

GEOMETRICAL CONSTRAINTS FOR LIGAND POSITIONING

Virginio Cantoni, Alessandro Gaggia, Riccardo Gatti and Luca Lombardi
University of Pavia, dept. of Computer Engineering and Systems Science, Via Ferrata 1, Pavia, Italy

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Abstract: The purpose of the activity here described is the morphological and subsequently the geometrical and topological analysis of the active sites in protein surfaces for protein-ligand docking. The approach follows a sequence of three steps: i) the solvent-excluded-surface is analyzed and segmented in a number of pockets and tunnels; ii) the candidate binding sites are detected through a structural matching of pockets and ligand, both represented through a suitable Extended Gaussian Image modality; iii) the loci of compatible positions of the ligand is identified through mathematical morphology. This representation of ligand and candidate binding pockets, the comparison of the morphological similarity and the identification of potential ligand docking are the novelties of this proposal.

1 INTRODUCTION

Much work has been done on the identification, the localization, and the analysis of the binding sites of proteins. The aim, for docking applications, is the search of sub-regions that are complementary (that is with concave and convex segments that match each others) between different molecules. When we have a large molecule (receptor) and a small molecule (ligand), docking takes place in a protein cavity; instead the protein-protein case is usually different, in fact the docking site is, in general, more planar than a cavity and the interface has different characteristics.

In this connection the first sub-problem to be solved, in protein-ligand interfaces, is to develop the representations and the data structures suitable to support the computational methods which consent a quantitative evaluation of the protein-protein and in particular the protein-ligand matching on the basis mainly of their 3D structure. Until now this problem has been pursued by 'ad hoc' descriptors of patterns like spin image (Shulman-Peleg, 2004), (Bock, 2007), context shape (Frome, 2004) and harmonic shape (Glaser, 2006). Some of these approaches are point-based and in general they look to us cumbersome with difficulties for management and processing.

In this paper a new method for representing the molecule is proposed, and the correspondent data

structure based on a first order statistic of the orientation is introduced. After the segmentation of the protein solvent excluded surface (SES) (Cantoni, 2010), the interface regions, which potentially can be active sites, are represented by a kind of Extended Gaussian Image (EGI) (Horn, 1984). The EGI represents the histogram of the orientations placed on the unitarian sphere and constitutes a compact and effective representation of a 3D object as a protein or one of its parts.

This paper is organized as follows: section two shows a survey of the EGI representations; in section three, the construction of CE-EGI is introduced; in section four is introduced the practical implementation and the data structure; in section five is described the matching problem and the possible discriminant functions based on different distance definition; in section six the detection of candidate positions is discussed; and finally in section seven the results for the new solution proposed for the identification of binding sites, together with a practical case are presented. The final section, provides a few concluding remarks and briefly describes our planned activity in the near future.

2 RELATED WORKS

The EGI was introduced for applications of photo

-metry by B.K.P. Horn (Horn, 1984) in the years '80 and has been extended by K. Ikeuci (the Complex-EGI) (Kang, 1993) in the years '90 to overcome the ambiguity risen in the representations by the convex parts. Later other improvements have been introduced in sequence: the More Extended Gaussian Image (MEGI) in 1994, the Multi-Shell Extended Gaussian Image (MSEGI) and the Adaptive Volumetric Extended Gaussian Image (A-VEGI) in 2007, and finally the Enriched Complex Extended Gaussian Image (EC-EGI) in 2010.

They have not up-to-now been applied on proteomics; starting from these, we propose a representation suitable for describing the matching between the ligand (here the protuberance) and the protein (the cavity under analysis).

Extended Gaussian Image (EGI). The EGI of a 3D object or shape is an orientation histogram that records the distribution of surface area with respect to surface orientation. Each surface patch is mapped to a point on the unit Gaussian sphere according to its surface normal. The weight for each surface normal (represented by a point on the Gaussian sphere) is the total sum of area of all the surface patches that are of that surface normal. Being a distribution related to surface orientation, EGI is in principle invariant to translation.

