A MULTIPHASE ACTIVE CONTOUR MODEL WITH DYNAMIC MEDIAL AXIS CONSTRAINT FOR MEDICAL IMAGE SEGMENTATION

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Abstract: A level-set based multiphase active contour model is proposed for medical image segmentation in this paper. The proposed method allows multiple objects with very different features to be jointly segmented by simultaneously evolving multiple active contours, each responsible for the segmentation of a single object. In this model, the forces exerted on each active contour mainly consists of two components. The first component makes use of boundary as well as regional information present in the input images. The second component is used to impose the so-called medial axis constraint, which is related to the force induced by interaction of multiple active contours. Experimental results on real medical images are also presented to show that the proposed method has good performances on topology preservation of multiple contours, as well as joint segmentation of similar objects in multiple images.

1 INTRODUCTION

Segmentation is a basic yet important problem in medical image processing, which is widely used in medical applications such as surgical planning, abnormality detection and treatment progress monitoring. Originally proposed in (Kass et al., 1988) as a tool for image segmentation, active contour models have attracted extensive research in the past two decades. The basic idea of the active contour is to iteratively evolve an initial curve towards the boundaries of the target objects driven by the combination of internal forces determined by the geometry of the evolving curve and the external forces induced from the image.

Image segmentation method using active contour is usually based on minimizing a functional which is so defined that for curves close to the target boundaries it has small values. To solve the functional minimization problem, a corresponding partial differential equation (PDE) can be constructed as the Gateaux derivative gradient flow to steer the evolution of the active contour.

Either explicit or implicit method can be applied to numerically approximate a curve evolution PDE. For explicit methods, an active contour can be represented in parametric form such as linear and cubic B-splines (Precioso et al., 2005). In that case, a finite number of points are sampled on the active contour and move according to the calculated forces, thereby causing the evolution of the entire contour.

For implicit (or level set) methods, an active contour is embedded as a constant level set (typically level zero) in a function defined in a higher dimensional space known as embedding function or level set function. The evolution of the active contour is carried out implicitly by the evolution of its embedding function. Thanks to its inherent capability to handle topological changes and straightforward extensibility to cope with high dimensional data, since the pioneering work of (Malladi et al., 1995), level set based segmentation has gained considerable research attention and has prompted the development of a large amount of models. Among these, two models, namely geodesic active contour model (Caselles et al., 1997) and Chan-Vese model (Chan and Vese, 2001), stand out respectively as the paradigms for boundary-based and region-based segmentation.

Most of the level-set algorithms in the aforementioned literature are focused on designing evolution functions for a single level set. According to these methods, although a single level set can embed multiple separate active contours, each active contour is

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driven independently by the same evolution rule. To simultaneously segment multiple objects with very different features, multiphase methods have been proposed to use more than one level set functions.

In (Brox and Weickert, 2006; Lankton and Tannenbaum, 2008), the evolution rule for each level set function consists of not only the traditional terms derived from specific functionals, but also the extra terms imposed by the proximity constraint. The proximity constraint ensures that each image pixel belongs to one and only one segmented region. To take the proximity constraint into account, these methods employ extra terms based on the concept of region competition, attempting to classify pixels on the region boundaries only to the most probable regions they can belong to.

In (Vese and Chan, 2002), the classic Chan-Vese model is extended from object/background segmentation to multiple region segmentation. In this method, different regions are binary coded by the signs of multiple level set functions so that the proximity constraint can be satisfied in an elegant and natural way. In (Cremers et al., 2006), a quite different multiphase method was proposed, where auxiliary labeling level set functions are introduced to dynamically divide an image into multiple regions which are labeled in a similar fashion as in (Vese and Chan, 2002) to keep the proximity constraint. The uniqueness of the method lies in that the labeled regions are not used directly as the segmentation result but used to identify different regions so that different evolution rules can be defined in differently labeled regions by using a single "segmenting" level set function.

