STRONG STABILIZATION BY OUTPUT FEEDBACK CONTROLLERS FOR INPUT-DELAYED LINEAR SYSTEMS

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Abstract: This paper addresses the strong stabilization problem for continuous-time linear systems with an unknown input delay using a dynamic output feedback. New criteria for output feedback stabilizability are proposed for the closed-loop system in terms of matrix inequalities with the separation of controller gain and not only Lyapunov matrix but also system matrices. Based on the new characterization, an iterative algorithm is given to design the strong output feedback controllers with the aid of an slack matrix introduced. The effectiveness and merits of the proposed approach are shown through a numerical example.

1 INTRODUCTION

It is well known that even a simple linear system with a single delay imposes difficulties and restrictions on the design of a stabilization controller. The stabilization problem for linear systems with an unknown delay in the input signal is still a difficult one as seen in (Chen and Zheng, 2006), (Respondek, 2008) and (Tadmor, 2000) (and the references therein). For this type of systems, few stabilization methods have been developed either with state feedback controllers or output feedback controllers, especially in strong stabilization analysis.

Strong stabilization, which is to design a stable stabilizing feedback controller for the given plant, is of great importance in the physical implementation of the control since unstable controllers may lead to unpredictable results in case of sensor faults and plant uncertainties/nonlinearities. The strong stabilization problem for linear delay-free systems has been studied in various frameworks (See (Cao and Lam, 2000), (Feintuch, 2008), (Vidyasagar, 1985)). For linear time-invariant systems, necessary and sufficient conditions shown in (Youla et al., 1974) for the existence of a stable stabilizing controller says that a rational plant is strongly stabilizable if and only if its number of unstable poles (counted according with their McMillan degrees) between every pair of right-half plane blocking zeros is even. Approaches utilizing $\mathcal{H}_2/LQG$ optimal control theory have been suggested subsequently to modify the cost function and Kalman filtering Riccati equation in order to guarantee the stability of the optimal controller. Campos-Delgado and Zhou (Campos-Delgado and Zhou, 2001) converted the stable $\mathcal{H}_\infty$ design problem into a 2-block standard $\mathcal{H}_\infty$ problem via the parametrization of all suboptimal $\mathcal{H}_\infty$ controllers, and reduced higher order controller designed in (Zeren, 1997) by a two-step reduction algorithm. Yoon and Kimura (Yoon and Kimura, 2006) presented a topological result on the robustness of nonstrong stabilizability and obtained two classes of nonstrongly stabilizable systems. However, little attention has been paid toward this issue for input-delayed systems since the stability constraints on the controllers are very hard to reflect on the cost functions and more difficult to implement than those without the strong stabilization requirement.

Related to the above remarks, a natural question to ask is how to design a dynamic output feedback (DOF) controller to strongly stabilize a system with an unknown input delay. This paper discusses in detail the output feedback stabilization problem for linear input-delayed systems using a new approach in the state space. A new stability condition of static output feedback (SOF) stabilization in terms of matrix inequalities is proposed first in Section 3. Advan-

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tages of such a characterization is twofold. First, the decoupling of the input and the gain-output matrix enables us to parameterize the controller matrix by a free matrix parameter. Second, the separation of the Lyapunov matrix and the controller matrix avoids imposing any constraint on the Lyapunov matrix when the controller matrix is parameterized. With the aid of the free-weighting matrix introduced and the separation of the Lyapunov matrix and the controller matrix, in Sections 4 and 5, delay-independent DOF strong stabilization of a general dynamic controller is realized though an iterative algorithm. The effectiveness and merits of the proposed approach are shown in Section 6 through numerical examples in the end of the paper.

