

OPTIMAL SPATIAL ADAPTATION FOR LOCAL REGION-BASED ACTIVE CONTOURS

An Intersection of Confidence Intervals Approach

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Abstract: In this paper, we propose within the level set framework a region-based segmentation method using local image statistics. An isotropic spatial kernel is used to define locality. We use the Intersection of Confidence Intervals (ICI) approach to define a pixel dependant local scale for the estimation of image statistics. The obtained scale is based on estimated optimal scales, in the sense of the mean-square error of a Local Polynomials Approximation of the observed image conditional on the current segmentation. In other words, the scale is 'optimal' in the sense that it gives the best trade-off between the bias and the variance of the estimates. The proposed approach performs very well, especially on images with intensity inhomogeneities.

1 INTRODUCTION

The introduction of the level set method (Osher and Sethian, 1988) as a general framework for segmentation has overcome many of the limitations of traditional image segmentation techniques, specifically active contours. Level set methods are parameter free, and provide a natural approach to handle the topological changes and the estimation of geometric properties of the evolving interface. They have become very popular and are widely used in segmentation with promising results (Osher and Paragios, 2003).

The use of statistical models in region-based image segmentation has a long tradition. Their introduction in active contour segmentation methods, mostly within the level set framework, led to a considerable improvement in efficiency and robustness. For instance, the CV model (Chan and Vese, 2001) and its variant (Rousson et al., 2003) both consider image background and foreground as constant intensities represented by their mean values. The mean separation method of Yezzi et al. relies on the assumption that foreground and background regions should have maximally different intensities (Yezzi et al., 2002). These methods have many advantages and perform better than edge-based models in handling the noise and weak boundaries. However, they cannot deal with the intensity inhomogeneities, which is almost unavoidable in real images.

Recently, some work has been carried out in utilizing local image statistics to solve segmentation problems within the level set paradigm. A local binary fitting method, drawn from the CV model, has been proposed in (Li et al., 2008). A general framework for local region-based segmentation models, with illustrations on how a local energy is derived from a global one, has been presented in (Lankton and Tannenbaum, 2008). An interpretation of the piecewise smooth Mumford-Shah functional using local Gaussian models has been proposed in (Brox and Cremers, 2009). All of the above methods prove that the segmentation using local statistics has the ability to capture the boundaries of inhomogeneous objects.

Local region-based segmentation models, however, are found to be more sensitive to noise than global ones. Such models may also be more sensitive to initialization if the size of the local window is not appropriate. This brings out several problems that need to be addressed such as: how to choose between the global and local methods to segment an image? Can global and local statistics be combined in one model? Is it possible to define a pixel dependant local scale of the estimation of image statistics? A first attempt within the level set framework has been proposed in (Wang et al., 2009). Their approach is straightforward in the sense that the proposed method adds two energy functions of the same nature, where the model parameters are estimated globally in one

and locally in the other. In fact, this is not the first time that global statistics and local statistics are combined together to solve a segmentation problem. To our knowledge, it is within the Bayesian framework that the first proposition has been introduced in (Boukerroui et al., 1999; Boukerroui et al., 2003). The work focused on the adaptive character of a Maximum A Posteriori segmentation algorithm and discussed how global and local statistics could be utilized in order to control the adaptive properties of the segmentation process.

Interestingly, we can learn a lot from the progress of denoising methods (Katkovnik et al., 2010). Indeed, traditional denoising techniques, such as filtering, are based on local averaging. Therefore, their performances depend on the number of averaged data. An increase of the size of the averaging window do not solve the problem as it introduces bias on image regions where the noise free data are not constant. In this perspective, the only possible solution is a data dependent increase of the number of the data or their effective weights in the filtering process. Recently an interesting solution based on the Intersection of Confidence Intervals (ICI) rule has been proposed. The ICI rule is used to optimise the size of the local window in order to achieve the best trade-off between a minimum variance and a minimum bias of the a Local Polynomial Approximation (LPA) denoising (Katkovnik et al., 2002; Katkovnik et al., 2006). The most general formulation of the LPA-ICI method can estimate not only the size of the local window, but also its shape when it is used in its anisotropic form.

