FREQUENCY EXTRACTION BASED ON ADAPTIVE FOURIER SERIES
Application to Robotic Yoyo

Tadej Petrič, Andrej Gams and Leon Žlajpah
Department of Automation, Biocybernetics and Robotics, Jožef Stefan Institute, Jamova cesta 39, 1000 Ljubljana, Slovenia

Keywords: Fourier series, Nonlinear dynamical systems, Adaptive frequency oscillators, Imitation, Yo-yo.

Abstract: We present a novel method to obtain the basic frequency of an unknown periodic signal with an arbitrary waveform. The method originates from nonlinear dynamical systems for frequency extraction, which are based on adaptive frequency oscillators in a feedback loop. While using several adaptive frequency oscillators in a loop results in extraction of separate frequency components, our method extracts the basic frequency of the input signal without any additional logical operations. The proposed method uses a whole Fourier series representation in the feedback loop. In this way it can extract the frequency and the phase of an unknown periodic signal, in real-time, and without any additional signal processing or preprocessing. The method also determines the Fourier series coefficients and can be used for dynamic Fourier series implementation. It can be used for the control of rhythmic robotic tasks, where only the extraction of the fundamental frequency is crucial. This is demonstrated on a highly nonlinear and dynamic task of playing the robotic yo-yo.

1 INTRODUCTION

Controlling rhythmic robotic tasks that require synchronization with the actuated device or interaction with the external environment is a difficult task, and requires complex sensory systems and advanced knowledge (Petrič et al., 2009). For example, such rhythmic tasks include handshaking (Kasuga and Hashimoto, 2005), locomotion (Ijspeert, 2008), drumming (Degallier et al., 2008), or playing with different toys, like the yo-yo(Žlajpah, 2006) or the gyroscopic device called Powerball (Gams et al., 2007). Controlling these tasks with robots requires both accurate trajectory generation and frequency tuning.

Determining the fundamental frequency of a task is a complex problem and can be achieved in different ways, e.g. with signal processing methods, such as FFT, or with the use of nonlinear oscillators (Matsuoka et al., 2005). Furthermore, trajectory generation and modulation are still difficult tasks in robotics. One possible approach to trajectory generation and the modulation is the imitation (Schaal, 1999), which can be preformed in several different ways, using encoding methods like splines (Ude et al., 2000) or dynamic movement primitives (DMP) (Schaal et al., 2007).

Not many approaches that combine both frequency extraction and waveform learning exist. One of them is the use of a two-layered imitation system based on nonlinear dynamical systems (Gams et al., 2009). In their work, the authors explained that the imitation system can be used for extracting the frequency of the input signal, learning its waveform, and imitating the waveform at the extracted or any other frequency. Similar, but with less properties for trajectory generation and modulation can be achieved by using only the first layer of this system for both frequency extraction and waveform learning (Righetti et al., 2006). The described systems are based on adaptive frequency oscillators in a feedback loop. Such an approach can determine several frequency components of an input signal. Despite favorable properties of these systems, there is a considerable drawback in determining the basic or fundamental frequency of the input signal.

For complex periodic signals with several frequency components, the first layer of the imitation system, referred to as the canonical dynamical system, has to include a high number of oscillators in the feedback loop. Using this system for movement imitation requires determining the basic frequency. This is accomplished by a logical algorithm that follows the feedback loop. With a high number of oscilla-

Petrič T., Gams A. and Žlajpah L. (2010).
FREQUENCY EXTRACTION BASED ON ADAPTIVE FOURIER SERIES - Application to Robotic Yoyo.
In *Proceedings of the 7th International Conference on Informatics in Control, Automation and Robotics*, pages 32-38
DOI: 10.5220/0002933900320038

tors, and when several of the oscillators tune to the same frequency, this can become extremely complex. A high number of oscillators is practically necessary and cannot be avoided.