2 NOTATION AND PRELIMINARIES

Notation: Throughout this paper, for real symmetric matrices X and Y, the notation $X \geq Y$ (respectively, $X > Y$) means that the matrix $X - Y$ is positive semi-definite (respectively, positive definite). 0 in a matrix inequality is a null matrix with an appropriate dimension. The superscript "T" represents the transpose of the matrix and the asterisk "$*$" in a matrix stands the term which is induced by symmetry. col$\{\cdot\}$ denotes a matrix column with blocks given by the matrices in $\{\cdot\}$. A block diagonal matrix with diagonal blocks A_1, A_2, $\ldots$, A_r will be denoted by diag$\{A_1, A_2, \ldots, A_r\}$. Matrices, if their dimensions are not explicitly stated, are assumed to have compatible dimensions for algebraic operations.

Consider the following linear time-invariant system with delayed and non-delayed inputs,

$$(\Sigma): \quad \begin{array}{rcl} \dot{x}(t) & = & Ax(t) + B_0 u(t) + B_1 u(t-d) \\ y(t) & = & Cx(t) \end{array}$$

where $x(t) \in \mathbb{R}^n$ is the state with the initial function $\phi(t)$ when $t \in [-d, 0]$, and $y(t)$ is the measurement output. Here, A, B_0, B_1, C are the system state, the control input and the measured output matrices, respectively, and $d > 0$ is an unknown constant input delay.

The following lemma is needed in the paper.

Lemma 1 (Finsler's Lemma). *Consider real matrices F and Ω such that $\Omega = \Omega^T$ and F has full row rank. $F^{\perp}$ is the orthogonal complement of F which is (possibly non-unique) defined as the matrix with maximum column rank satisfying $FF^{\perp} = 0$ and $F^{\perp T}F^{\perp} > 0$. Then the following statements are equivalent:*

1. *There exists a vector $\xi(t) \in \mathbb{R}^n$ such that $\xi^T(t)\Omega\xi(t) < 0$ and $F\xi(t) = 0$;*
2. *There exists a scalar $\mu \in \mathbb{R}$ such that $\Omega + \mu F^T F < 0$;*
3. *The following condition holds: $F^{\perp T}\Omega F^{\perp} < 0$.*

3 SOF STABILITY ANALYSIS

An SOF controller under consideration is of the form,

$$(C_1): \quad u(t) = Ky(t)$$

where K is the controller gain to be designed. When SOF controller (C_1) is applied to (Σ), the closed-loop system is

$$(\Sigma_{c1}): \quad \dot{x}(t) = (A + B_0KC)x(t) + B_1KCx(t-d)$$

The following delay-independent criterion utilizes a free matrix $P_2 > 0$ to describe the stabilizability of system (Σ) associated with controller (C_1) in a special form.

Theorem 1. *The closed-loop system (Σ_{c1}) is asymptotically stable, if there exist matrices $P_1 > 0$, $P_2 > 0$, $S > 0$, and K such that*

$$\Upsilon_1 = \begin{bmatrix} \mathbf{P}^T\mathbf{A} + \mathbf{A}^T\mathbf{P} + \mathbf{S}_1 & \mathbf{P}^T\mathbf{B}_1 \\ \mathbf{B}_1^T\mathbf{P} & \mathbf{S}_2 \end{bmatrix} < 0 \qquad (1)$$

where $\mathbf{A} = \begin{bmatrix} A & B_0 \\ KC & -I \end{bmatrix}$, $\mathbf{B}_1 = \begin{bmatrix} 0 & B_1 \\ 0 & 0 \end{bmatrix}$, $\mathbf{S}_1 = \begin{bmatrix} S & 0 \\ 0 & 0 \end{bmatrix}$, $\mathbf{S}_2 = \begin{bmatrix} -S - C^TK^TP_2KC & C^TK^TP_2 \\ P_2KC & -P_2 \end{bmatrix}$, *and* $\mathbf{P} = \begin{bmatrix} P_1 & 0 \\ -P_2KC & P_2 \end{bmatrix}$.