Motivated by the LPA-ICI method, this paper proposes a segmentation method based on local statistics with an adaptive size of local region. The size or the scale of the local isotropic window is optimal in the sense that it gives the best trade-off between the bias and the variance of the estimates. This new method provides promising segmentation results on images with intensity inhomogeneities.

The paper is organized as follows. We briefly introduce the local segmentation method in Sec. 2 and the LPA-ICI rule in Sec. 3. In Sec. 4, we give details on the proposed method for the local ‘optimal’ scales selection and its use in the segmentation algorithm. In Sec. 5, illustrative results are presented in order to demonstrate the new development. Finally, the authors’ conclusions are summarized in Sec. 6.

2 LOCAL REGION-BASED SEGMENTATION METHOD

Recently, Brox and Cremers (2009) derived a statistical interpretation of the full (piecewise smooth) Mumford-Shah functional (Mumford and Shan, 1989) by relating it to recent works on local region statistics. They showed that the minimization of the piecewise smooth Mumford-Shah functional is equivalent to a first order approximation of a Bayesian a-posteriori maximization based on local region statistics. Precisely, it is the approximation of the Bayesian setting with an additive noise with local Gaussian distribution, which can be expressed by the minimization of the following energy function:

$$E(\phi) = \int_{\Omega_i} \left[\frac{(I(\mathbf{x}) - \mu_i(\mathbf{x}))^2}{2\sigma_i^2(\mathbf{x})} + \frac{1}{2} \log(2\pi\sigma_i^2(\mathbf{x})) \right] d\mathbf{x} + \int_{\Omega_o} \left[\frac{(I(\mathbf{x}) - \mu_o(\mathbf{x}))^2}{2\sigma_o^2(\mathbf{x})} + \frac{1}{2} \log(2\pi\sigma_o^2(\mathbf{x})) \right] d\mathbf{x} + \lambda \int_{\Omega} \delta(\phi(\mathbf{x})) |\nabla \phi(\mathbf{x})| d\mathbf{x} + \nu \int_{\Omega} H(\phi(\mathbf{x})) d\mathbf{x}, \quad (1)$$

where the subscripts ‘*i*’ and ‘*o*’ represent inside and outside the segmentation contour C , and \mathbf{x} are the spatial positions in $\Omega \subset \mathbb{R}^2$. The image $I: \Omega \rightarrow \mathbb{R}$ is divided into foreground Ω_i and background Ω_o by C . H is the Heaviside function of the level set function ϕ . And the last two terms form the regularization term, whose contribution does not depend on image statistics. In Eq. (1) the local means and variances are functions of the spatial position \mathbf{x} , and can be estimated using normalized convolutions:

$$\mu_r(\mathbf{x}) = \frac{\int_{\Omega} K_{\tilde{h}}(\mathbf{x} - \zeta) H_r(\phi(\zeta)) I(\zeta) d\zeta}{A_r(\mathbf{x})}, \quad \sigma_r^2(\mathbf{x}) = \frac{\int_{\Omega} K_{\tilde{h}}(\mathbf{x} - \zeta) H_r(\phi(\zeta)) (I(\zeta) - \mu_r(\mathbf{x}))^2 d\zeta}{A_r(\mathbf{x})}, \quad (2)$$

$$A_r(\mathbf{x}) = \int_{\Omega} K_{\tilde{h}}(\mathbf{x} - \zeta) H_r(\phi(\zeta)) d\zeta,$$

where $r = \{i, o\}$, $H_i = H(\phi(\zeta))$, $H_o = 1 - H(\phi(\zeta))$. $K_{\tilde{h}}(\cdot)$ can be any appropriate local kernel, and \tilde{h} is a scaling parameter. The minimization of Eq. (1) is obtained when each point on the curve C has moved, such that the local interior and local exterior of each point along the curve are best approximated by local means. The exact shape gradient with respect to the level set function can be computed by the Gâteaux derivative. This leads to a fast implementation using recursive filtering. More details can be found in (Brox and Cremers, 2009, and references therein).