The contribution of this paper is the novel design of the canonical dynamical system for the two-layered imitation system. The proposed approach does not require a logic algorithm to determine the fundamental frequency of the input signal, as in the original approach (Gams et al., 2009). We use a single adaptive phase oscillator in a feedback loop. The oscillator is followed by a complete Fourier series approximation, with a built-in algorithm to determine the Fourier coefficients. The combination of an adaptive phase oscillator and the adaptive Fourier series allows us to extract the fundamental frequency of the input signal and use it to control rhythmic robotic tasks.

With this approach we essentially implemented a real-time, adaptive Fourier series analysis. Our system is able to calculate the Fourier coefficients of an unknown periodic signal in real-time and is computationally inexpensive. The usefulness of this system is presented on the case of playing the yo-yo. Controlling the yo-yo has already been a subject of several studies (Žlajpah, 2006; Jin et al., 2009), which mostly rely on complex, specially designed controllers based on the models of the device. The task of playing yo-yo is highly non-linear and requires on-line frequency adaptation. The proposed approach simplifies the synchronization between the upward jerk of the robot and the movement of the yo-yo by determining the frequency of the up-down motion from a measurable periodic quantity.

The paper is organised as follows. In section 2, we give a brief description of the original two-layered imitation system with an emphasis on the first layer - the canonical dynamical system. In section 3 we describe the novel approach using the Fourier series in the feedback loop. In section 4 we evaluate the proposed approach in simulation, and on a real-world experiment of playing the yo-yo. Conclusions and summary are in section 5.

2 TWO-LAYERED IMITATION SYSTEM

The two-layered imitation system was presented in detail in (Gams et al., 2009). In their work the authors explained that the system can be used for extracting the frequency spectrum of the input signal, learning the waveform of one period, and imitating the desired waveform at an arbitrary frequency. The system structure is presented in Figure 1. The first layer, i.e. the canonical dynamical system, is used for frequency extraction. It is based on a set of adaptive frequency oscillators in a feedback loop. The second layer is called the output dynamical system and is used for learning and repeating the desired waveform. The latter is based on dynamic movement primitives - DMPs, e.g. (Schaal et al., 2007).

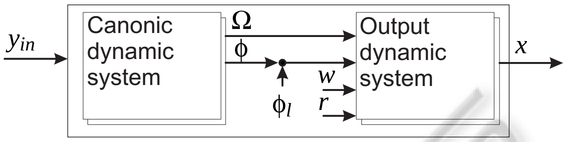

Figure 1: Two-layered structure of the imitation system. The input y_{in} is a measured quantity and the output is the desired trajectory x of the robot. The input ϕ_l is the additional phase lag and r is the amplitude of the trajectory. The system can work in parallel for an arbitrary number of dimensions.

The first layer of the system has two major tasks. It has to extract the fundamental frequency Ω of the input signal and it has to exhibit stable limit cycle behavior in order to provide the phase signal Φ. The basis of the canonical dynamical system is a set of adaptive phase oscillator with applied learning rule as introduced in (Buchli and Ijspeert, 2004). In order to accurately determine the frequency, it is combined with a feedback structure (Buchli et al., 2008) (see Figure 2). The feedback structure of M adaptive frequency oscillators is governed by

$$\dot{\phi}_i = \omega_i - K \cdot e \cdot \sin\phi_i, \tag{1}$$

$$\dot{\omega}_i = -K \cdot e \cdot \sin\phi_i, \tag{2}$$

$$e = y_{in} - \hat{y}, \tag{3}$$

$$\hat{y} = \sum_{i=1}^{M} \alpha_i \cos\phi_i, \tag{4}$$

$$\dot{\alpha}_i = \eta \cdot e \cdot \cos\phi_i, \tag{5}$$

where K is the coupling strength, ϕ_i, $i = 1...M$ is the phase of separate oscillators, y_{in} is the input signal, M is the number of oscillators, α_i is the amplitude associated with the i-th oscillator, and η is the learning constant.