Proof: System (Σ_{c1}) is asymptotically stable (Gu et al., 2003) if there exist matrices $P_1 > 0$ such that

$$\Upsilon_2 \triangleq \begin{bmatrix} \begin{array}{c} S + P_1(A + B_0KC) \\ +(A + B_0KC)^TP_1 \end{array} & P_1B_1KC \\ * & -S \end{bmatrix} < 0 \quad (2)$$

In the following, we establish the equivalence of (1) and (2).

(Sufficiency) By pre- and post multiplying (1) by $\begin{bmatrix} S_1^T & 0 \\ 0 & S_1^T \end{bmatrix}$ and $\begin{bmatrix} S_1 & 0 \\ 0 & S_1 \end{bmatrix}$ with $S_1 = \begin{bmatrix} I \\ KC \end{bmatrix}$, respectively, we have

$$\begin{bmatrix} S_1^T\mathbf{P}^T\mathbf{A}S_1 + S_1^T\mathbf{A}^T\mathbf{P}S_1 + S_1^T\mathbf{S}_1S_1 & S_1^T\mathbf{P}^T\mathbf{B}_1S_1 \\ * & S_1^T\mathbf{S}_2S_1 \end{bmatrix} < 0 \qquad (3)$$

and in that

$$\begin{bmatrix} I \\ KC \end{bmatrix}^T \begin{bmatrix} P_1 & -C^TK^TP_2 \\ 0 & P_2 \end{bmatrix} \begin{bmatrix} A & B_0 \\ KC & -I \end{bmatrix} \begin{bmatrix} I \\ KC \end{bmatrix}$$
$$= \begin{bmatrix} P_1 & 0 \end{bmatrix} \begin{bmatrix} A + B_0KC \\ 0 \end{bmatrix} = P_1(A + B_0KC)$$

$$\begin{bmatrix} I \\ KC \end{bmatrix}^T \begin{bmatrix} S & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} I \\ KC \end{bmatrix} = S$$

$$\begin{bmatrix} I \\ KC \end{bmatrix}^T \begin{bmatrix} P_1 & -C^TK^TP_2 \\ 0 & P_2 \end{bmatrix} \begin{bmatrix} 0 & B_1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} I \\ KC \end{bmatrix}$$
$$= P_1B_1KC$$
$$\begin{bmatrix} I \\ KC \end{bmatrix}^T \begin{bmatrix} -S-C^TK^TP_2KC & C^TK^TP_2 \\ P_2KC & -P_2 \end{bmatrix} \begin{bmatrix} I \\ KC \end{bmatrix}$$
$$= -S$$

Thus inequality (3) is in fact (2).

(Necessity) Assume that (2) holds. There must be a sufficiently large matrix $P_2 > 0$, such that

$$- \begin{bmatrix} B_0^T \\ B_1^T \end{bmatrix} \begin{bmatrix} P_1 & 0 \end{bmatrix} \Upsilon_2^{-1} \begin{bmatrix} P_1 \\ 0 \end{bmatrix} \begin{bmatrix} B_0 & B_1 \end{bmatrix}$$
$$- \begin{bmatrix} 2P_2 & 0 \\ * & P_2 \end{bmatrix} < 0$$

Then straightforward manipulation and Schur complement equivalence yields that

$$T_1^T \begin{bmatrix} \Upsilon_2 & \begin{bmatrix} P_1B_0 \\ 0 \end{bmatrix} & \begin{bmatrix} P_1B_1 \\ 0 \end{bmatrix} \\ * & -2P_2 & 0 \\ * & * & -P_2 \end{bmatrix} T_1$$
$$= \begin{bmatrix} \bar{S}_1^T & 0 \\ * & \bar{S}_1^T \end{bmatrix} \Upsilon_1 \begin{bmatrix} \bar{S}_1 & 0 \\ * & \bar{S}_1 \end{bmatrix} < 0$$

where $T_1 = \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & 0 & I \end{bmatrix}$ and $\bar{S}_1 = \begin{bmatrix} I & 0 \\ KC & I \end{bmatrix}$ are nonsingular matrices. This completes the proof. □