3 ADAPTIVE WINDOW SIZE BASED ON LPA-ICI

The LPA-ICI estimation, proposed as a nonparametric denoising method is not as the more traditional parametric ones which pursue the unbiased estimation. Instead, it controls the value of the varying window size in order to find a compromise between the bias and the variance of estimation.

a) LPA Estimation. Suppose f is a function of the spatial position $\mathbf{x} = (x, y)$, $f(\mathbf{x}) : \mathbb{R}^2 \rightarrow \mathbb{R}$. We wish to reconstruct this function using the noisy observations $I(\mathbf{x}_s) = f(\mathbf{x}_s) + \varepsilon_s$, $s = 1, \dots, n$, where the observations coordinates \mathbf{x}_s are known, and the ε_s are zero-mean random errors with variance σ^2 . Assume f can be well approximated locally by members of a simple class of parametric functions. Motivated by the Taylor series, the LPA provides estimates in a point-wise manner, which finds the weighted least-square fitting in a sliding window. The standard LPA minimizes the following criteria with respect to \mathbf{C} :

$$\mathbf{J}_h(\mathbf{x}) = \sum_s w_h(\mathbf{x}_s - \mathbf{x}) (I(\mathbf{x}_s) - \mathbf{C}'\boldsymbol{\psi}_h(\mathbf{x}_s - \mathbf{x}))^2,$$

where the sliding window w_h satisfies the conventional properties of kernel estimates and h is a scaling parameter. $\boldsymbol{\psi}_h$ is a vector of independent 2D polynomials of order from 0 to m . Using an appropriate set of polynomials, the estimate of the function f is given as $\hat{f}_h(\mathbf{x}) = \widehat{\mathbf{C}}_1(\mathbf{x}, h)$ and its l^{th} derivative is given by $\widehat{\mathbf{C}}_l(\mathbf{x}, h)$ (Katkovnik et al., 2002; Katkovnik et al., 2006). The estimate given by the LPA can be written as the kernel operator on the observations:

$$\hat{f}_h(\mathbf{x}) = \sum_s g_h(\mathbf{x}, \mathbf{x}_s) I(\mathbf{x}_s), \quad (3)$$

where the kernel g_h is defined by the window w_h and the set of polynomials $\boldsymbol{\psi}_h$. When the grid is assumed to be regular, the kernel $g_h(\mathbf{x}, \mathbf{x}_s)$ become shift-invariant on \mathbf{x} and the solution is given by a convolution operation.

Assuming an additive and identically distributed independent zero mean noise for all (local) observations, the Mean Square Error (MSE) of the LPA estimate is given by (Katkovnik et al., 2002):

$$\begin{aligned} \text{MSE}\{\hat{f}_h(\mathbf{x}, h)\} &= \text{E} \left\{ (f(\mathbf{x}) - \hat{f}_h(\mathbf{x}))^2 \right\} \\ &= m_{\hat{f}_h}^2(\mathbf{x}, h) + \sigma_{\hat{f}_h}^2(\mathbf{x}, h). \end{aligned} \quad (4)$$

The analysis of MSE demonstrates that, the bias of the estimation $m_{\hat{f}_h}$ is a monotonically increasing function of h , while the variance $\sigma_{\hat{f}_h}^2$ is a monotonically

decreasing one. This means that there exists a bias-variance balance giving the ideal scale h^* , which can be found by the minimization of Eq. (4). The ideal scale depends, however, on the $(m+1)^{\text{th}}$ derivative of the unknown function $f(\mathbf{x})$. The minimization of Eq. (4) leads us to following inequalities:

$$|m_{\hat{f}_h}(\mathbf{x}, h)| \begin{cases} \leq \gamma \cdot \sigma_{\hat{f}_h}(\mathbf{x}, h) & \text{if } h \leq h^*, \\ > \gamma \cdot \sigma_{\hat{f}_h}(\mathbf{x}, h) & \text{if } h > h^*, \end{cases} \quad (5)$$

which shows that the ideal bias-variance trade-off is achieved when the ratio between the absolute value of the bias to the variance is equal to γ . This inequality is the starting point for the development of a hypothesis testing for a data driven adaptive-scale selection.