Complex EGI (Kang, 1991). CEGI encodes each surface patch's signed perpendicular distance from the reference coordinate center.

It uses a complex number, as opposed to a scalar in EGI, as the weight for the corresponding point on the Gaussian sphere. The magnitude and phase of the complex number are the area and signed perpendicular distance of the patch (from the origin of the reference coordinate frame), respectively. The use of complex numbers allows the area and position information to be decoupled. Furthermore, the translation component of the pose can be determined more readily.

More Extended Gaussian Image (MEGI) (Matsuo, 1994). The MEGI model consists of a set of position vectors X_i for surfaces originating from an object center and their normal vectors p_i . Each length of a normal vector also corresponds with surface area, as in the EGI. Also this model is shift-invariant since it is expressed by an object-oriented coordinate. The MEGI model is an extended EGI modeling which is able to represent concave objects.

Multi-Shell Extended Gaussian Image (MSEGI) (Wang, 2007) or Volumetric Extended Gaussian Image (VEGI) (Zhang, 2006). The VEGI captures the volumetric distribution of a triangulated 3D model by connecting the vertices of each triangle

with the geometry centroid of the object to form a tetrahedron as the elementary volume unit. Then the 3D model is decomposed into a number of N_s concentric spheres. Each sphere surface is subdivided in cells, each one identified through their polar and longitudinal angles (θ_i, φ_i). The quantized volume of each tetrahedron and its associated direction (the outward surface normal) are mapped to the corresponding cell of the concentric sphere with radius ρ , obtaining N_s spherical distribution functions $\eta(\rho, \theta, \varphi)$. These functions are expanded into spherical harmonics to achieve a features vector. The VEGI and this representation, without canonical alignment, maintains the property of translation, scaling, rotation invariance and facilitate multiple scale approximation. An improvement to fix the irregular sampling of the polar and longitudinal coordinate system (in the poles there is a higher sampling density than in the equator) has been proposed with the *Adaptive Volumetric Extended Gaussian Image (A-VEGI) (Wang, 2007).*

Enriched CEGI (Hu, 2010). The EC-EGI encodes each surface patch's signed 3D position. It uses three complex numbers, as the weight for the corresponding point on the Gaussian sphere. The resultant weight at the point is then the sum of the contributions of all surface patches that are of the corresponding surface normal referred to each one of the coordinate planes. The magnitude part of the EC-EGI representation is translation-invariant. This is an important property that allows the rotation part of pose, in the pose estimation application, to be determined separately from the translation. The EC-EGI can be viewed as three independent Gaussian spheres, each encoding the 3D position information along the x-, y- and z-axes, respectively.

In this paper we propose the adoption of this last EC-EGI for ligand and cavity to evaluate quantitatively the matching between candidate active sites and ligand. ρ

3 CONSTRUCTION OF CE-EGI

A given 3D molecule, modeled through its Solvent Excluded Surface in a triangular mesh, is described by the set of triangles:

$$T = \{T_1, \dots, T_m\}, T_l \subset R^3 \quad (1)$$

where each T_i consists of a set of three vertices:

$$T_l = \{P_{A,l}, P_{B,l}, P_{C,l}\} \quad (2)$$

Center, normal and area of each triangle T_l , namely g_l , \vec{d}_l and A_l , respectively, can be computed by:

$$g_l = (P_{A,l} + P_{B,l} + P_{C,l}) / 3 \quad (3)$$

$$\vec{d}_l = (P_{C,l} - P_{A,l}) \times (P_{B,l} - P_{C,l}) \quad (4)$$

$$A_l = |(P_{C,l} - P_{A,l}) \times (P_{B,l} - P_{C,l})| / 2 \quad (5)$$

while the total area of the mesh A is given by cumulating the area of each single triangle:

$$A = \sum_{l=1}^m A_l \quad (6)$$

where the Gaussian sphere is partitioned into a number of cells m .

Then all the triangle T of the target molecule are mapped onto the corresponding cells on the basis of the orientation \vec{d} .