Remark 1. *Theorem 1 provides an equivalent form to design a static output feedback controller for the input-delayed systems. The advantage of Theorem 1 lies in not only the separation of B_0, B_1 and KC, but also in the separation of Lyapunov matrix P_1 and the controller matrix K. This feature enables us to parametrize K by a tuning matrix $P_2 > 0$, independent of the Lyapunov matrix used for checking stability or performances directly. Therefore, less conservative results will be obtained, comparing with previous approaches, since no additional constraints induced to deal with the nonconvex terms of the Lyapunov matrix and the controller matrix when it is parametrized.*

Remark 2. *Intuitively, if d is known, the stabilization problem of systems $\dot{x}(t) = Ax(t) + Bu(t)$ and $y(t) = Cx(t)$ can be treated by a delayed output feedback controller $u(t) = K[y(t) + Zy(t-d)]$, where Z is a tuning matrix satisfying $KZ = ZK$, in the sense that the system $\dot{x}(t) = Ax(t) + Bu(t) + BZu(t-d)$ can be stabilized by an SOF controller $u(t) = Ky(t)$.*

4 DOF STRONG STABILIZATION

Consider a general form of dynamic controller as follows:

$$(C'): \quad \begin{aligned} \dot{\vartheta}(t) &= K_A\vartheta(t) + K_By(t) \\ u(t) &= K_c\vartheta(t) + K_Dy(t) \end{aligned}$$

The input-delayed system (Σ) with controller (C') gives the following closed-loop system $(\Sigma_{c'})$:

$$\begin{bmatrix} \dot{x}(t) \\ \dot{\vartheta}(t) \end{bmatrix} = (\hat{A} + \hat{B}_0\hat{K}\hat{C}) \begin{bmatrix} x(t) \\ \vartheta(t) \end{bmatrix} + \hat{B}_1\hat{K}\hat{C} \begin{bmatrix} x(t-d) \\ \vartheta(t-d) \end{bmatrix}$$

where $\hat{A} = \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix}$,
$\hat{B}_0 = \begin{bmatrix} 0 & B_0 \\ I & 0 \end{bmatrix}$, $\hat{B}_1 = \begin{bmatrix} 0 & B_1 \\ 0 & 0 \end{bmatrix}$, $\hat{C} = \begin{bmatrix} 0 & I \\ C & 0 \end{bmatrix}$
and $\hat{K} = \begin{bmatrix} K_A & K_B \\ K_C & K_D \end{bmatrix}$ is the controller gain matrix to be determined. Only the system data are involved in the above shorthands, and $\hat{A} + \hat{B}_0\hat{K}\hat{C}$ and $\hat{B}_1\hat{K}\hat{C}$ are affine in the controller gain $\hat{K}$.

The problem of strong stabilization is regarded as searching one or more common positive definite matrices to guarantee the stability of both the closed-loop system $(\Sigma_{c'})$ and its stabilizing controller (C'). For (C'), it is asymptotically stable if and only if there exists a matrix $S_c > 0$ such that $S_cK_A + K_A^TS_c < 0$. A delay-independent criterion utilizing the free matrix $P_2 > 0$ to describe the strong stabilizability of system (Σ) associated with controller (C') is given as follows.

Theorem 2. *Controller (C') strongly stabilizes (Σ) if there exist matrices $P_1 > 0$, $S > 0$, $P_2 = \mathrm{diag}\{P_{21}, P_{22}\} > 0$, L, N satisfying*

$$\Phi(N) \triangleq \begin{bmatrix} \begin{matrix} P_1\hat{A} + \hat{A}^TP_1 \\ +S+2M \end{matrix} & P_1\hat{B}_0 + 2\hat{C}^TL^T & 0 & P_1\hat{B}_1 \\ * & -2P_2 & 0 & 0 \\ * & * & -S+M & \hat{C}^TL^T \\ * & * & * & -P_2 \end{bmatrix} < 0 \quad (4)$$

$$\begin{bmatrix} 0 & I \end{bmatrix} (L + L^T) \begin{bmatrix} 0 \\ I \end{bmatrix} < 0 \quad (5)$$

where $M = -N^TL\hat{C} - \hat{C}^TL^TN + N^TP_2N$. Under the above conditions, the gain matrix of a stabilizing controller (C') can be parametrized as $\hat{K} = P_2^{-1}L$.