b) ICI Rule. Let \mathbf{h} be a set of the ordered scale values $\mathbf{h} = \{h_1 < h_2 < \dots < h_J\}$. The estimates $\hat{f}_h(\mathbf{x})$ are calculated for $h \in \mathbf{h}$ and compared. The ICI rule, which uses the estimates and their variances, identifies a scale closest to the ideal one. The estimation error of the LPA satisfies the following inequality (Katkovnik et al., 2002):

$$|f(\mathbf{x}) - \hat{f}_h(\mathbf{x})| \leq |m_{\hat{f}_h}(\mathbf{x}, h)| + |e_f^0(\mathbf{x}, h)|. \quad (6)$$

Given the noise model assumptions, the random estimation error, $e_f^0(\mathbf{x}, h)$, follows a Gaussian probability distribution $\mathcal{N}(0, \sigma_{f_h}^2(\mathbf{x}, h))$, where,

$$\sigma_{f_h}^2(\mathbf{x}, h) = \sigma^2 \sum_s g_h^2(\mathbf{x} - \mathbf{x}_s).$$

Suppose all samples are independent to each other, the following inequality holds with probability $p = 1 - \beta$:

$$|e_f^0(\mathbf{x}, h)| \leq u_{1-\beta/2} \cdot \sigma_{\hat{f}_h}(\mathbf{x}, h), \quad (7)$$

where $u_{1-\beta/2}$ represents the $(1 - \beta/2)$ th quantile of the normal distribution $\mathcal{N}(0, 1)$. It means that the values of the random error belong to the interval with a probability p . Combining inequalities (7) and (5), with equation (6), it can be obtained that for all $h \leq h^*$ (Katkovnik et al., 2002):

$$|f(\mathbf{x}) - \hat{f}_h(\mathbf{x})| \leq (\gamma + u_{1-\beta/2}) \cdot \sigma_{\hat{f}_h}(\mathbf{x}, h).$$

From the equations above, the confidence interval $Q(h)$ of the estimate is given by:

$$Q(h) = \left[\hat{f}_h(\mathbf{x}) - \Gamma \cdot \sigma_{\hat{f}_h}(\mathbf{x}, h), \hat{f}_h(\mathbf{x}, h) + \Gamma \cdot \sigma_{\hat{f}_h}(\mathbf{x}, h) \right].$$

where $\Gamma = \gamma + u_{1-\beta/2}$. This is equivalent to: $\forall h_i \leq h^*(\mathbf{x})$, $f_{h_i}(\mathbf{x}) \in Q(h_i)$ holds with probability p . Therefore, for all $h_i < h^*$, the $Q(h_i)$ have a point in common, namely $f(\mathbf{x})$. If the ICI is empty, it indicates $h_i > h^*$. In this way, the ICI rule can be used to test the existence of this common point and to obtain the adaptive window size. The ICI algorithm is defined by the following steps:

1. Define a sequence of confidence intervals $Q_i = Q(h_i)$ with their lower bounds L_i and upper bounds U_i :

$$L_i = \widehat{f}_{h_i}(\mathbf{x}) - \Gamma \cdot \sigma_{\widehat{f}_{h_i}}(\mathbf{x}, h_i) ,$$

$$U_i = \widehat{f}_{h_i}(\mathbf{x}) + \Gamma \cdot \sigma_{\widehat{f}_{h_i}}(\mathbf{x}, h_i) .$$

2. For $i = 1, 2, \dots, J-1$, let

$$\bar{L}_{i+1} = \max\{\bar{L}_i, L_{i+1}\}, \quad \bar{L}_1 = L_1 ,$$

$$\bar{U}_{i+1} = \min\{\bar{U}_i, U_{i+1}\}, \quad \bar{U}_1 = U_1 .$$

According to these formulas, \bar{L}_{i+1} and \bar{U}_{i+1} are respectively nondecreasing and nonincreasing sequences.