Based on the authors' two earlier papers (Zhang et al., 2008; Zhang and Matuszewski, 2009), we propose a level-set based multiphase active contour model dedicated to a different type of constraint — the topology constraint. The model consists of two major components. The first component, making use of both boundary and regional information derived from the input image, describes how each active contour evolves independently. The second component takes the interaction of multiple active contours into account by using inter-object medial axes.

The rest of the paper is organized as follows. Section 2 discusses the theory of the method. Section 3 shows results of applying the method on medical images, whereas the conclusions are drawn in Section 4.

2 THEORY

2.1 Hybrid Active Contour Model

First consider the case of a single active contour. Let C denote an active contour and $\mathbf{x}$ denote a point in the image domain Ω. Then, as illustrated in Figure 1, the level set function $\phi(\mathbf{x})$ can be defined to have the following properties: (1) $C = \{\mathbf{x} : \phi(\mathbf{x}) = 0\}$; (2) $\phi(\mathbf{x}) > 0$ for $\mathbf{x}$ inside the contour and $\phi(\mathbf{x}) < 0$ for $\mathbf{x}$ outside. The normal of the active contour $\vec{N}$ is defined as the unit direction expanding the contour.

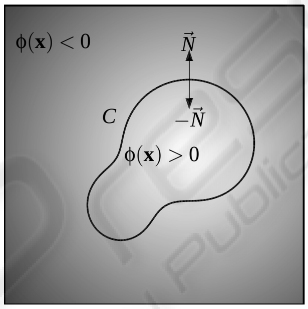


Figure 1: An active contour and its level set function.

The proposed functional to be minimized can be written as

$$E(\phi(\mathbf{x})) = -\int_{\Omega} P(I(\mathbf{x}))H(\phi(\mathbf{x}))\,d\mathbf{x} + \alpha \int_{\Omega} g(|\nabla I(\mathbf{x})|)|\nabla H(\phi(\mathbf{x}))|\,d\mathbf{x} \qquad (1)$$

where: $P(x)$ and $g(x)$ are the regional and boundary mapping functions related to the input image $I(\mathbf{x})$, $H(x)$ represents the Heaviside function which has value 1 when $x \geq 0$ and 0 otherwise, and α is a scalar factor used to balance the two terms. The first term of the functional is the regional term, wherein the function $P(x)$ is designed to map image intensities, expected to be typical for the object, to positive values with all other image intensities being mapped to negative values. The selection of the regional mapping function $P(x)$ is image dependent but flexible. The second term is the classical geodesic boundary term as proposed in (Caselles et al., 1997). The boundary mapping function $g(x)$ is often chosen to be an image edge indicator function which is a nonnegative decreasing function of the image gradient.

By deriving the Gateaux derivative of the proposed functional, the implicit PDE, describing the evolution of active contour that minimizes the functional, can be expressed as

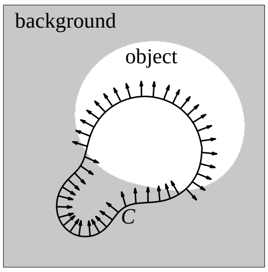


Figure 2: Regional term and its flow.

$$\frac{\partial\phi(\mathbf{x})}{\partial t} = P(I(\mathbf{x}))|\nabla\phi(\mathbf{x})| + \alpha\cdot\nabla\cdot\left(g(|\nabla I(\mathbf{x})|)\frac{\nabla\phi(\mathbf{x})}{|\nabla\phi(\mathbf{x})|}\right)|\nabla\phi(\mathbf{x})| \quad (2)$$

The corresponding explicit PDE can be written as

$$\frac{\partial C}{\partial t} = P(I(C))\cdot\vec{N} + \alpha\left(g(|\nabla I(C)|)\kappa - <\nabla g(|\nabla I(C)|),\vec{N}>\right)\cdot\vec{N} \quad (3)$$