Proof: Expanding inequality (1), with A, B_0, B_1, C,

K replaced by $\hat{A}$, $\hat{B}_0$, $\hat{B}_1$, $\hat{C}$, $\hat{K}$, yields that

$$\begin{bmatrix} P_1\hat{A}+\hat{A}^TP_1+S-2\hat{C}^T\hat{K}^TP_2\hat{K}\hat{C} & P_1\hat{B}_0+2\hat{C}^T\hat{K}^TP_2 & 0 & P_1\hat{B}_1 \\ * & -2P_2 & 0 & 0 \\ * & * & -S-\hat{C}^T\hat{K}^TP_2\hat{K}\hat{C} & \hat{C}^T\hat{K}^TP_2 \\ * & * & * & -P_2 \end{bmatrix} < 0 \quad (6)$$

Here, it suffices to prove that the feasibility of (6) is equivalent to that of (4) in terms of their respective variables.

(Sufficiency) Assume (4) holds. It follows that $P_2 > 0$, and let that $\hat{K} = P_2^{-1}L$ is well defined, and $L = P_2\hat{K}$. Substituting it into (4) and noting, for any real matrix N with appropriate dimension,

$$(N-\hat{K}\hat{C})^TP_2(N-\hat{K}\hat{C}) \geq 0$$

we have (6) holds with property that all the terms $-\hat{C}^T\hat{K}^TP_2\hat{K}\hat{C}$ in the diagonal.

$$-\hat{C}^T\hat{K}^TP_2\hat{K}\hat{C} \leq N^TP_2N - N^TL\hat{C} - \hat{C}^TL^TN = M$$

(Necessity) Assume (6) holds. Then, by setting $N = \hat{K}\hat{C}$, we obtain

$$\begin{aligned} & -\hat{C}^T\hat{K}^TP_2\hat{K}\hat{C} \\ = \; & -\hat{C}^T\hat{K}^TP_2\hat{K}\hat{C} + (N-\hat{K}\hat{C})^TP_2(N-\hat{K}\hat{C}) \\ = \; & -N^TP_2\hat{K}\hat{C} - \hat{C}^T\hat{K}^TP_2N + N^TP_2N \end{aligned}$$

Substituting it into (6), and denoting $L = P_2\hat{K}$, (4) is obtained.

Due to $P_2 = \mathrm{diag}\{P_{21}, P_{22}\} > 0$, from $L = P_2\hat{K} = \begin{bmatrix} P_{21}K_D & P_{21}K_C \\ P_{22}K_B & P_{22}K_A \end{bmatrix}$, the inequality (5) is used to ensure the matrix K_A stable, meaning the stability of the controller (C'). This completes the proof. □

Remark 3. *It is worth pointing out that the parametrization of the controller matrices by our approach is fairly flexible. Indeed, the parametrization of the strongly stabilizing controller (C') mainly depends on the free parameter P_2, which can be set to be any positive definite matrix without loss of generality. Thus more synthesis problems such as simultaneous stabilization, structural controller synthesis can be treated readily in the same framework.*

5 PARAMETRIZATION DESIGN OF CONTROLLER

We are now in a position to design controller gains via an effective algorithm. When N is fixed, (4) becomes a strict LMI problem, which can be verified easily by conventional LMI solver. The remaining problem is how to select the matrix N. It can be seen from the proof of Theorem 2 that the left hand side of (4), $\Phi(N)$ achieves its minimum when $N = P_2^{-1}L\hat{C}$, which can be used to construct an iteration rule. We summarize briefly our analysis on N in the following proposition.