3. The ICI rule is finding the largest i , when $\bar{L}_i \leq \bar{U}_i$, $i = 1, 2, \dots, J$, is still satisfied.

4 PROPOSED METHOD

We propose applying the ICI approach to optimize the spatial adaptation for local region-based active contours. For each point, it finds an optimal kernel size that meets the trade-off between the bias and the variances of estimate. This optimal local scale is then used for the estimation of the local means and variance of the segmentation model as given in Eq. (2).

Suppose we are given a noisy image I with intensity inhomogeneities within its foreground and background. We define an initial zero level set C , as the yellow contour that is shown in Fig. 1. Given a finite set of scale values \mathbf{h} , we calculate the g_h for each element. Then utilize the LPA Eq. (3) to get the local estimations of the regions inside, Ω_i , and outside, Ω_o , respectively. It means that if a point \mathbf{x} is inside of C , its approximation uses the overlapping area of its local kernel $g_h(\mathbf{x})$ and Ω_i , and vice versa. As introduced in Sec. 3, we can calculate the confidence intervals of this estimation, then apply the ICI algorithm for each point. After that, we obtain the optimal kernel size that well balances the estimate bias-variance.

To find out the relation of these data adaptive scales with the position of segmentation contour, we picked out several typical points for analysis. As we are only interested on a narrow band of C , within which we select four pairs of neighbor points where one is inside (marked with blue '+') and the second is outside (marked with black 'o') the contour C (see Fig. 1). The corresponding estimated scales are also illustrated on the same figure with circles.

The leftmost pair P_4 lays around a region with very low contrast between Ω_i and Ω_o , where the local statistics are very similar. Also the contour near P_4 is the correct boundary. In order to maintain this partition, we tend to consider more information, which is corresponding to the larger kernel size obtained by LPA-ICI algorithm.

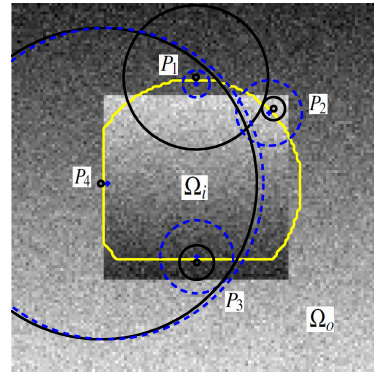


Figure 1: Adaptive kernel sizes obtained by LPA-ICI algorithm (image size 128×128 , SNR = 10 dB).

For the pair P_1 around the top of C , the inside one, lays between C and the real boundary, has small kernel size. Because in that position, larger window will introduce greater estimation bias. While its symmetric point within Ω_o has larger window, because in this neighboring region in Ω_o , the image is relatively homogenous. Reversely, the pairs P_2 and P_3 have larger window inside and smaller one outside. Therefore, if we directly use these kernel sizes in the segmentation algorithm, as the contour C gets closer to the real boundary, the local regions of points between them become smaller and smaller, and so will be the estimated local scale. It brings out the problem that the closer C is to the correct segmentation, slower the evolution speed is. Analyzing case P_1 (P_2), the scale in the inside (outside) has to be at least as bigger as the outside (inside) in order to increase the force driving the segmentation process. To overcome this problem, one possible solution is to set a minimum scale in \mathbf{h} , bigger enough for a correct estimation of local image statistics. An alternative solution is to run a max filter, of a small size 3×3 , on the estimated local scales, so that near the contour, the estimated scales have similar values. This filtering operation is necessary only when the algorithm is in progress. Indeed, the estimated scales, given the correct segmentation, are very appropriate as it can be seen on the point P_4 .