As shown in Figure 2, each of the oscillators in the feedback structure receives the same input, i.e. the difference between the input signal and the weighted sum of separate frequency components. Such a feedback structure preforms a kind of Fourier analysis. The number of extracted frequencies depends on how many oscillators are used. As only the fundamental frequency is of interest, the feedback structure is followed by a logic algorithm. Determining the correct frequency and the phase is crucial, because they are

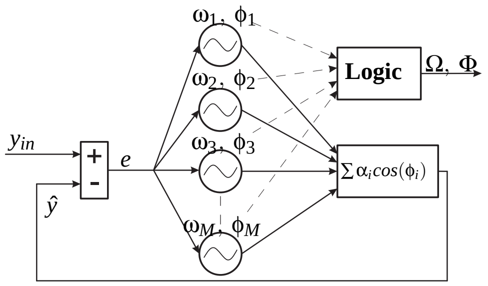

Figure 2: Feedback structure of M nonlinear adaptive frequency oscillators. Note the logic algorithm that follows the feedback loop.

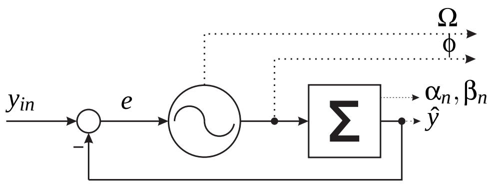

Figure 3: Feedback structure of nonlinear adaptive frequency oscillator combined with dynamic Fourier series. Note that no logic algorithm needed.

the basis for the output dynamic system and the desired behavior of the actuated device.

One possible approach is to choose the first nonzero frequency as was presented in (Gams et al., 2009). However, it has a drawback that when more than one oscillators converge to, or oscillate, around the same frequency, the logic algorithm switches between the oscillators, and consequently the phase will not be smooth, leading to oscillations in the output trajectory.

3 CANONICAL DYNAMICAL SYSTEM BASED ON FOURIER SERIES

In this section a novel architecture for canonical dynamical system is presented. As the basis of the canonical dynamical system we use a single nonlinear phase oscillator with applied learning rule (Buchli and Ijspeert, 2004). This is combined with a feedback structure based on an adaptive Fourier series in order to accurately determine the frequency. A feedback structure with an adaptive frequency oscillator combined with an adaptive Fourier series is shown in Figure 3. The feedback structure of an adaptive frequency phase oscillator is governed by

$$\dot{\phi} = \Omega - K \cdot e \cdot \sin\phi, \tag{6}$$

$$\dot{\Omega} = -K \cdot e \cdot \sin\phi, \tag{7}$$

$$e = y_{in} - \hat{y}, \tag{8}$$

where K is the coupling strength, ϕ is the phase of the oscillator, e is the input into the oscillator and y_{in} is the input signal. If we compare Eqs. (1, 2) and Eqs. (6, 7), we can see that the frequency Ω and the phase ϕ are now clearly defined. The feedback loop $\hat{y}$ is now represented by the Fourier series

$$\hat{y} = \alpha_0 + \sum_{i=1}^{M} (\alpha_i \cos(i\phi) + \beta_i \sin(i\phi)), \tag{9}$$

and not by the sum of separate frequency components as in Eq. 4. M is the size of the Fourier series and α_0 is the amplitude associated with the first segment of the series, it is governed by

$$\dot{\alpha}_0 = \eta \cdot e, \tag{10}$$

here η is a learning constant. The amplitudes associated with the other terms of the Fourier series are determined by

$$\dot{\alpha}_i = \eta \cos(i\phi) \cdot e, \tag{11}$$

$$\dot{\beta}_i = \eta \sin(i\phi) \cdot e, \tag{12}$$

where $i = 1...M$. As shown in Figure 3, the oscillator of the feedback structure receives the difference between the input signal and the Fourier series. Since a negative feedback loop is used, the difference approaches zero when the Fourier series representation approaches the input signal. Such a feedback structure preforms an adaptive Fourier analysis, where the phase difference between the harmonics can only be 0, $\pi/2$, π or $3\pi/2$. This is not the case in the original approach (Righetti and Ijspeert, 2006), where the phase difference can be arbitrary.

