# MULTIMETRIC NETWORK TOMOGRAPHY

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Keywords: Network tomography, Link delays, Multimetric, NTF, NNMF.

Abstract: We introduce a novel concept of multiple metric network tomography in this paper. The conventional network tomography observes a single parameter directly and infers another parameter indirectly from the the directly measured parameter. We consider observing two parameters (packet loss rate (PLR) and path delays) directly and use both of these parameters to infer a single parameter indirectly. We applied a variation of NTF1 model of non negative tensor factorization (NTF) for this purpose and estimated link delay. Simulation results should show a better correlation between the estimated and measured link delays when duplex of metrics is used as compared to using only the path level link delays for estimating the link delays on a test bed.

#### **1** INTRODUCTION

Network tomography presents a good means to measure the statistics of interest that may not be measured directly. Network tomography measures a parameter actively or passively (that is not desired for network management), and the desired parameter is indirectly measured by applying statistical techniques over an inverse model. Such parameters are essential for network management.

Vardi (Vardi, 1996) was the first one to introduce the term of network tomography for such kind of indirect inference of interested statistics. The research on network tomography has always concentrated on estimating a single parameter indirectly from another parameter that is directly measured. Various categories of network tomography have been mentioned in the literature in this context (Castro et al., 2004) (Coates and Nowak, 2001).

The simplest model of network tomography, that represents the above two examples is shown by the following equation,

$$Y = AX, \tag{1}$$

linking the measured parameters matrix (Y) with the matrix of unknown parameters (X) with dependence on the routing matrix (A) of the network. If Y has I rows and X has J rows, then the size of the routing matrix (A) is  $I \times J$ . The rows of A (A<sub>i</sub>) correspond to

paths from the sender to the receivers and the columns  $(A_j)$  correspond to individual links in those paths. An element  $(A_{ij})$  of the routing matrix is 1 if the link j is included in the path i and 0 otherwise.

In contrast to the conventional tomography model as discussed above, we propose the idea of direct measurements of multiple metrics to recover indirectly a single parameter with expectation of getting a better estimate as compared to using a single directly measured parameter to estimate a parameter indirectly. The new model is represented by the equation below, where  $Y_1$  and  $Y_2$  are directly observed in order to estimate X indirectly by solving the following inverse equation.

$$Y_1 Y_2 = AX \tag{2}$$

For example, instead of recovering link delays from merely end to end path delays, we can estimated link delays from a combination of path delays ( $Y_1$ ) and PLR ( $Y_2$ ). The idea behind this innovation is that a better input in terms of two interdependent metrics should produce better estimation than using only one parameter such as path level delay. This correlation of two network parameters has been discussed in the literature. For example, the authors of (Moon et al., 1998) report on the correlation between delay and loss observed by a continuous-media traffic source. This study is to determine the extent to which one performance measure could be used as a predictor of the

H. Raza M., Robertson B., J. Phillips W. and Ilow J. (2010).

- MULTIMETRIC NETWORK TOMOGRAPHY.
- In Proceedings of the International Conference on Data Communication Networking and Optical Communication Systems, pages 80-84 DOI: 10.5222/000292000800084

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future behavior of the other (for example, whether observed increasing delay is a good predictor of future loss) so that an adaptive continuous media application might take anticipatory action based on observed performance.

For this purpose, we have applied one of the variations of NTF1 model of nonnegative tensor factorization (NTF) (Cichocki et al., 2009).

The rest of the paper is organized as follows. Section 2 reviews the related work. Section 3 discusses NTF. Section 4 details simulation arrangements. Section 5 presents and discusses results. Section 6 concludes the paper.

#### 2 RELATED WORK

This section investigates related work in the domains of multiple metric network tomography concept and the interdependence of link delays and PLR.