In the approach described in this paper it has been adopted the EC-EGI solution. In this framework, in the Gaussian sphere are mapped the surface patches according to their orientation with a weight composed of three complex numbers:

$$\begin{aligned} W_{x,\vec{d}} &= \sum_{l=1}^{N_{\vec{d}}} A_{l,\vec{d}} e^{ig_{l,x}}; \\ W_{y,\vec{d}} &= \sum_{l=1}^{N_{\vec{d}}} A_{l,\vec{d}} e^{ig_{l,y}}; \\ W_{z,\vec{d}} &= \sum_{l=1}^{N_{\vec{d}}} A_{l,\vec{d}} e^{ig_{l,z}}; \end{aligned} \quad (7)$$

where \vec{d} is the direction associated with a point on the Gaussian sphere, $N_{\vec{d}}$ the total number of surface patches with normal \vec{d} , $A_{l,\vec{d}}$ the area of the l th surface patch with normal \vec{d} , and $[g_{l,x}, g_{l,y}, g_{l,z}]$ are the 3D coordinates of the mass center of the l th surface patch. Note that the EC-EGI representation can be seen as three CEGI representations, one for each one of the main axis. Moreover, if the object is convex the mass center of the three W_x , W_y , and W_z distributions on the Gaussian sphere coincides with the center of the sphere. In fact, this is true also for the EGI (and for the CEGI), it is:

$$\begin{aligned} \sum_{\vec{d}} |W_{x,\vec{d}}| \vec{d} &= \sum_{\vec{d}} |W_{y,\vec{d}}| \vec{d} = \\ &= \sum_{\vec{d}} |W_{z,\vec{d}}| \vec{d} = \\ &= \sum_{\vec{d}} A_{\vec{d}} \vec{d} = 0 \end{aligned} \quad (8)$$

Since for convex object $|W_{x,\vec{d}}| = |W_{y,\vec{d}}| = |W_{z,\vec{d}}| = A_{\vec{d}}$, being $A_{\vec{d}}$ the area of the surface patch with normal \vec{d} .

It is also easy to show that $[|W_{x,\vec{d}}|, |W_{y,\vec{d}}|, |W_{z,\vec{d}}|]$ is translation invariant (i.e. the magnitude of the EC-EGI representation is translation invariant).

The Extended Gaussian Image does not encode any position information; the Complex EGI encodes the signed distance of each surface patch, and finally the EC-EGI encodes the 3D position. With the richer information included, the EC-EGI could remove some of the ambiguities that CEGI has.

4 IMPLEMENTATION

A tessellated sphere with uniform, and isotropic subdivision is needed. These properties are satisfied by the projection of regular polyhedron onto the sphere. Adopting the highest order regular polyhedron, the icosahedron with twenty triangular cells as a basis (that provides a too coarse sampling of the orientations), and proceeding further with precision, by dividing iteratively the triangular cells into four smaller triangles according to the well known geodesic dome constructions, the required level of resolution can be achieved: being n the number of iterative subdivision steps, the cells number is $m=10 \cdot 2^{2n+1}$, and the area (solid angle) of the single cells is $\pi/10 \cdot 2^{2n-1}$ respectively. The corresponding data structure is consequently a hierarchical one (in which each cell of one level contains, other than the specific orientation, the four pointers to cells of the subsequent level) and hierarchical is the searching strategy of the orientation histogram values.

5 THE MATCHING PROBLEM

Given two candidates dual parts of proteins (i.e. a cavity and a ligand) the aim is to find if they are geometrically compatible, that is about findings the rigid motion that could bring the protrusion into the cavity. On this purpose we apply a preliminary coarse constraint given by the mass of the EC-EGI of the cavity and the ligand: $A_{cav} > A_{lig}$, this constraint is not theoretically supported, but in practice works in all the considered cases. Satisfied this constraint, as a matching index we experimented four parameters:

- ❖ the Minkowski distance:

$$M = \left\{ \sum_{l=1}^m |A_{l,cav} - A_{l,lig}|^p \right\}^{1/p} \quad (9)$$

in for $p=1$ and $p=2$ we obtain the Manhattan and the Euclidean distances respectively;

- ❖ the Bray Curtis distance:

$$B = \frac{\sum_{l=1}^m |A_{l,cav} - A_{l,lig}|}{\sum_{l=1}^m |A_{l,cav} + A_{l,lig}|} \quad (10)$$

obviously $0 \leq B \leq 1$;

- ❖ the Hausdorff distance:

$$H = \max \left(\begin{aligned} &||\max_{v_l} A_{l,cav} - \min_{v_l} A_{l,lig}||, \\ &||\max_{v_l} A_{l,lig} - \min_{v_l} A_{l,cav}|| \end{aligned} \right) \quad (11)$$

- ❖ the EC-EGI distance:

for a given threshold θ , being n the number of triangles for which:

$$\left[\begin{aligned} &\left(\frac{|A_{l,cav} - A_{l,lig}|}{\max(A_{l,cav}, A_{l,lig})} \geq \theta \right) \\ &\bigcup_{l=1}^m (\max(A_{l,cav}, A_{l,lig}) = 0) \end{aligned} \right] \quad (12)$$

the distance is given by $E = n/m$, i.e. the percentage of the triangles satisfying the threshold criteria.

For each couple candidate protein-ligand these parameters are applied to detect the best k candidates active sites.

6 LIGAND POSITIONING

The candidate positions of a ligand into a cavity is determined on the basis of two steps:

- alignment of ligand and cavity;
- detection of the set $V \equiv \{v\}$ for which $L_v \subseteq C$;

The set V can be easily obtained through the erosion operator \blacksquare of mathematical morphology:

$$V = C \blacksquare L \quad (13)$$

being L the structural element of the erosion.

7 RESULTS

A first experimentation of the proposed technique has been applied to a number of proteins (e.g PDB

IDs 1KIM, 1TNL, 2OH4, 3EHY, 3L62). The analysis has been done with a resolution of 0.25 \AA , which entails a van der Waals radius of more than five voxels to the smallest represented atoms. The SES is obtained from the van der Waals surface, after the execution of a closure operator, using a sphere with radius of 1.4 \AA , approximately 6 voxels (corresponding to the conventional size of a water molecule), as structural element.

For what concerns the pockets detection the three parameters quoted in (Cantoni, 2010) have been set as follows: the minimum travel depth of the local tops TD_{LT} within a range of $[25,50]$ voxels; the nearness of others, more significant, local tops to $\tau_1=200$ voxels and the relative values of the local-top travel-distance to $\tau_2=2000$ voxels. Moreover, the volume of the water molecule has been set to XX voxels.

In particular we will show here the results for protein with PDB ID 3EHY. In this case the quoted parameter TDLT has been set to 47 voxels. particular structure.

In figure 1 it is shown the final result of the segmentation process of the protein 3EHY for the detection of pockets and tunnels. Note that among the 25 pockets that have been detected, we have considered only the five most extensive.

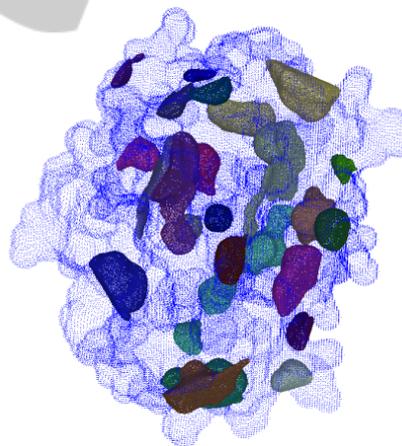


Figure 1: Result of the segmentation process of PDB ID 3EHY for the detection of pockets.

Referring to computational performance, our algorithm runs on an Intel Q6600 Processor with 4 GB of Ram. The analysis of pockets and protuberances on 3EHY protein has been done in 45 seconds starting from the PDB file. All the matching of the ligand with all the pockets of the protein has been done in 125 ms.