with κ representing the curvature of C and $<\cdot,\cdot>$ denotes the inner product of two vectors. A synthetic $P(\mathbf{x})$, where object pixels are mapped to 1 and background pixels are mapped to -1, is shown in Figure 2 in order to illustrate the geometric interpretation of the regional term. Superimposed on the image is an active contour with arrows showing the motion of the contour according to the regional term $P(\mathbf{x})\vec{N}$ in Equ. (3). It is clear that, in this case, the regional term can always steer the active contour to the object boundary (where the minimization of the regional term in Equ. (1) is obtained) as long as the initial contour has common interior area with the object. This illustrates the robustness of the regional term which can hardly be achieved by the boundary term alone due to its narrow effective range and high sensitivity to image noise. The value of the boundary term lies in the fact that it can normally lead to more accurate segmentation results when relatively strong boundaries are presented in images. In case when boundary information is weak or ambiguous, the boundary mapping function can be chosen as $g(x) = 1$. Then the boundary term in Equ. (3) is simplified to a curvature flow term $\kappa\vec{N}$ whose role is simply to smooth curve.

2.2 Multiphase Framework

In the proposed model, to jointly segment multiple objects, multiple active contours, each associated with a single object, need to be applied. If each active contour evolves independently according to the evolution rule in Equ. (3), a set of PDEs can be written as

$$\frac{\partial C_k}{\partial t} = V_k(C_k)\cdot\vec{N}, \quad k = 1,...,n \quad (4)$$

with n denoting the number of active contours and C_k denoting the k^{th} active contour. V_k represents the velocity along the normal direction of C_k, which can be written as

$$V_k(\mathbf{x}) = P_k(I(\mathbf{x})) + \alpha_k\left(g_k(|\nabla I(\mathbf{x})|)\kappa - <\nabla g_k(|\nabla I(\mathbf{x})|),\vec{N}>\right) \quad (5)$$

where the selections of $P_k(x)$, $g_k(x)$ and α_k depend on the corresponding object to be segmented.

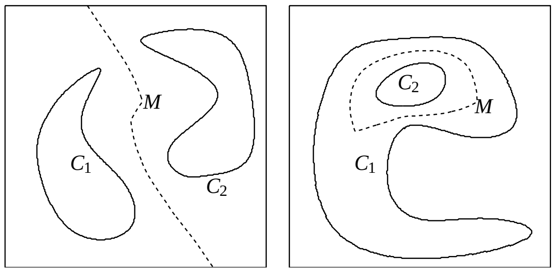
In order to make active contours interact with each other so that they can evolve in a more controllable way, a constraining component in the form of $D_k(C_k;\mathbf{C})$ is introduced into Equ. (4)

$$\frac{\partial C_k}{\partial t} = V_k(C_k)\cdot\vec{N} + \beta_k D_k(C_k;\mathbf{C})\cdot\vec{N}, \quad k = 1,...,n \quad (6)$$

where $\mathbf{C}$ represent the entire set of active contours, i.e, $\mathbf{C} = \{C_1,\ldots C_n\}$. In our study, medial axes are found to be an elegant way to constrain the evolution of multiple curves. The following two subsections describe how the medial axis can be related to the constraining component to impose topology and dynamic shape constraints.

2.2.1 Topology Constraint

In medical images, topology of objects to be segmented is often known and can be used as prior information. By using the prior, a set of initial contours satisfying the topology can be established. The topology constraint requires that the initial topology of the contours should be preserved in the process of curve evolution.



(a) Exclusion (b) Inclusion

Figure 3: Two fundamental topological relationship studied in this paper.

For any pair of curves, there are two fundamental topological structures studied in this paper, namely, exclusion and inclusion. The exclusion structure is illustrated in Figure 3(a), where the two curves C_1 and

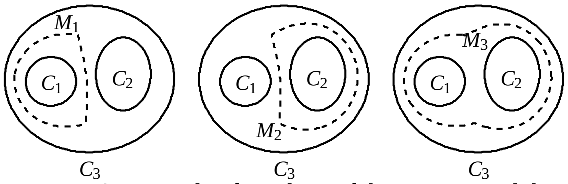


Figure 4: An example of topology of three curves and their associated medial axes.