## 2.1 Multimetric versus Additive Metrics

Up to best of our knowledge, there has never been an implicit consideration of directly measured multiple metrics for indirect estimate of a network metric. Though, we found an evidence (Bhamidi et al., 2006) of considering multiple metrics in the form of additive metrics. A framework was proposed for analyzing topology using ideas and tools from phylogenetic inference in evolutionary biology. The phylogenetic inference problem determines the evolutionary relationship among a set of species. The framework is built upon additive metrics. Under an additive metric the path metric (path length) is expressed as the summation of the link metrics (link lengths) along the path. The basic idea is to use (estimated) distances between the terminal nodes (end hosts) to infer the routing tree topology and link metrics. Based on the framework some inference algorithms have been presented as an alternative to network tomography.

They (Bhamidi et al., 2006) consider that G = (V, E) denotes the topology of the network, which is a directed graph with node set V (end hosts, internal switches and routers, etc.) and link set E (communication links that join the nodes). For any nodes *i* and *j* in the network, if the underlying routing algorithm returns a sequence of links that connect *j* to *i*, they say *j* is reachable from *i*. They call this sequence of links a path from *i* to *j*, denoted by P(i, j).

As per their terminology, d(e) can be viewed as the length of link e, and d(i, j) can be viewed as the distance between nodes *i* and *j*. Basically, an additive metric associates each link on the tree with a finite positive link length, and the distance between two nodes on the tree is the summation of the link lengths along the path that connects the two nodes. Suppose T(s, D) = (V, E) is a routing tree with source node s and destination nodes D. Let

$$d(E) = d(e) : e\varepsilon E \tag{3}$$

denotes the link lengths of T(s, D) under additive metric d. Remember  $U = s \bigcup D$  is the set of terminal nodes on the tree. Let

$$d(U^2) = d(i, j) : i, j \in U$$
(4)

denote the distances between the terminal nodes.

The above review makes it clear that considering additive metric is different from multiple metric based network tomography. Actually this phylogenetic based technique is claimed to be an alternative of network tomography (Bhamidi et al., 2006). Therefore, our idea of considering multiple metric stays as a novel way of improving the conventional monometric network tomography.

#### 2.2 Correlation of Link Delays and PLR

The authors of (Moon et al., 1998) examine the correlation between packet delay and packet loss experienced by a continuous media traffic source. Their goal is to study the extent to which one performance measure can be used to predict the future behavior of the other (for example, whether observed increasing delay is a good predictor of future loss) so that an adaptive continuous media application might take anticipatory action based on observed performance. They provide a quantitative study of the extent to which such correlation exists. There are two examples in this regard.

When the buffer reaches its capacity, packet losses begin to occur. The receiver of the continuous-media application thus sees increased delay, and eventually losses.

When packets from a continuous-media application arrive at a buffer that is already full, they are dropped. As other sources (for example, TCP connections) detect congestion and decrease their transmission rate, the queue length at the buffer will decrease, and packets from the continuous-media application will start to be queued, rather than dropped. The receiver sees losses followed by high, but possibly decreasing, packet delays.

They introduce a lag, loss-conditioned average delay, in calculating the average delay conditioned on loss. Specifically, the average packet delay, conditioned on a loss occurring at a time lag j packets in the past, is the average delay of all packets in the trace that have a loss j packets before them in the trace. That is,

$$E[d_i \mid l_{i-j} = 1] = \sum_{k \in P} d_k / \mid P \mid,$$
 (5)

where  $P = k : l_{k-j} = 1$  and  $l_k = 0$ .

If the loss-conditioned average delay at a positive lag of j is higher than the unconditional average delay (that is, the delay averaged over all received packets), then the packets that arrive j packets after a loss have a higher average delay than the unconditional average delay. That is, a loss occurring j packets in the past can be taken as a precursor to a higher delay later.

This discussion shows that delay and PLR are interdependent and correlated based on lossconditioned average delay. This evidence motivated us to consider multimetric network tomography. We have applied NTF tool to carry out the multiple metric network tomography and NTF is briefly described in the next section.

# 3 NONNEGATIVE TENSOR FACTORIZATION (NTF)

We researched for a mathematical technique that could deal with multiple metrics and is capable of matrix factorization. Matrix factorization is an important area in signal processing and linear algebra, with applications in many other areas. Blind source separation (BSS) and related methods, for example, independent component analysis (ICA), employ a wide range of unsupervised learning algorithms and have found important applications from engineering to neuroscience (Cichocki et al., 2009).

