfaces).



Figure 1: System architecture.

#### 2.1 Data Layer

#### Pollution Concentration (from sensors)

We consider as environmental data in our case study the concentrations of air pollutants in the city of Milan. The network of air pollution monitoring stations, in operation since 80s, counts 9 monitoring stations, spread in all the city. Each station is equipped with a variable number of sensors, for a total of 37 sensors in the whole town. Each sensor measures the concentration (in  $\mu g/m^3$ ) of one among: benzene, nitrogen dioxide, sulphur dioxide, carbon monoxide, nitrogen oxide, total nitrogen oxide, ozone, PM10 (Particulate Matter), PM2.5, TSP (Total Suspended Particulate). The station calculates every hour the mean of pollutant concentration and sends it to a control centre, where the data are manually validated to filter outliers and further aggregated to obtain a daily measure.

These data are publicly available at the web site of the environment protection agency of the Lombardy region (ARPA Lombardia); they can be downloaded as CSV files at the url: www.arpalombardia.it/qaria/doc\_RichiestaDati.asp. Most of the data are available from the 80s-90s, but for many stations they are available only after year 2000, in one case from 2007.

The series of data contains missing values, since some stations have been under maintenance for months or have been switched off. In building our decision models we selected the time series with less missing values, since not all the data series could be analyzed together for long time periods. The data have been downloaded and imported into a CSV file.

#### Pollution Concentration (from Lenvis services)

In Lenvis are developed a set of services that given the actual concentration of air pollutants, weather conditions and geographical locations of the emission sources allows forecasting the pollution concentrations in a given area for the next days. The data produced by these services are used by our system as soft sensors, i.e. virtual sensors that produce data for the future. Based on this data, we can provide health impact forecast for a longer period.

#### Admission Data

The health indicator that we address is the daily number of hospitalizations in the city of Milan for two principal classes of disease that can be related to air pollution: respiratory (asthma) and cardio vascular (myocardial infarction, ischemic heart and deep vein thrombosis). In these data each patient is characterized by his associated ID and his diagnosis. The number of hospitalizations for each pathology are collected by the local government of the Lombardy and region published on http://www.aleeao.it/. For the setup and tuning of health models we obtained specific data from two important general hospitals accredited by the National Health Service, one with general audience and one with focus on geriatrics patients. These two hospitals have provided a small quantity of very detailed data, including also ages of the patients, sex, detailed diagnosis, which allows to perform classspecific analysis (for e.g. age classes). Data have been downloaded and imported into a CSV file.

#### 2.2 Integration Layer

Input data are collected through web services and local databases. The DAC is a software component that provides integrated access to multiple, heterogeneous and distributed data sources. Its functionalities are: object-oriented representation of the domain data through a meta-model definition of the types of data supported; submission of structured queries; return of the query result as a searchable and navigable data structure. It allows executing cross-source queries on temporal, spatial and logical intervals, supporting data analysis and presentation activities. Each query, despite the traditional SQL syntax, specifies a target but not the data sources from which to extract the data.

The platform defines the data types that the data sources can provide. Each data type is a structured object, containing multiple fields (e.g. for a sensor reading, the source of the data, the value collected, the timestamp to which it refers etc.) and it is linked to other data types through hierarchical relations. Each query also specifies the constraints on results (temporal intervals, values of some attributes...).

This querying mechanism hides the heterogeneity and distribution of the data sources. Moreover, it is responsibility of the DAC to identify, select and query all the sources needed to reply to user requests and also to prepare and present the output in the common format. A fundamental feature is the possibility to reply to queries not only accessing to persistent data but also using streaming data produced online by sensors (e.g. sensor networks) or by the models which perform online inference (forecast).

The main components of the DAC are: the query processor that takes as input a complex query formulated by the user and produces a set of simpler queries, each of which can be satisfied by a single wrapper. The query processor analyzes also the final conditions that have to be satisfied before to return to the user the results produced by the Data Aggregator. The Wrapper component is able to manage wrappers belong to different classes; each class is specific for a type of data source (relational database, web service, text file...). A wrapper processes a simple query, it is connected to a data source and extracts from it the data needed to reply to the query, implementing specific protocols of the source. Finally, Data aggregator component merges the partial results produced by different wrappers. Analogously to relational databases, this merging can consist in set union or join.