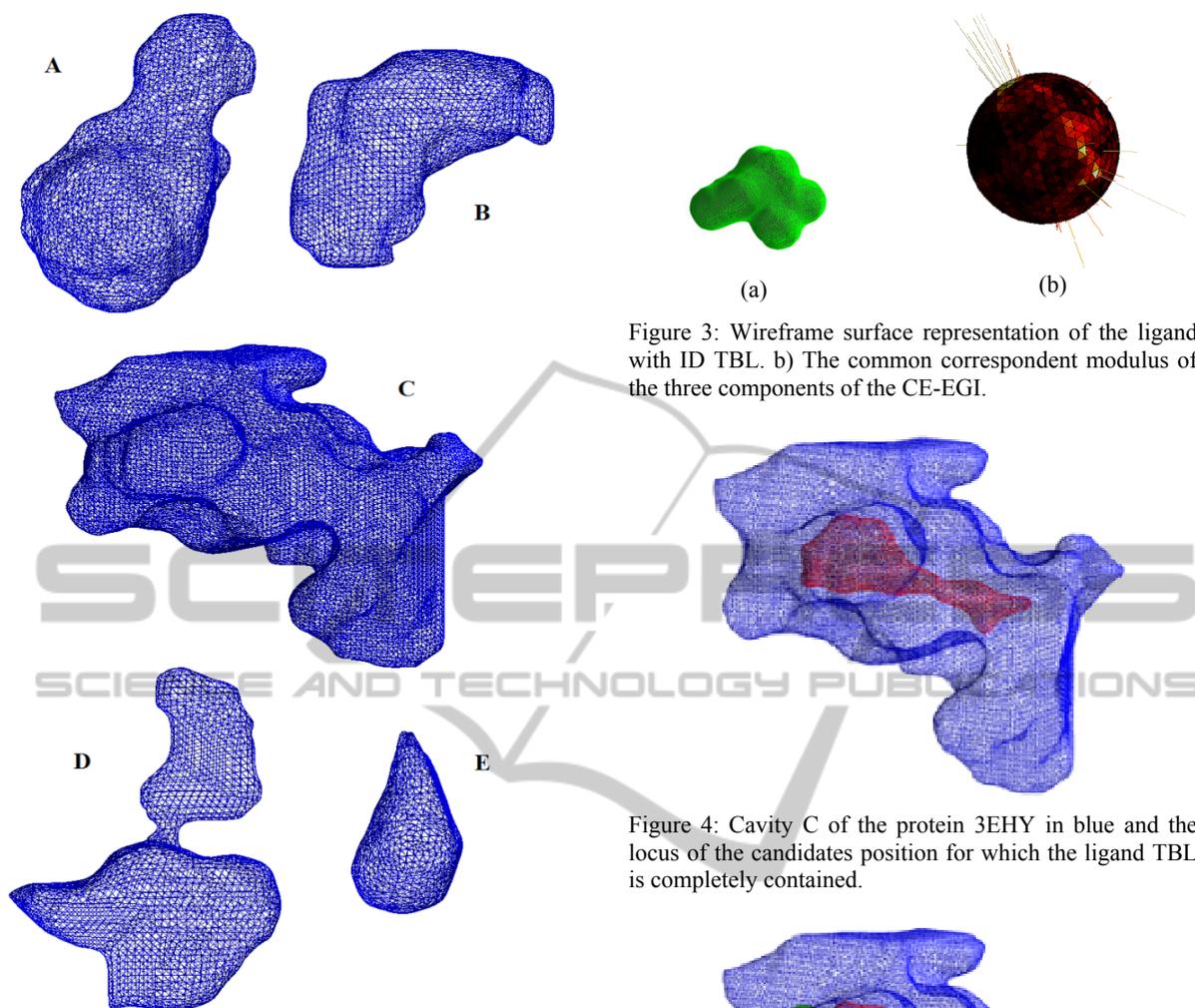


Figure 3: Wireframe surface representation of the ligand with ID TBL. b) The common correspondent modulus of the three components of the CE-EGL.