C_2 are disjoint, i.e., $\Omega_1 \cap \Omega_2 = \emptyset$ with Ω_k denoting the interior area of C_k throughout the paper. The inclusion structure is illustrated in Figure 3(b), where one curve contains the other, i.e., $\Omega_1 \subset \Omega_2$ or $\Omega_2 \subset \Omega_1$. More complicated topology can be built based on these two fundamental structures. A example of three curve configuration is shown in Figure 4. For the shown configuration, the topological constraints can be written as follows: $\Omega_1 \cap \Omega_2 = \emptyset$, $\Omega_1 \subset \Omega_3$ and $\Omega_2 \subset \Omega_3$.

For each evolving curve C_k, a medial axis denoted as M_k is associated with it. The medial axis, dependent on multiple curves, is defined as

$$M_k(\mathbf{C}) = \left\{ \mathbf{x} : \mathrm{d}(\mathbf{x}; C_k) = \min_{j \neq k} \mathrm{d}(\mathbf{x}; C_j) \right\} \tag{7}$$

with $\mathrm{d}(\mathbf{x}; C)$ representing the Euclidean distance between the point $\mathbf{x}$ and the curve C. The medial axes are shown as the dash curves in Figure 3 and Figure 4. Note that the medial axes are identical if there are only two curves. The details of an efficient way to calculate medial axes can be found in (Zhang and Matuszewski, 2009).

A medial axis divides the image domain into two regions — the region that only contains the associated active contour and the region that contains all the other active contours. The topological structure of multiple active contours is guaranteed to be preserved if none of the active contours 'leak' through their corresponding medial axes to the other sides of the regions. To prevent the leakage from happening, forces induced from the medial axes can be exerted on the corresponding active contours. Hence, the constraining term in Equ. (6) can be defined as

$$D_k(\mathbf{x}; \mathbf{C}) = f_k(\mathrm{sdf}(\mathbf{x}; M_k(\mathbf{C}))) \tag{8}$$

where $\mathrm{sdf}(\mathbf{x}; M_k)$ represents the signed distance function which equals to $\mathrm{d}(\mathbf{x}; M_k)$ for the point $\mathbf{x}$ belonging to the region that contains C_k and $-\mathrm{d}(\mathbf{x}; M_k)$ otherwise. $f_k(x)$ in Equ. (8) is the so-called distance-to-force transfer function which maps the signed distance to medial axis to the force exerted on the associated active contour. An example of the transfer function is $f_k(x) = x - \gamma$ with the positive distance threshold γ, which indicates that the force exerted on the associated active contour becomes more and more repulsive (negative) as the active contour approaches to its medial axis from its own side.

2.2.2 Dynamic Shape Constraint

Medial axis can also be used as a dynamic shape constraint for joint segmentation of multiple objects with similar shapes in different images.

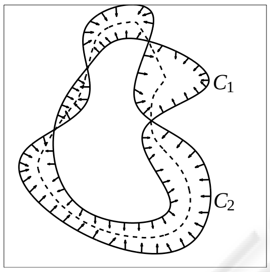


Figure 5: Basic idea of using medial axis as a dynamic shape constraint.

The basic idea of dynamic shape constraint is illustrated in Figure 5, where C_1 and C_2 are two active contours taken from two different images and put into the same figure for demonstration purpose. In this case, the medial axis of the two curves, shown as the dash curve in the figure, is defined as

$$M(\mathbf{C}) = \{ \mathbf{x} : \mathrm{d}(\mathbf{x}; C_1) = \mathrm{d}(\mathbf{x}; C_2) \text{ and } \mathbf{x} \notin \Omega_1 \cap \Omega_2 \} \tag{9}$$

Consequently, the constraining term in Equ. (6) can be defined as

$$D_k(\mathbf{x}; \mathbf{C}) = \mathrm{sdf}(\mathbf{x}; M(\mathbf{C})) \tag{10}$$

where $\mathrm{sdf}(\mathbf{x}; M)$ represents the signed distance function that equals to $\mathrm{d}(\mathbf{x}; M)$ for the point inside M and $-\mathrm{d}(\mathbf{x}; M_k)$ otherwise. The medial axis can be considered as the mean shape of the two active contours and it is dynamic in the sense that it also evolves as the active contours evolve. The directions of the forces imposed on the active contours are indicated as arrows in Figure 5. It can be seen that the forces tend to move the contours towards their dynamic mean shape.