### 2.3 Analysis Layer

At this layer are deployed the data analysis components of our system, which forecast the number of hospitalization in the next days through Autoregressive Hidden Markov Models and Multiple Linear Regression (see Sect. 3). An innovation aspect of the DAC is that for each class of computational components is defined a Wrapper, in the DAC format, which allows to query the components and extract data to reply to user's queries. An example of query is the request for the number of hospitalization from the whole week, being on Friday. The reply to this query involves the extraction of historical data (from Monday to Friday) from the Admission Database, while the forecast for Saturday and Sunday is produced online, merged with the historical data and presented to the user through e.g. a graphical histogram (see sect. 2.4).

### 2.4 Application Layer

In this layer are included the components for the interaction between users and the DAC, meaning "user" as both human users and external applications. The Presentation and User Interaction set of modules includes the Service Oriented Business Application that, based on the DAC for the extraction and production of data, are responsible to collect user queries and present results through graphs and tables in the web interface of the Lenvis Portal.

The other paradigm for the interaction with the DAC are the Lenvis Data Services, web services that expose the DAC functionalities for querying and data extraction, which can be easily integrated in external applications.

# **3 COMPUTATIONAL ENGINE**

The algorithmic core of our system is the forecasting engine of the analysis layer. While the literature shows mostly the application of regression models, which we used as benchmarks in our experiments, the authors feel that the dynamic of the target process is better captured by probabilistic statespace models, namely Hidden Markov Models (HMM) (Rabiner 1989). When data are generated by a process that we cannot directly observe, or is too complex, HMM allows ignoring or partially simplify its nature, focusing the attention on the data generation process, which is indeed our final objective. A HMM is a stochastic process whose evolution is governed by a Markov Chain whose state variable can assume a finite number of values  $s_j$ , j=1,2,..,N, which are often called *states* and cannot been directly observed. Each  $s_j$  corresponds to one of the possible "conditions" in which the process can be.

The model is characterized by a probability distribution for the initial value of the state  $\pi(s_j)$ , a set of state transition probabilities *A* whose elements  $a_{ij}=P(s_j/s_i)$  measure the probability of being in state  $s_i$  at time *t* and  $s_j$  at t+1. We apply *Autoregressive HMM* (*AHMM*), a class of HMM which assumes that an observation depends on the current state and also on the previous *p* values. The formulation of the model is completed by a set *B* of Probability Density Functions (PDF) which describe the generation (*emission*) of the observed values in each state.

The observation  $o_t$  at time t is generated by  $s_j$  according to:

$$o_t = b_j(O) = P(o_t/s_j, o_{t-1}..o_{t-p})$$
 (1)

K subsequent observations are grouped in vectors, which define an autoregression process of order p:

$$o_k = \sum_{i=1}^p r_i o_{k-i} + e_k , \qquad k = 1, 2, ..., K$$
(2)

Where  $e_k$  is a Gaussian noise component, with zero mean and variance  $\sigma^2$ , and  $r_i$  (*i*=1,2,...,*p*) are the autoregression coefficients. Note that, the PDF of the observed variable O for state  $s_j$  is a mixture of *M* components:

$$b_j(O) = \sum_{m=1}^{M} c_{jm} b_{jm}(O), \quad \sum_{m=1}^{M} c_{jm} = 1, \quad c_{jm} \ge 0$$
 (3)

Where  $c_{jm}$  are a set of weight, whose sum is 1, and  $b_{jm}(O)$  is the observation density for state *j* and  $m^{\text{th}}$  mixture component, computed as follows:

$$b_{jm}(O) = \left(\frac{2\pi}{K}\right)^{-K/2} \exp\left(-\frac{K}{2}\delta(O, r_{jm})\right)$$
(4)

where:

$$\delta(O, r_{jm}) = R_{r_{jm}}(0)R(0) + 2\sum_{i=1}^{p} R_{r_{jm}}(i)R(i)$$
(5)

$$R_{r_{jm}}(i) = \sum_{n=0}^{p-1} r_{jmn} r_{jm,n+1} \ (r_{jm0} = 1), \ l \le i \le p$$
(6)

$$R(i) = \sum_{n=0}^{K-i-1} o_n o_{n+1}, \quad 0 \le i \le p$$
(7)

 $r_{jmn}$  is the autoregression coefficient for state *j*, mixture component *m* and element *n* of the autoregression process.  $R_{jm}(i)$  is the autocorrelation of the autoregression coefficients for state *j* and mixture component *m* and R(i) is the autocorrelation of the observations.