Tensors are generalizations of vectors and matrixes, for example, a third-order tensor (or three-way array) has three modes (or indices or dimensions).

A tensor is a multi-way array or multidimensional matrix. The order of a tensor is the number of dimensions, also known as ways or modes. Tensor can be formally defined as following. Let  $I_1$ ,  $I_2, \ldots, I_N \in N$  denote index upper bounds. A tensor  $\underline{Y} \in \mathbb{R}^{I_1, I_2, \ldots, I_N}$  of order N is an N-way array where elements  $y_{i_1, i_2, \ldots, i_N}$  are indexed by  $i_n \in 1, 2, \ldots, I_n$  for  $1 \le n \le N$ .

Unfolding or matricization or flattening is a process of reordering the elements of an N-th order tensor into a matrix. Two of the most commonly used decompositions are the Tucker decomposition and PARAFAC, which are often considered as higher order generalizations of the matrix singular value decomposition (SVD) or principal component analysis (PCA). A model which imposes nonnegativity on factor matrices is called the NTF (Nonnegative Tensor Factorization) or Nonnegative PARAFAC (Cichocki et al., 2009).

Figure 1 illustrates one of the three ways of the basic 3D NTF1 model, which is an extension of the NTF model. As per NTF1 model, given a three-way (third-order) tensor formed by a set of matrices  $Y_q \in \mathbb{R}_+^{I \times T_q}$  (q = 1, 2, ..., Q), formulates a set of nonnegative and sparse matrices A  $\in \mathbb{R}_+^{I \times J}$ , C  $\in \mathbb{R}_+^{Q \times J}$ , and A  $X_q \in \mathbb{R}_+^{I}$  for q = 1, 2, ..., Q with reduced dimensions ( $J << I < T_q$ ).

Global matrix representation using row-wise unfolding of the three-way array is shown in Figure 1 and is expressed (error free model) as  $Y_q = AD_qX_q$ . In this case the sub-matrices are defined as  $X_q \triangleq D_qX_q$ . Thus, only the mixing matrix A and the set of scaled source matrices  $X_q$  need to be found whereas due to scaling ambiguity the matrix C does not need to be calculated explicitly (Cichocki et al., 2009).

There are several possible approaches to find or identify extended NTF1 model such as global strategy, or local strategy, or a combination of both. A global strategy based on alternating minimization of cost function is shown in the following equation.

$$D_F(\overline{Y} \| A\overline{X}) = \frac{1}{2} \| \overline{Y} - A\overline{X} \|_F^2$$
(6)

A local strategy based on alternating minimization of cost function is shown in the following equation.

$$D_F(Y_q || AX_q) = \frac{1}{2} ||Y_q - AX_q||_F^2 \quad (q = 1, 2, ., ., ., ., ., Q)$$
(7)



Figure 1: Decomposition into two matrices using row-wise unfolding (Cichocki et al., 2009).

We plan to employ local strategy based on alternating minimization of cost function in the simulations. For solving the model,  $Y_1Y_2 = AX$ , NTF will be applied to recover link delays (X) from a duplex of metrics (path delays (Y<sub>1</sub>) and PLR (Y<sub>2</sub>)). Two matrices (Y<sub>1</sub> and Y<sub>2</sub>) will be input to NTF and a matrix, X, will be determined as per row wise decomposition shown in Figure 1. The parameters (Y<sub>1</sub>, Y<sub>2</sub>, and X (for bench marking)) of the multimetric network tomography model will be determine from a laboratory test bed. The setup of the test bed and simulations is discussed in the next section.

# 4 SIMULATION ARRANGEMENT FOR MULTIMETRIC NETWORK TOMOGRAPHY

We collected data from a test with two options. Firstly, we determined link delay from path delays by using non negative matrix factorization (NNMF) by using Cisco Service Level Agreement (CSLA). Secondly, we determined link level delay from a combination of path delay and PLR by using CSLA and Real Time Transport Protocol (RTP) (RFC 1889) by using row wise unfolding of NTF1 model. This section discuses the test bed and simulation arrangement, delay estimation by using path delay input to NNMF, and link delay estimation from the combined data of path delay by CSLA and RTP.