This approach can be easily extended to the case of more than two curves as long as the mean shape of multiple curves can be properly defined.

3 EXPERIMENTAL RESULTS

A couple of experiments on real medical images, using medial axis as topology constraint and dynamic shape constraint respectively, are presented in this section.

In the first experiment, the objective was to segment two ventricles as well as the entire heart from the MRI image of a mouse shown in the top-left image of

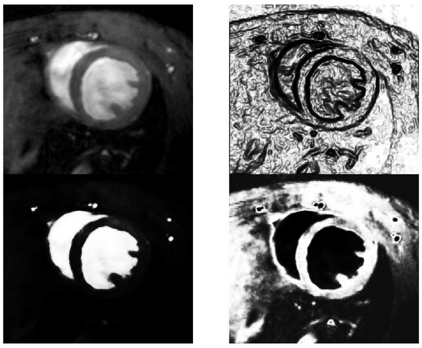


Figure 6: Original MRI image of mouse heart (left top); its boundary term (right top) and regional terms (bottom). (Image was kindly provided by the FUGE Molecular Imaging Centre, Trondheim, Norway.)

Figure 6. The rest of the Figure shows the three images, obtained from the MRI image, encoding boundary and regional terms (as g_k and P_k in Equ. (5)). From the image slice, it can be seen that the objects to be segmented have the same topological structure as those in Figure 4, i.e., the heart includes both ventricles and the ventricles are mutually exclusive. Therefore three curves with the same topological structure were initialized as shown in the first image of Figure 7. For comparison, curve evolution processes with and without medial axis constraint are shown in Figure 7 and Figure 8 respectively. Curve evolution without medial axis constraint clearly failed as the outer contour collapsed, due to lack of well-defined information of the outer heart wall, resulting in topological changes.

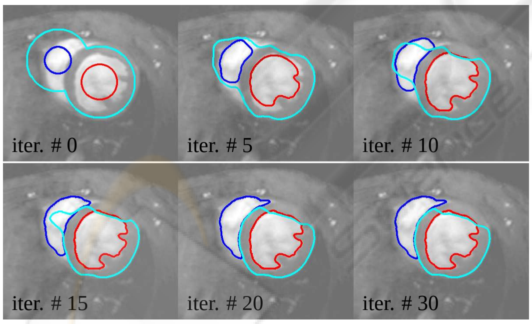


Figure 7: Curve evolution without medial axis constraint.

In the second experiment, two knee images were extracted from a volumetric MRI data. The objective was to segment bones from these images. The original images as well as the processed images used for regional and boundary terms are shown in Figure 9. Figure 10 shows the curve evolution when the segmentation is performed independently. It can be seen that the results for both images are not acceptable — in the first image, leakage happened due to weak boundary; in the second image, the resulting

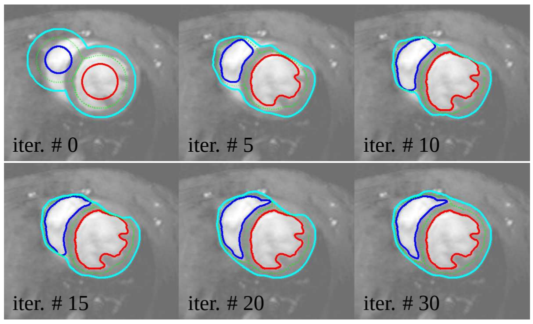


Figure 8: Curve evolution with medial axis constraint.