The proposed approach has the ability to adapt to the basic frequency of the input signal. The number of harmonic frequency components it can accurately extract depends on how many terms of the Fourier series are used. Since in this structure only one oscillator is used and the harmonics are encoded in the Fourier series, the basic frequency and phase are clearly defined. This is an important improvement, especially for the usefulness of the imitation system when performing rhythmic tasks.

The new architecture of the canonical dynamic system can be used as an imitation system by itself, as it is able to learn arbitrary periodic signals. After convergence, e reaches zero (with an accuracy that depends on the number of elements of the Fourier series). Once e is zero, the periodic signal stays encoded in the Fourier series. The learning process is embedded and is done in real-time. There is no need for any external optimization process or learning algorithm.

Adding the output dynamical system enables us to synchronise the motion of the robot to a measurable periodic quantity of the task we would like to preform. The measured signal is now encoded into the Fourier series and the desired robot trajectory is encoded in the output dynamic system. Sice adaptation of the frequency and the learning of the desired trajectory can be done simultaneously, all of the system time-delays can be automatically included. Furthermore, when a predefined motion pattern for the trajectory is used, the system time-delays can be adjusted with a phase lag parameter ϕ_l. This enables us to either predefine the desired motion or to teach the robot how to preform the desired rhythmic task online.

The output dynamical system also ensures greater robustness against perturbations and smooth modulation. Specially greater robustness to perturbation is crucial when performing fast, dynamic tasks.

4 EVALUATION

In the following section we evaluate of the proposed imitation system with the new canonical dynamical layer. In the Section 4.1 the numerical results from the original and the novel architectures are presented. In Section 4.2 a real-world experiment of playing the yo-yo with the use of the proposed imitation system is shown.

4.1 Simulation

In this numerical experiment the proposed architecture for the canonical dynamical system learns an arbitrary signal. The populating of the frequency spectrum is done without any signal processing, as the whole process of frequency extraction and adaptation of the waveform is completely embedded in the dynamics of the adaptive frequency oscillator combined with the adaptive Fourier series. Unless stated otherwise, we use the following parameters: $\mu = 2$, $K = 20$, $M = 10$.

Frequency adaptation results from time- and shape-varying signals are illustrated in Figure 4. The input signal itself is of three parts: a periodic pulse signal, a sinusoid, and a sawtooth wave signal. Transition between the signal parts is instant for both frequency and waveform. We can see that after the change of the input signal, the output frequency stabilises very quickly.

A single adaptive frequency oscillator in a feedback loop is enough, because the harmonics of the input signal are encoded with the Fourier series in the feedback loop. As can be seen from the bottom plots in Figure 4, the input signal and the feedback signal are very well matched. The approximation error depends only on M. A comparison with the original approach as proposed in (Buchli et al., 2008) is given in Figure 5. In their approach, if there are not enough oscillators to encode the input signal, the system will only learn the frequency components with more power. Thus, the output signal will only be an approximation.

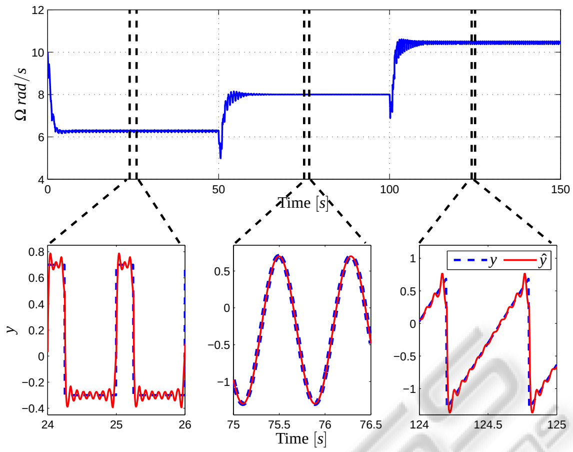

Figure 4: Typical convergence of an adaptive frequency oscillator combined with an adaptive Fourier series, driven by a periodic signal with different waveforms and frequencies. Frequency adaptation is presented in the top plot and the comparison between the input signal y and the approximation $\hat{y}$ in the bottom plot.