We set up a test bed in the Advanced Internetworking Laboratory (AIL) at Dalhousie University that consists of six 38 series Cisco routers, Agilent Router Tester (N2X), and a Multi Router Traffic Grapher (MRTG) capable workstation. OSPF routing has been implemented on routers and N2X. The test bed is of smaller size and has limited number of links, because we have to collect the actual values of the link delays for bench marking the accuracy of estimated link delays. We intent to prove that the estimated link delays are close to the actual link delays. In contrast to this test bed, the practical networks are larger in scale, but scalability is not an issue as NNMF and NTF can handle larger sizes of matrices (Cichocki et al., 2009).

The Echopath option of the CSLA was implemented to send four probes and collect the cumulative round trip time (RTT) from source to each hop. All probes were grouped together. All the probes in the group start at the same time. The group of probes was repeated 100 times with a time difference of 10 sec between two consecutive repetitions. The MRTG enabled workstation verified the end to end RTT. The selected links were stressed by two sources: extended ping on selected links and traffic injected from the N2X.

Figure 2 shows the test bed with the four probes and two of the links (1 and 6) were stressed with an extended ping of 200 Bytes. The other source of disturbance was the traffic from the Agilent router tester (N2X). The module 1 of N2X was generating a variable packet size from 1000 Bytes to 1500 Bytes. The size of the probes in CSLA was 10 Bytes for this scenario. In this case also, the condition of the network remains unchanged during the CSLA operation.



Figure 2: Testbed Setup with a mixture of extended pings and N2X traffic.

# 4.1 Estimation of Link Delay from Path Delays

In this first part of the simulations, we estimate link delays from path level delays and then determine the correlation between the estimated and measured link delays. The data obtained from the CSLA is in the form of accumulative hop-wise round trip time, the following steps are followed to process the data for obtaining two matrices; a matrix of end to end delays and a matrix of link level delays:

- A parsing software, written in java, extracts link delays and end to end (path) delays in the form of two matrices. From the accumulative round trip time from source to each hop, hop to hop delays are calculated to form the link delay matrix. From the accumulative round trip time from the source to the destination, end to end delay matrix is determined.
- 2. Path level delays (V) are input to NNMF. The Matlab tool NMFpack (Hoyer, 2004) has been used for NNMF factorization. The NMFpck Matlab package implements and tests various versions of NNMF with the feature of sparsity. The sparsity for the routing matrix is kept fixed at 0.5 in all the tests and the sparsity of the link delays varies from 0.1 to 0.9.
- 3. The coefficient of correlation between the estimated link delay (H) and actual link delay (X) is determined by using a modified component of EEGLAB. The EEGLAB is an interactive Matlab toolbox for processing continuous and eventrelated EEG, MEG and other electrophysiological data.

For finding the correlation coefficient, matching rows in two matrices (H and X) are found and their correlation is determined. As a result a column vector of correlation coefficients between the best-correlating rows of matrices H and X is obtained along with other by-products.

Figure 3 shows a correlation between the estimated and true link delays.



Figure 3: Correlation between H and X with a mixture of extended pings and N2X traffic.

# 4.2 Estimation of Link Delay from a Combination of Path Delays and RTP Data

In the second part of the simulations, we intend to estimate link delays from a combination of path level delays and PLR data. The correlation between estimated and measured link delays will be measured again and we expect this correlation to be better than the correlation shown in Figure 3. In the same test bed, we will inject two types of traffic; CSLA and RTP. We are in a process of getting a combination of path level delay and PLR. This data will be used to recover link level delays by inputting this data to row wise unfolding of NTF1 model with local strategy based on alternating minimization of the cost function.

## **5** CONCLUSIONS

We introduced a novel concept of multiple metric network tomography in this paper. We estimated link delays from path delay for the mono-metric network tomography. The correlation between the estimated and measured link delays was close to 1 depending on the sparsity. We are in a process to estimate link delays from two input parameters; path delays and RTP data. We expect the correlation between the estimated link delays (by using multimetric) and measured link delay to be better than using a mono-metric (path delays).

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