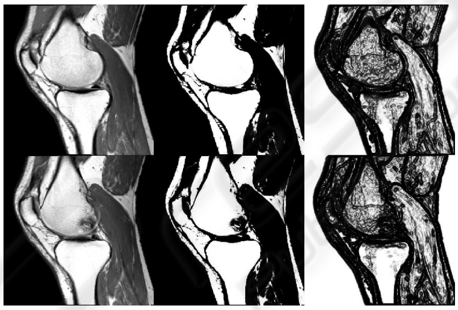


Figure 9: First row: image 1; second row: image 2. Columns from left to right: original image, regional term, boundary term.

segmentation failed to include the lower right corner of the upper bone due to dark shadow. As shown in Figure 11, much better results were achieved by using the medial axis constraint with the two evolving contours helping each other to converge to the correct bone structures.

4 CONCLUSIONS

This paper proposes a multiphase active contour method for simultaneous segmentation of multiple objects in one or multiple images. In the method, each object to be segmented is associated with an active contour. The evolution PDE associated with each active contour involves two main components — the component responsible for analysis of the image and each separate contour properties, and the component encoding interaction between different evolving curves. For the former component, a model using both boundary and regional information from input images is proposed in order to achieve robust and accurate results. In the model, medial axis plays an essential role in composing the interaction components to impose topology constraint or dynamic shape constraint to achieve robust results.

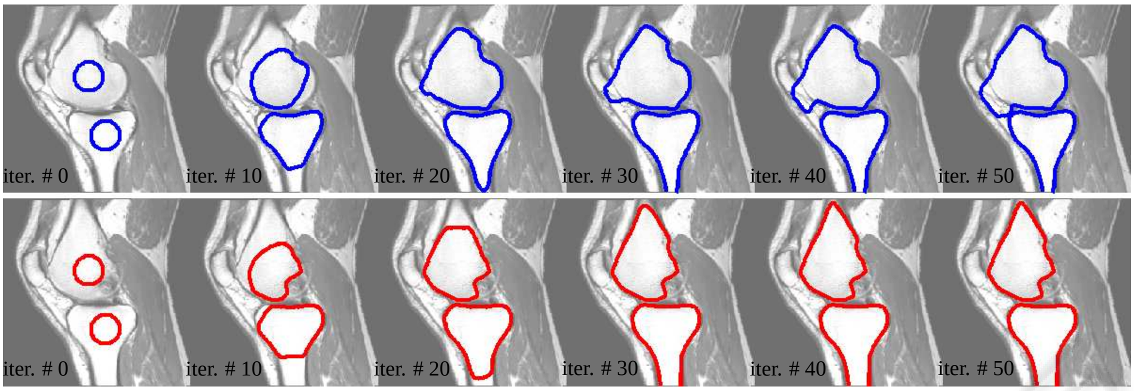


Figure 10: Curve evolution without medial axis constraint.

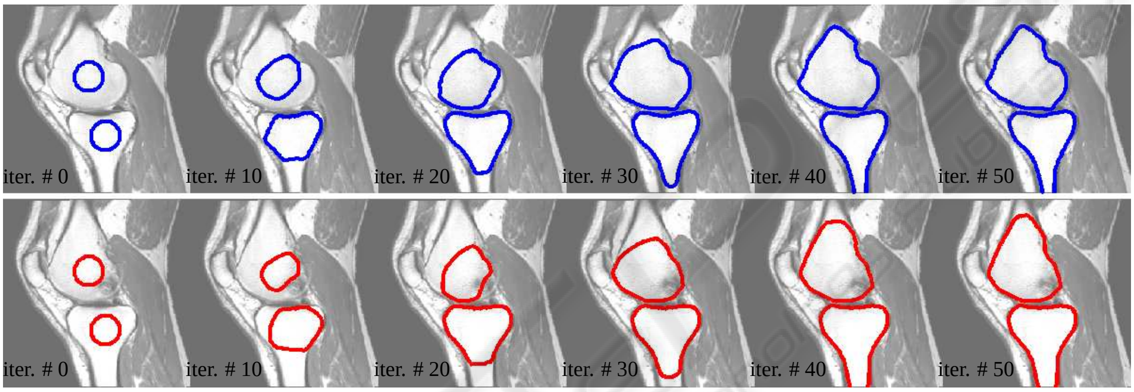


Figure 11: Curve evolution with medial axis constraint.

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