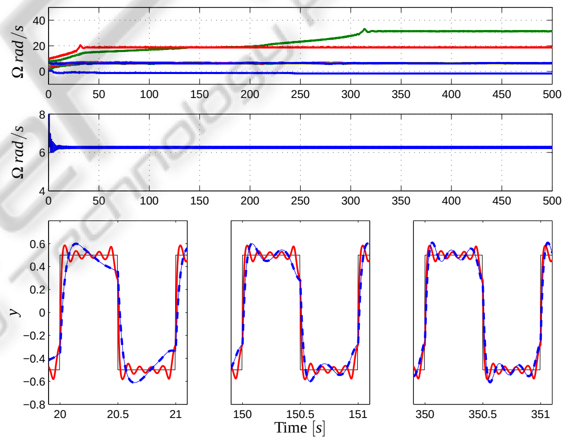

Figure 5: Comparison between the pool of the adaptive oscillators and our proposed approach. First plot shows evolution of frequency distribution using a pool of 10 oscillators. Middle plot shows the extracted frequency using an adaptive frequency oscillator combined with Fourier seres. The comparison of approximated signals is presented in the bottom plot. The thin solid line presents the input signal, the solid line presents our new proposed approach and the dotted line presents the pool of adaptive oscillators.

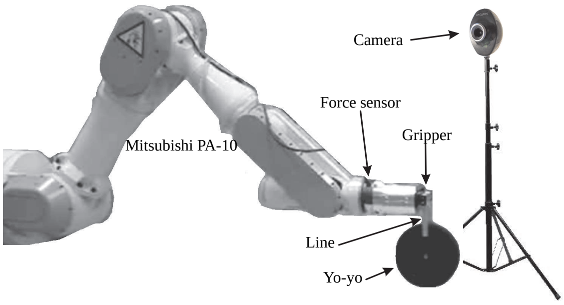

Figure 6: Experimental setup.

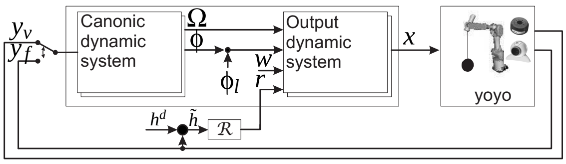

Figure 7: Proposed two-layered structure of the control system for controlling the peak height of the yo-yo. The input is either the force y_f or the visual feedback y_v.

However, if there are more oscillators than the frequency components to learn, either some of them will not converge to any frequency or the same frequency components will be coded by several oscillators, as shown in the top plot in Figure 5, where a pool of ten oscillators was used. In this particular experiment, five of the oscillators converge to the basic frequency of the signal.

Choosing the right oscillator from that pool is a very difficult task and requires a complex logic algorithm. On the other hand, using our new approach, where the feedback is encoded with a Fourier series, the oscillator converges to the basic frequency of the input signal. Therefore, the basic frequency and the phase are clearly defined. Furthermore, the approximation and the convergence of the feedback signal is quicker, as it is shown in the bottom plots in Figure 5. Even after 350 s, the original architecture from (Righetti and Ijspeert, 2006) did not produce as good an approximation as it was after 20 s when using our new proposed canonical dynamical system.

4.2 Application to Robotic Yo-yo

To illustrate the proposed approach we implemented it on a real robot playing yo-yo.

Playing yo-yo with a robot can be achieved in different ways, depending on what one can measure. It can be the length of the unwound string, which can be effectively measured by a vison system. As described in (Žlajpah, 2006), using vison is also one of the ways humans do it, even though approaches using only the measured force were described (Jin et al., 2009). With our proposed system, playing yo-yo can be accomplished either with force feedback or with visual feedback. Furthermore, the proposed system is able to synchronise even if the input signal is changed from one measurable quantity to another during the experiment.

We preformed the experiment on a Mitsubishi PA-10 robot as presented in Figure 6. A force sensor (JR3), was attached to the end effector to measure the impact force of the yoyo, and a USB camera was used to measure the length of the unwound string.