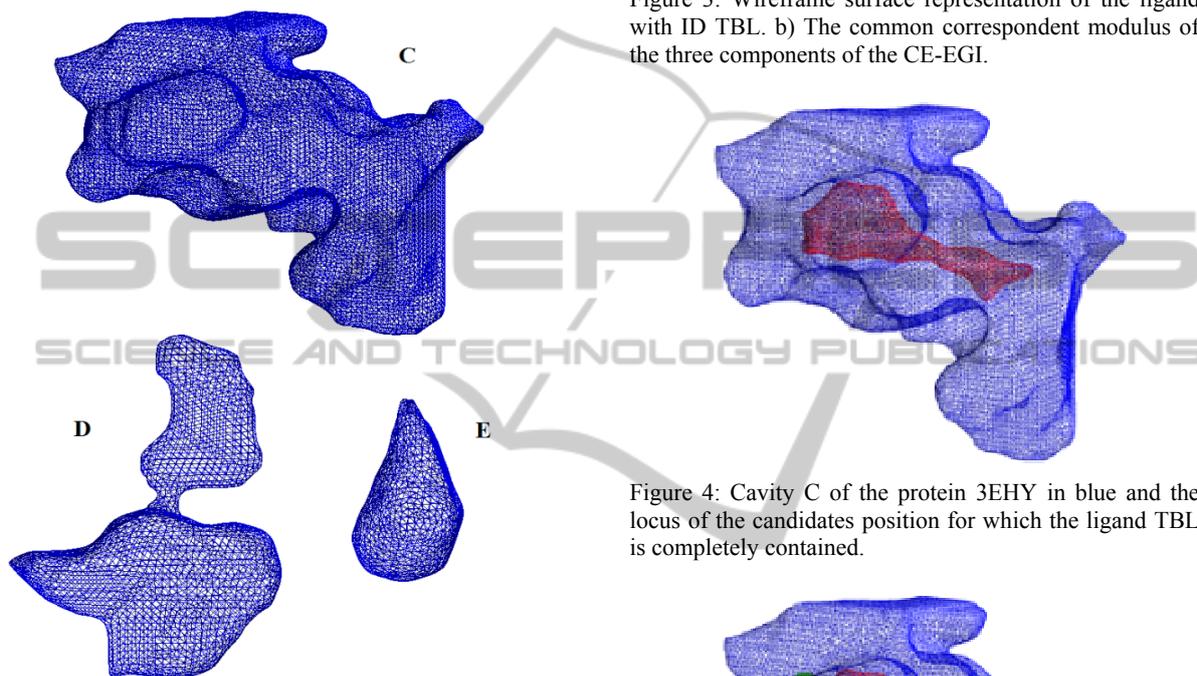


Figure 4: Cavity C of the protein 3EHY in blue and the locus of the candidates position for which the ligand TBL is completely contained.

Figure 2: The five most extensive pockets for protein 3EHY.

8 CONCLUSIONS

The aim of this activity is the identification of candidate locations for a given ligand in a given protein. The approach is based on an evaluation up-to-now only geometrical and topological, but we are now working for the introduction of the biochemical aspects.

For the morphological analysis we are proposing the technique of the Enriched Complex Extended Gaussian Image.

The achieved results look very promising as it seems to improve something not only from the computational point of view. We started an extensive experimentation phase to validate our solution and to identify the best practice for our new approach.

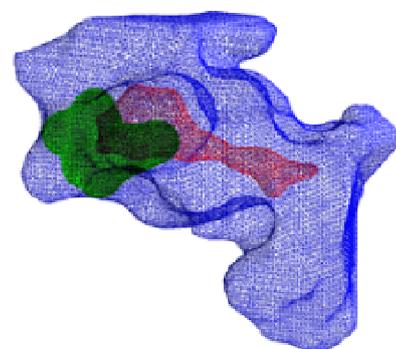


Figure 5: Cavity and a generic candidate position of the ligand corresponding to the point indicated with a star in figure 4.

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