Our algorithm is organized as follows:

1. **Initialization.** Given an image I , an initial segmentation C or \emptyset , a finite set of half window sizes \mathbf{h} and a vector of polynomials ψ . For each $h \in \mathbf{h}$, calculate the LPA kernel g_h .
2. **Optimal Spatial Kernel Size Estimation.** LPA-ICI algorithm: with g_h , estimate the image inside and outside of C by the LPA; apply the ICI rule on the estimation, in order to get the adaptive window size h^+ for this C .
3. **Local Region-based Segmentation.** Use the lo-

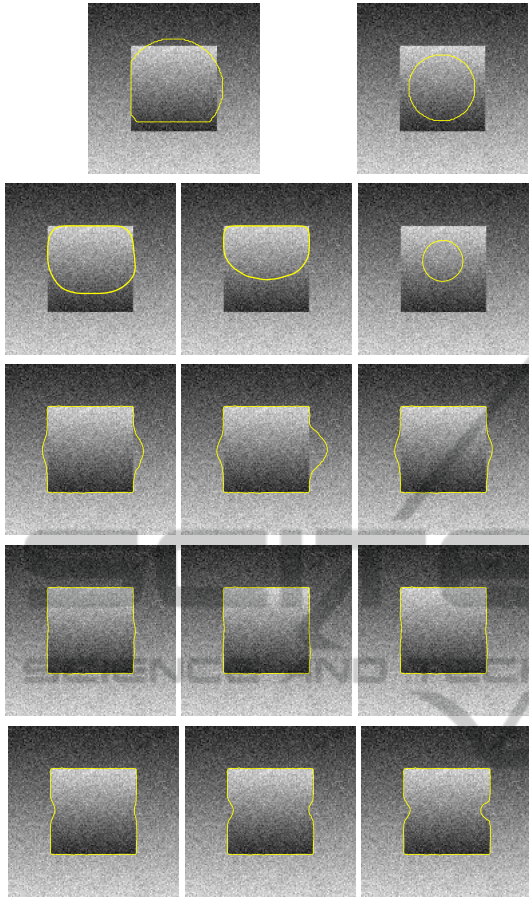


Figure 2: Segmentation results of Fig. 1 for two initializations. The top row shows the initial contours. From the 2nd row to the bottom row using: the global CV model, local MS method with $h = 30$, $h = 26$ and $h = 22$. Left column, results after 100 iterations. Middle and right columns, results after 200 iterations.

cal statistics within h^+ for local piecewise smooth MS segmentation (Brox and Cremers, 2009).

4. Repeat steps 2 and 3 until convergence.

5 EXPERIMENTS & DISCUSSION

In this section we analyze the performance of the proposed segmentation algorithm and compare it with the global and the single local scale segmentation methods. We utilize the Chan-Vese model (Chan and Vese, 2001) as a global method and the local piecewise smooth MS method (Brox and Cremers, 2009) as a local one. For local methods, we use a Gaussian kernel with a standard deviation $h = h^+ / 3$.

Fig. 2 shows the segmentation results on the synthetic image of Fig. 1, obtained using the CV and

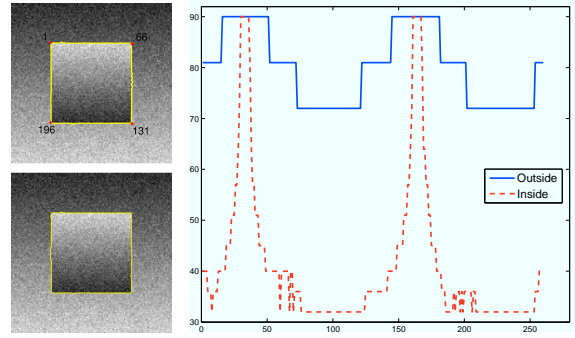


Figure 3: Segmentation results of the proposed method. Left column: results for the two initializations shown in Fig. 2. Right column: the estimated optimal spatial kernel sizes for the left top segmentation contour, clockwise along the contour, for the inside and the outside of C .

the local MS model for three different scales. The CV models fails to produces correct results because the global model supposes a constant foreground and background. It is, therefore, inappropriate to segment images with intensity inhomogeneities. Three different scales are used to test and study the influence of the kernel scale in the local MS method. As expected, the underling model is more appropriate, as two of the three results, obtained with the smaller kernel size, are better than the global. Notice, however, unless we choose an appropriate kernel scale, here around $h = 26$, the local MS model is not able to distinguish the parts with a very low contrast between inside and outside of C . Notice also that both methods converge to different solutions given different initializations.