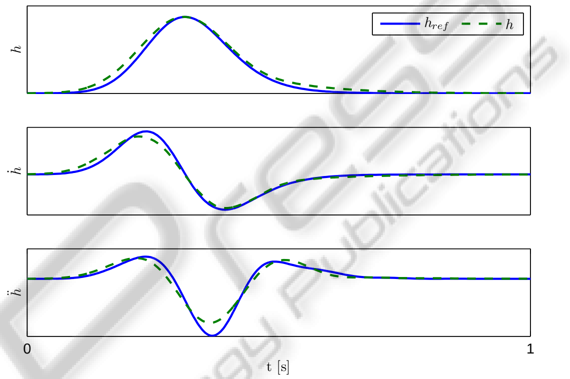

Figure 8: Pre-defined hand motion pattern for playing yo-yo.

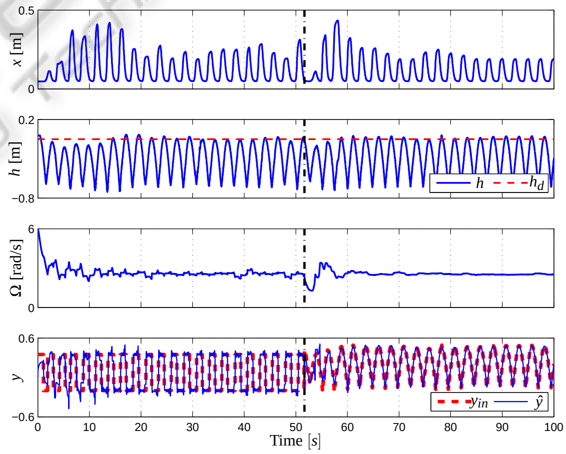

Figure 9: Robot trajectory x in the top plot, height h of the yo-yo in the second plot, extracted frequency in the third plot and signal adaptation $\hat{y}$ in the bottom plot. At 52 s the input signal is switched form force feedback to visual feedback. Yo-yo parameters in this case are: axle radius $r_a = 0.01$ m and mass $m = 0.2564$ kg.

The two layered imitation system with the novel canonical dynamical system was implemented in Matlab/Simulik. The control scheme is presented in Figure 7. As we can see, the imitation system, based on a nonlinear oscillator combined with dy-