The model is fully characterized by the set of parameters  $\lambda = [\pi(s_j), a_{ij}, r_{jmn}, R(i)]$ , which have been

estimated using the customized version of the Baum-Welch algorithm described in (Messina and Toscani, 2008).

Since there is no analytical process to estimate the optimal order p of the autoregression process, given a sequence of T training values  $v_{1..}v_{T}$ , we estimate multiple autoregression processes  $\theta_p$  for different p by maximizing the conditional distribution  $f(v_{1..}v_T | \theta_p)$ , i.e. their likelihood (MLE - Maximum Likelihood Estimation). Then, we select the optimal p through the AIC (Akaike Information Criterion) (Akaike, 1973). The initialization of autoregression parameters is performed by analyzing the relationship between autoregression parameters and the autocorrelation function of the observed data by solving a system of Yule-Walker equations (Tsay, 2005).

The formulation of AHMM described here can deal only with uni-variate series. To manipulate multiple pollutants and health indicators at the same time we use multiple models, one for each series, as in (Messina and Toscani, 2008). We capture the relations among series by estimating the Cholesky decomposition  $R_D$  of their *correlation matrix* (Russel and Norvig, 2002).

The generation of data is as follows: each AHMM is applied to sample T data. For each time t=1,2,..,T we produce a vector of forecast (one value for each series): given the current state  $s_t$  we sample the next state  $s_{t+1}$  according to the state transition probability and the observation  $o_t$  from the distribution  $b_j(O)$ . Finally, as in (Hoyland et al., 2003), we restore the correlations among series by multiplying the forecasts at the same time by  $R_D$ .

### 4 CASE STUDY: MILAN AREA

We selected as case study the air pollution in the metropolitan area of Milan, which is the Italian area with the highest population (10<sup>th</sup> in Europe), very high population density and it is affected by air stagnation; all these conditions have brought to considerable air pollution problems. This represents the ideal test bed for our DSS, since we can observe a high number of pollution peaks.

We collect through the DAC the environmental data and health indicators. For each series, we obtain a scale-free summary of changes in the data during time by calculating the *log-variation* among two consecutive values  $v_{t-1}$  and  $v_t$ , i.e.  $log(v_t/v_{t-1})$ . Log-variations are the observations  $o_t$  for our AHMM.

We grouped pollutants into three classes of homogeneous substances; for each class we selected a representative series, resulting in 9 triples of series whose mutual correlation coefficient is <0.8. Each triple has been used to predict cardio-vascular admissions and, separately, respiratory ones. The resulting configurations of the experiments (E1..E18) are detailed in Table 1.

We analyzed series of about 3000 values. We determined experimentally the optimal structure of the AHMM, which achieves highest model likelihood with respect to training data, having 2 states, 5 mixture components and an autoregression process with order variable from 2 to 5.

In each experiment we trained the model on a sliding window of 500 observations and we forecast the number of hospitalizations from 1 to 6 days. For each series we measure the performance by calculating the *Mean Absolute Percentage Error* (MAPE) between the real number of hospitalizations and the forecast produced by AHMM.

In order to assess the model reliability, the forecast is repeated 6 times for each model. We present the results in terms of global MAPE, calculated as mean of all the test executed with the same series.

For evaluating the performance of our forecasting model we used as benchmark the *Multiple Linear Regression* (MLR) (Montgomery et al., 2006), a multivariate statistical technique widely used to capture the linear correlations between some predictor variables  $v_{1..}v_{L-1}$  (i.e. concentrations of pollutants) and a single dependent variable  $v_L$  (*response variable*, i.e. health indicator).

Table 1: Configuration of data series in each experiment.