The result of the proposed scale adaptive segmentation algorithm on the same test image is presented in Fig. 3). A very satisfactory result is obtained. For a further analysis, plots of the estimated local window sizes along the final contour, for the inside and the outside regions, are also shown. The plots follow a pixel clockwise parametrization of the curve starting from the top left corner. Notice for example that the estimated scales in Ω_i are smaller than the outside region. This can be explained by the fact that the inhomogeneity in Ω_i is stronger than in Ω_o . This difference of scales, between the inside and the outside, is important for the forces in competition around low contrasted boundaries. This experiment demonstrates that local models perform better on images with intensity inhomogeneities, and also that the selection of the size of the local kernel is of a high importance in order to achieve acceptable results.

Finally, we consider the influence of the noise level on the estimation of the spatial kernel size. For this experiment, we use the same synthetic image, and study the LPA-ICI behavior on three typical pairs of

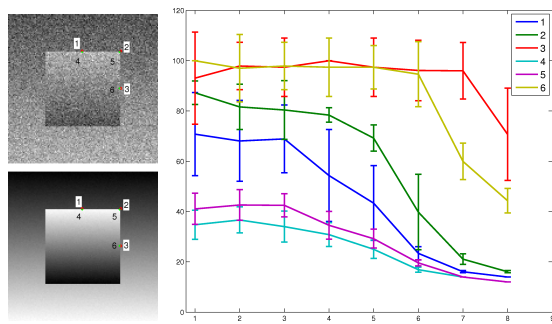


Figure 4: Adaptive scale obtained by ICI rule for the same image with decreasing noises. Left column: test images for SNR= 2 and SNR= 32. '1','2','3' are points in Ω_o ; '4','5','6' are the points in Ω_i . Right column: estimated kernel size error bars versus increasing SNR values for the 6 chosen points. SNR $\in \{.5, 1, 2, 4, 8, 16, 25, 32\}$. 20 values are used for mean and variance estimation.

points (similar as in Fig. 1). A number in 1 to 6 is assigned to each point as shown in Fig. 4. The study is carried out for the special case of an ideal segmentation (to discard the influence of bias estimation). In order to obtain statistically meaningful estimations, we run the experiment 20 times and for 8 different SNR values. The means and the standard deviations, calculated with the 20 estimated kernel sizes, are visualized as error bars versus increasing SNR values on Fig. 4, a curve for every point.

First, we observe that the kernel size is inversely proportional to the SNR value. Second, the variance of the estimation is also inversely proportional to the SNR value. These observations imply that, when the image SNR increases, the corresponding optimal kernel size decreases, and the proposed segmentation method tends to be more local. Reversely, if the SNR of image is small, the proposed method will adaptively use large kernel sizes. As expected, bigger kernel sizes are obtained for the exterior points in comparison to their adjacent interior points. The pairs of points '3' and '6' have the largest kernel estimated sizes. Notice that for SNR values ≤ 8 dB, the obtained sizes are almost equal to the maximum value of the set \mathbf{h} . This implies that the behavior of the proposed method on such situation will be similar to global methods, but locally. However, around points '1' and '4' the proposed method will always use local image statistics. The curves also show that the noise level has small influence on the estimated kernel size at this location. This can be explained by the higher inhomogeneity of the noise free image at this position. In other words, the optimal scale is more governed by the bias of the local region.

6 CONCLUSIONS

In this paper, we proposed an adaptive local region-based segmentation method within the level set framework. To our knowledge, this is the first work for data driven local scale selection, in the context of region based level set segmentation. The ICI rule is used to derive an optimal scale for interior and exterior points of the segmentation contour. The optimality is in the sense of the mean-square error minimization of a Local Polynomials Approximation of the observed image conditional on the current segmentation. Although the presented results are preliminary, they however, illustrate well the improvement on the state of the art of segmentation methods. The continued development and refinement of the proposed method, with more experiments, should be researched in the foreseeable future.

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