When $P_1 > 0$, $P_2 > 0$, S, L are fixed, the following relationship holds for any real matrix N,

$$\Phi(P_2^{-1}L\hat{C}) \leq \Phi(N)$$

It follows that the scalar ε satisfying $\Phi(N) < \varepsilon I$ achieves its global minimum only if $N = P_2^{-1}L\hat{C} = \hat{K}\hat{C}$. Therefore, the following iteration algorithm is constructed to solve the condition of Theorem 2.

Algorithm OFSS (Output Feedback Strong Stabilization):

- Step 1. Set $m = 1$, and $\varepsilon_0^* > 0$, $c > 0$ be three prescribed initial values. Select an initial matrix N_1 such that the closed-loop system ($\Sigma_{c'}$), where $\hat{K}\hat{C}$ is substituted by N_1, is stable.
- Step 2. For the fixed N_m, solve the following convex optimization problem with respect to L_m, $P_{1m} > 0$, $P_{2m} > 0$, $S_m > 0$:

$$\begin{aligned} \min \quad & \varepsilon_m \\ \text{s.t.} \quad & \Phi(N_m) < \varepsilon_m I \\ & \varepsilon_m > -c \end{aligned} \quad (7)$$

  where $\Phi(N_m)$ is the function $\Phi(N)$ defined in(4). Denote ε_m^* as the minimized value of ε_m satisfying (7). If $\varepsilon_m^* \leq 0$, the system (Σ) is stabilizable via the DOF controller (C'). The gain matrix $\hat{K}$ of (C') can be obtained as $\hat{K} = P_{2m}^{-1}L_m$, STOP, else, go to next step.
- Step 3. If $|\varepsilon_m^* - \varepsilon_{m-1}^*| \leq \delta$, a prescribed tolerance, then go to Step 4, else update N_{m+1} as

$$N_{m+1} = (P_{2m})^{-1}L_m\hat{C}$$

  and set $m = m+1$, then go to Step 2.
- Step 4. The system may not be strongly stabilizable via the controller (C'). STOP. (Or choose another initial value N_1, then run the algorithm again.)

Remark 4. *It follows from that the sequence $\{\varepsilon_m^*\}$ is monotonic decreasing with respect to m and has a lower bound c. Therefore, the stopping of the iteration is guaranteed.*

Remark 5. *The initial value of N_1 can be considered as a state feedback stabilizing controller matrix, which can be found by existing stabilization approaches. Like many other iteration algorithms, the* *sequence of iterations depends on the selection of initial values, and appropriate selection will improve the solvability. Here, we attempt to get a relaxing state feedback controller N_1 which just satisfy $\hat{A}+\hat{B}_0$ or $\hat{A}+(\hat{B}_0+\hat{B}_1)N$ stable, as the initial value N_1 for system $(\Sigma_{c'})$. There has much conservatism since it is only a delay-independent approximative solution. If failed, Zhang* et.al *in (Zhang et al., 2005) gave a further method to obtain a new state- and input-delay-dependent state feedback controller to ensure the stability of the closed-loop system. The numerical examples in the following section will illustrate that Algorithm OFSS is relaxed to rely on the initial matrix N_1.*