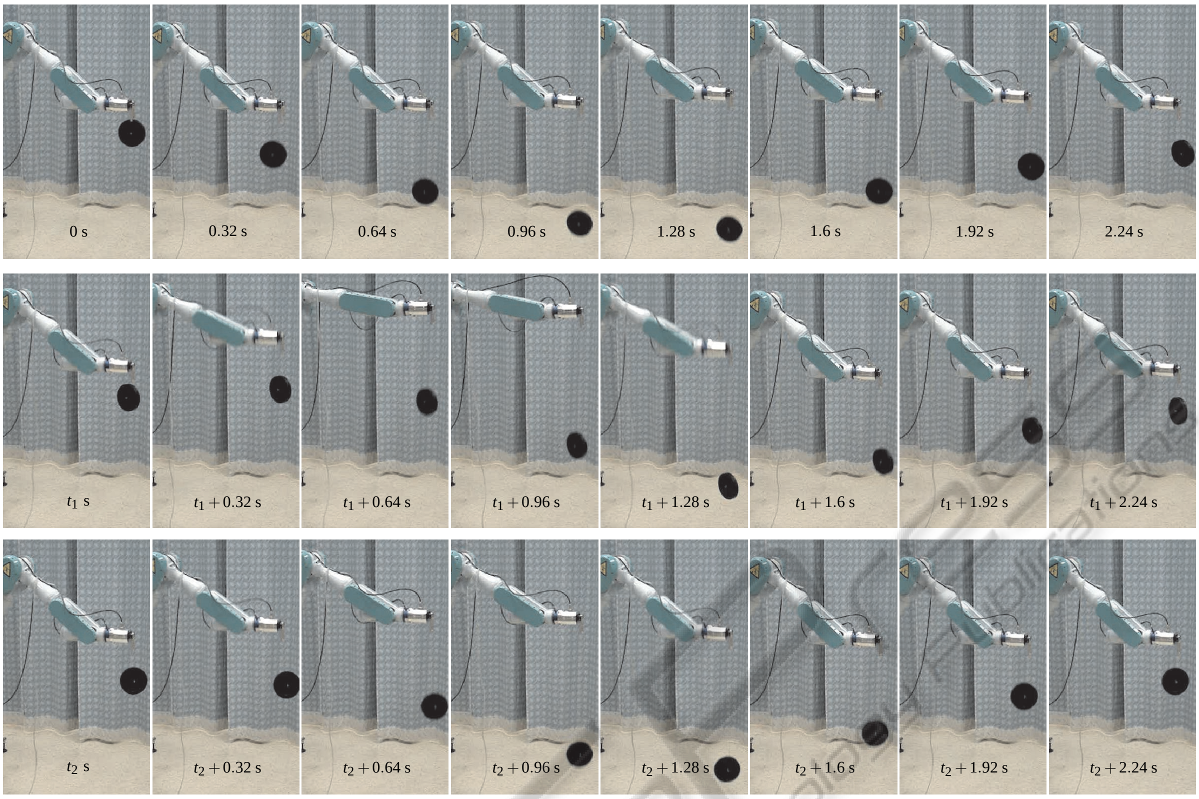

Figure 10: Image sequence of a robotic yoyo. Top sequence present the decent of the yoyo from the gripper. In the middle sequence a behavior of the system after switching from force feedback to vision feedback is shown ($t_1 = 54$ s) and in bottom sequence the behavior in steady state is presented ($t_2 = 81$ s).

namic Fourier series, provides the desired trajectory for the robot with a yo-yo attached at the top. The motion of the robot was constrained to up-down motion using inverse kinematics. The length of the string or the force from the top of the robot can be used as the input into the system. Since a measurable force difference appears only as a spike, when the yo-yo hits the end of the string, we modify the signal in a way that it carries more energy. In our particular case, we use the measured spike to create a short pulse.

To preform the task, we first determine the waveform of the required motion pattern. We chose the motion pattern described in (Žlajpah, 2006), which satisfies the required criteria for playing the yo-yo. The hand motion pattern encoded into the output dynamic system (dashed line), and the desired hand motion pattern (solid line) are presented in Figure8.

The frequency of the task depends on the parameters of the yo-yo itself, and on how high the yo-yo rolls up along the string. The height can be influenced by the amplitude of the hand motion, which can be easily modified using the amplitude parameter r of the motion, see Figure 7. PI controller was used to control the peak height of the yo-yo. The controller is given by

$$u(t) = k_p e(t) + k_i \int e(t)dt, \tag{13}$$

where $k_p = 2$, and $k_i = 0.4$ were determined empirically. Figure 9 shows the results of frequency adaptation and yo-yo height during the experiment.

As we can see, the frequency of the imitated motion quickly adapted to the motion of the yo-yo and stable motion was achieved. At approximately 52 s the input into the imitation system was switched from force feedback to visual feedback. At that point some oscillation in the frequency and the approximation of the input signal can be observed because they have to adapt to the new waveform of the input signal. Furthermore, form the middle sequence in Figure 10 we can see that the amplitude of hand motion is higher after switching from the force feedback to the vision feedback. Despite the change, the imitation system still manages to extract the correct frequency and the robot motion returns to steady-state oscillations. Note that in the bottom sequence in Figure 10 the hand amplitude is smaller than immediately after the switch.

As far as we know, this is the first system which has the capability of playing the yo-yo by force feedback or by vision feedback, without changing the system parameters. Furthermore, switching from one to another measured quantity can even be done during the experiment. This shows that the proposed system is adaptable and robust.