Pollutants	Experiment / Pathology
Nitrogen oxides, Sulfur dioxide, PM 10	E1 Card., E2 Resp.
Nitrogen oxides, Ozone, PM10	E3 Card., E4 Resp.
Nitrogen oxides, Carbon monoxide, PM 10	E5 Card., E6 Resp.
Nitrogen oxides, Benzene, PM10	E7 Card., E8 Resp.
Nitrogen dioxide, Sulphur dioxide, Benzene	E9 Card., E10 Resp.
Nitrogen dioxide, Sulphur dioxide, Total PM	E11 Card., E12 Resp.
Nitrogen monoxide, Ozone, Total PM	E13 Card., E14 Resp.
Sulphur dioxide, Ozone, Carbon monoxide	E15 Card., E16 Resp.
Ozone, Total PM, PM10	E17 Card., E18 Resp.

#### 4.1 Experimental Results

The results from the high number of computational experiments shows that AHMM achieve lower average error on forecast than MLR. This can be interpreted as indicator of the benefits introduced by the application of state-space probabilistic modelling which, despite regression models, has the ability of better capture the variability in data patterns. In the DSS, the measurement of the forecast error can help the user in selecting the "best" mix of pollutants, i.e. the set of sensors whose monitoring can give the lower error in forecast and hence the best advice on health indicators.

Fig. 3 and Fig. 4 show the results on cardiovascular diseases; experiments for AHMM are labelled with "H", for MLR with "R". Each experiment is constituted by multiple tests; for each experiment we report the overall minimum, maximum and average value of MAPE obtained in the tests. AHMM achieves a mean value from 7.424 to 19.64, while the best result for MLR is 15.86.

While MLR achieves a relatively high error, which is approximately constant in all the experiments, the result for AHMM vary depending on the experiment. This variation can be explained by the nature of the learning process of AHMM. In fact, while MLR is a deterministic model, which can be trained on a small quantity of data, training procedures for AHMM are non deterministic optimization techniques; the quality of the resulting model depends on the reaching of the local maxima and, definitively, on the nature of data.

The performance of AHMM can be improved by executing multiple times the learning process and selecting the maximum likelihood model, or even changing the structure of the model and the quantity of data.

Another remarkable feature of AHMM is that in most experiments the maximum and minimum value for MAPE in different tests is very close to the mean; this shows the robustness of the approach, which demonstrates that the low error can be achieved in different conditions.

Fig. 5 and Fig. 6 show the MAPE in forecast respiratory diseases. As for cardio-vascular, in most cases the MAPE for AHMM is lower than for MLR, ranging from 7.505 to 23.84 for AHMM and from 19.56 to 21.01 for MLR. The variation among maximum and minimum MAPE in the test for each experiment is generally lower for AHMM, showing as in the previous case that this model can provide more stable results.



Figure 2: MAPE of experiments E1..E9 on cardio-vascular diseases.

# **5** CONCLUSIONS

In this paper we present a software architecture for the collection and integration of historical and streaming data from multiple sources, which includes also as data data analysis algorithms as providers. We apply our system to the evaluation of health impact of air pollution on the number of hospitalization through Autoregressive Hidden Markov Models. In particular, given the measurements of the concentration of some pollutants in a given time period, the objective is to forecast the number of hospitalizations for acute cases of the related pathologies (of the cardiovascular and respiratory systems).

We compare AHMM with traditional regression methods, showing that the probabilistic modelling achieves a lower forecasting error.

Our results have obtained appreciation and interest, expressed by researchers of European universities in the area of environment, computer science and health, and also by the medical staff of public hospitals. The type of environmental and health information that is typically available to them is related to epidemiological statistics, while the short term link between environment and health has not been exploited yet. In fact, information is sometimes available on dedicated web sites, but in many case specialized papers and journals are the only source of information.



Figure 3: MAPE of experiments E11..E17 on cardio-vascular diseases.



Figure 4: MAPE of experiments E2..E10 on respiratory diseases.



Figure 5: MAPE of experiments E12..E18 on respiratory diseases.

The substantial innovation of this paper is to provide an automatic system that allows to have continuously updated forecasts about the demand for health services, i.e. hospitalizations and emergency accesses.

Our work is still in progress: the links with experts in environment and health are supporting us in understanding the data, choosing the models and interpreting results. Further analysis has to be performed in order to expand the possibilities of data integration and the robustness of the software components of the computational engine. We are actually expanding the application field of our DSS to water pollution, to monitor pathologies of the digestive system, skin and subcutaneous tissue.

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