Remark 6. *The approach proposed in the paper is in fact not a conservative one. The direct iterative procedure (D-K iteration) may generate a feasible solution. However, the success rate may be low. As is well known, even for LTI systems without delay, the DOF controller design is a non-convex problem, and is likely to be NP-hard. To cope with a nonconvex problem via convex approach, there are generally two recipes. One is the so called* relaxation*, and the other is the* local optimization*. The relaxation approach is easy to implement, but may introduce conservatism in some cases. LMI approaches can be regarded as one kind of relaxation. For the local optimization, one wants to seek a point that is only locally optimal, which means that it minimizes the objective function among feasible points that are near it. Therefore, the initial values are critical to such optimization problems, and good initial values may generate a globally optimal solution. Most exact approaches to DOF synthesis, including CCL, ILMI, alternating projection, D-K iteration, nonsmooth optimization for instance, involve local optimization. However, few approaches have systematic procedures to even determine an initial value. Obviously, finding an initial stabilizing state-feedback gain is more desirable than guessing a stabilizing DOF one. In this sense, the selection of initial values in this paper is more desirable than a direct iterative procedure (D-K iteration). In fact, as we have shown in the proposition, a globally optimal solution of conditions (4) and (5) is obtained only if N is a stabilizing state-feedback gain, which means that our iteration begins with a set of necessary N for the matrix inequalities conditions (4) and (5) to be feasible rather than random guesses.*

6 NUMERICAL EXAMPLE

This section presents a numerical example to demonstrate the validity of the proposed method in this paper to design a DOF strong stabilization controller. Consider a linear input-delayed system (Σ) with the parameters as follows:

$$A=\begin{bmatrix}0.9926 & 0.1443\\ 0 & -0.3698\end{bmatrix},\ B_0=\begin{bmatrix}-1\\ 0\end{bmatrix},\ B_1=\begin{bmatrix}0\\ 1\end{bmatrix}$$

The input delay d is constant and it has a particular form with $C=I$. Now we apply the proposed approach to find DOF controllers to stabilize this system. An initial matrix N_1 is chosen for DOF controller (C') which is obtained directly by solving state stabilization conditions for a system pair $(\hat{A},\hat{B}_0)$ defined in $(\Sigma_{c'})$, $\hat{A}X+\hat{B}_0Y+(\hat{A}X+\hat{B}_0Y)^T<0$ and $X>0$, with setting $N_1=YX^{-1}$. Two cases are considered as follows with $\varepsilon_0^*=10$:

- a. Full order DOF controller

$$N_1=\begin{bmatrix}0.0013 & 0.0066 & -0.4916 & 0.0038\\ 0.0091 & 0.0068 & 0.0052 & -0.4918\\ 1.4990 & 0.1517 & 0.0015 & 0.0017\end{bmatrix}$$

is chosen as the initial matrix in Algorithm OFSS. After 1 iteration, a desired strong DOF controller (C') is obtained as

$$\begin{cases}\dot{\vartheta}(t)=\begin{bmatrix}-0.8119 & 0.0034\\ 0.0046 & -0.8125\end{bmatrix}\vartheta(t)\\ \quad+\begin{bmatrix}-0.0068 & 0.0112\\ 0.0089 & 0.0112\end{bmatrix}y(t)\\ u(t)=\begin{bmatrix}-0.0021 & 0.0003\end{bmatrix}\vartheta(t)\\ \quad+\begin{bmatrix}1.4883 & 0.1834\end{bmatrix}y(t)\end{cases}$$

The eigenvalues of the controller matrix K_A are -0.8082 and -0.8162.

- b. Lower order DOF controller

$N_1=[0.0057\ 0.0086\ -0.4982; 1.5019\ 0.1460\ 0.0024]$ is chosen as initial matrix, and a desired strong DOF controller (C') is obtained after 1 iteration,

$$\begin{cases}\dot{\vartheta}(t)=-0.8030\vartheta(t)+\begin{bmatrix}0.0012 & 0.0145\end{bmatrix}y(t)\\ u(t)=-0.0012\vartheta(t)+\begin{bmatrix}1.4763 & 0.1784\end{bmatrix}y(t)\end{cases}$$

Furthermore, consider the same model with a different output matrix $C=[0.9556\ 0.1132]$. With the same method to calculate initial matrix N_1 as the above model, two kinds of DOF stabilizing controllers are given by applying Algorithm OFSS again with 1 iteration.