5 CONCLUSIONS

We presented a new architecture for the canonical dynamical system which is a part of a two layered imitation system, but can be used as an imitation system by itself. The dynamical system which, is used to extract the frequency, is composed of a nonlinear phase oscillator combined with a Fourier series. This system essentially implements an adaptive Fourier series of the input signal. It can extract the frequency, phase and the Fourier series coefficients of an unknown periodic signal. This is done in real-time without any additional processing of the input signal. Integrating this system into the imitation system based on dynamic motion primitives enables simple and computationally inexpensive control of rhythmic tasks with at least one measurable periodic quantity.

Furthermore, we presented the use of the imitation system to preform a rhythmic task that requires synchronization with the controlled device. For playing the yo-yo, we have shown that the information on how high the yo-yo rolls up along the string, or the force feedback is enough to achieve stable performance. The proposed approach enables to play yo-yo by measuring either the force or the yo-yo position. Furthermore we also showed that the system has the capability of changing the measured quantity in a single experiment without loosing the synchronization between the robot and the yo-yo.

REFERENCES

Buchli, J. and Ijspeert, A. (2004). A simple, adaptive locomotion toy-system. In Schaal, S., Ijspeert, A., Billard, A., Vijayakumar, S., Hallam, J., and Meyer, J., editors, *From Animals to Animats 8. Proceedings of the Eighth International Conference on the Simulation of Adaptive Behavior (SAB'04)*, pages 153–162. MIT Press.

Buchli, J., Righetti, L., and Ijspeert, A. J. (2008). Frequency Analysis with coupled nonlinear Oscillators. *Physica D: Nonlinear Phenomena*, 237:1705–1718.

Degallier, S., Righetti, L., Natale, L., Nori, F., Metta, G., and Ijspeert, A. (2008). A modular bio-inspired architecture for movement generation for the infant-like robot iCub. In *Proceedings of the 2nd IEEE RAS / EMBS International Conference on Biomedical Robotics and Biomechatronics (BioRob)*.

Gams, A., Ijspeert, A. J., Schaal, S., and Lenarcic, J. (2009). On-line learning and modulation of periodic movements with nonlinear dynamical systems. *Auton. Robots*, 27(1):3–23.

Gams, A., Žlajpah, L., and Lenarčič, J. (2007). Imitating human acceleration of a gyroscopic device. *Robotica*, 25(4):501–509.

Ijspeert, A. J. (2008). Central pattern generators for locomotion control in animals and robots: A review. *Neural Networks*, 21(4):642–653.

Jin, H.-L., Ye, Q., and Zacksenhouse, M. (2009). Return maps, parameterization, and cycle-wise planning of yo-yo playing. *Trans. Rob.*, 25(2):438–445.

Kasuga, T. and Hashimoto, M. (2005). Human-robot handshaking using neural oscillators. In *ICRA*, pages 3802–3807.

Matsuoka, K., Ohyama, N., Watanabe, A., and Ooshima, M. (2005). Control of a giant swing robot using a neural oscillator. In *ICNC (2)*, pages 274–282.

Petrič, T., Gams, A., and Žlajpah, L. (2009). Modeling and control strategy for robotic powerball. In *RAAD*, page 7.

Righetti, L., Buchli, J., and Ijspeert, A. J. (2006). Dynamic hebbian learning in adaptive frequency oscillators. *Physica D*, 216(2):269–281.

Righetti, L. and Ijspeert, A. J. (2006). Programmable Central Pattern Generators: an application to biped locomotion control. In *Proceedings of the 2006 IEEE International Conference on Robotics and Automation*.

Schaal, S. (1999). Is imitation learning the route to humanoid robots? *Trends in cognitive sciences*, 6:233–242.

Schaal, S., Mohajerian, P., and Ijspeert, A. (2007). Dynamics systems vs. optimal control–a unifying view. *Progress in Brain Research*, 165:425–445.

Ude, A., Atkeson, C. G., and Riley, M. (2000). Planning of joint trajectories for humanoid robots using b-spline wavelets. In *ICRA*, pages 2223–2228.

Žlajpah, L. (2006). Robotic yo-yo: modelling and control strategies. *Robotica*, 24(2):211–220.