- a. Full order DOF controller

$$N_1=\begin{bmatrix}0.0019 & 0.0010 & -0.4929 & 0.0086\\ 0.0010 & 0.0020 & 0.0089 & -0.4932\\ 1.4995 & 0.1504 & 0.0060 & 0.0078\end{bmatrix}$$

$$\begin{cases} \dot{\vartheta}(t) = \begin{bmatrix} -0.7880 & 0.0416 \\ 0.0389 & -0.7743 \end{bmatrix} \vartheta(t) \\ \qquad + \begin{bmatrix} 0.0257 \\ 0.0379 \end{bmatrix} y(t) \\ u(t) = \begin{bmatrix} 0.0010 & 0.0024 \end{bmatrix} \vartheta(t) + 1.5792y(t) \end{cases}$$

The eigenvalues of the controller matrix K_A are -0.8220 and -0.7403.

- b. Lower order DOF controller

$$N_1 = \begin{bmatrix} 0.0054 & 0.0020 & -0.4944 \\ 1.4926 & 0.1487 & 0.0039 \end{bmatrix}$$

$$\begin{cases} \dot{\vartheta}(t) = -0.7926\vartheta(t) + 0.0289y(t) \\ u(t) = 0.0054\vartheta(t) + 1.5683y(t) \end{cases}$$

It is known from the above computational cases that the algorithm converges to the feasible solutions quickly for the arbitrarily chosen of initial matrix N_1 very much while designing any order strong DOF controllers.

7 CONCLUSIONS

This paper has developed the strong output feedback control problem for an input-delayed system from a new perspective. Input-delay-independent stabilization criteria for output feedback controllers are derived from a new equivalent characterization on stabilizability of the system in terms of matrix inequalities by introducing a slack positive definite matrix, and an iterative algorithm is developed to solve these conditions. Although common to other approaches, the proposed approach is not guaranteed to find a solution even it exists, it is quite effective since there is no need to introduce additional constraints to linearize the product term of Lyapunov matrix and controller gain when parametrized.

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REFERENCES

Campos-Delgado, D. U. and Zhou, K. (2001). H_∞ strong stabilization. *IEEE Trans. Autom. Control*, 46(12):1968–1972.

Cao, Y. and Lam, J. (2000). On simultaneous H_∞ control and strong H_∞ stabilizations. *Automatica*, 36(6):859–865.

Chen, W. and Zheng, W. (2006). On improved robust stabilization of uncertain systems with unknown input delay. *Automatica*, 42:1067–1072.

Feintuch, A. (2008). On strong stabilization for linear continuous time time-varying systems. *Systems & Control Letters*, 57:691–695.

Gu, K., Kharitonov, V., and Chen, J. (2003). *Stability of Time-Delay Systems*. Birkhauser, Boston, MA.

Respondek, J. S. (2008). Approximate controllability of the n^{th} order infinite dimensional systems with controls delayed by the control devices. *Int. J. Syst. Sci.*, 39(8):765–782.

Tadmor, G. (2000). The standard H_∞ problem in systems with a single input delay. *IEEE Trans. Autom. Control*, 45(3):382–397.

Vidyasagar, M. (1985). *Control System Synthesis: A Factorization Approach*. MIT Press, Cambridge, MA.

Yoon, M. G. and Kimura, H. (2006). A topological result on strong stabilization problem. *IEEE Trans. Autom. Control*, 51(4):657–661.

Youla, D. C., Bongiomo, J. J., and Lu, N. C. (1974). Single-loop feedback stabilization of lincar multivariable dynamical plants. *Automatica*, 10(2):159–173.

Zeren, M. (1997). On stable H_∞ controller design. In *Proceedings of American Control Conference*, pages 1302–1306, Albuquerque, NM.

Zhang, X., Wu, M., She, J., and He, Y. (2005). Delay-dependent stabilization of linear systems with time-varying state and input delays. *Automatica*, 41(8):1405–1412.