

A NEW METHOD AND METRIC FOR QUANTITATIVE RISK ANALYSIS

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Abstract: Quantitative risk analysis provides practitioners a deeper understanding of the risks in their projects. However, the existing methods for impact assessment are inaccurate and the metrics for risk prioritization also can not properly prioritize the risks for certain cases. In this paper, we propose a method for measuring risk impact by using AHP. We also propose a new indicator, risk intensity (RI), to prioritize the risks of a project. Compared with the widely used metric Risk Exposure (RE), the contours of RI show a convex pattern whereas the contours of RE show a concave pattern. RI allows practitioners weight probability and risk impact differently and can better satisfy the needs of risk prioritization. Through a case study, we found that RI could better prioritize the risks than RE.

1 INTRODUCTION

Nowadays, Information Technology (IT) projects become more and more complicate, and face many challenges and uncertain factors. To guarantee the success of IT projects, effective risk management is necessary. Most of the failed projects are caused by poor risk management (Sherer, 2004).

As one of the key processes of risk management model proposed by Project Management Institute (PMI, 2008), quantitative risk analysis is important since one can not manage what one does not measure. One key output of the quantitative risk analysis is a prioritized list of quantified risks. For accurate risk prioritization, two preconditions are needed: 1) accurate assessment of the probability of the occurrence of risk and the impact of the risk, 2) a good metric to determine the priority of risks.

There are some easy to use guidelines and principles for assessing probability. (Mcmanus, 2004; Pandian, 2007; Ferguson, 2004; Boehm, 1991) Compared with the assessment of risk probability, the assessment of risk impact is more complicate since the impact may affect different aspects of a project, such as schedule, cost, scope, and quality of the product and service. Our investigation on the existing methods for assessing risk impact found that the existing methods are inaccurate.

Risk Exposure (RE) is a commonly used metric for quantitative risk analysis and risk prioritization.

However, it can not properly prioritize the risks for certain cases. Boehm (1989) also pointed out this problem when he proposed RE.

In summary, in order to properly prioritize the risks, we need a method for accurately assessing risk impact and a new way to address the priority of the risk. In this paper, we will propose a method for measuring risk impact by using AHP. Then, we proposed a new indicator, risk intensity (RI), to prioritize the risks of a project. RI could overcome the shortcoming of RE in risk prioritizing and properly address the priority of the risk. The aim of this paper is to develop a better method and a new metric that help practitioners to assess the risks accurately and prioritize the risks properly, thus supporting more effective risk management.

The rest of the paper is organized as follow. We present the related work in section 2. Then, we propose a method for accurately assessing risk impact and a new metric for properly prioritizing the risks in section 3 and section 4 respectively. At last, we draw a conclusion and address the future study in section 5.

2 RELATED WORK

2.1 Definition of Risk

Glutch (1994) defined risk as:

Risk is the combination of the probability of an abnormal event or failure and the consequence(s) of that event or failure to a system's operators, users, or its environment.

Although there are other definitions (Boehm, 1989; Mcmanus, 2004; Pandian, 2007), a risk has two basic attributes, probability (P), and impact (I), where probability is the probability of risk occurrence, and impact is the level of damage if risk occurs. Recent risk management literatures have broadened the definition of risk to include *opportunity* (PMI, 2008; Kähkönen, 2001). According to PMI (2008), a project risk is an event that can have either positive or negative effect on project objectives. An event offers *risk* if $I > 0$, and it offers *opportunity* if $I < 0$.

According to probability theory, P theoretically ranges in $[0, 1]$. The range of I does not have any theoretical boundaries. However, we can assess it on a relative scale which range from $-i$ to $+i$, or normalize the scale to $[-1, 1]$. In this paper, we assess the impact with the latter scale.

Not all events can be considered as risks. White (2006) argues that three kinds of events are not risk. An event is not a risk if it:

- never happens ($P = 0$);
- happens without any impact ($I = 0$);
- surely happens ($P = 1$).

In summary, we can use $R:(P, I)$ to denote a risk, where P is a real number in $(0, 1)$, and I is a real number in $[-1, 1]$ and does not equal to 0 ($I \in [-1, 0) \cup (0, 1]$).

For convenient, the risk impact is considered as negative if we do not specify otherwise. All the results which are based on the negative impact can easily extend to the positive impact, since the formulas can be directly extended from $(0, 1]$ to $[-1, 0)$ and the discussion based on the range of $(0, 1]$ is also suitable for $[-1, 0)$.

In risk management, those risks with very high impacts are called *hazards*. According to Pandian (2007), in hazard analysis we do not discount a hazard. Instead we apply Murphy's law: If something can go wrong, it *will* go wrong. Similarly, those risks with high probability are considered as *constraints* of the project since there is no surprise element (Pandian, 2007). In summary, the risks with high probability or high impact should have a higher priority than those risks with relatively low probability and impact (Boehm, 1989; Mcmanus, 2004; Pandian, 2007).

Risk matrix is a widely used qualitative method for ranking risks (PMI, 2008; Cox, et al, 2005; Cox, 2008). A risk matrix is a table that has several categories of probability for its rows (or columns) and several categories of risk impact for its columns

(or rows) respectively. The gray level indicates the priority of the risks. The deeper gray means higher risk. The risk level of each region in the risk matrix should reflect the opinions of stakeholders. Although people may argue that risk matrix may not rank the risks accurately (Cox, et al, 2005; Cox, 2008), it can serve as the basis of quantitative risk analysis. It provides us the distribution pattern of risks' priority at least. Although different projects may use different risk matrix, the risks with high probability and/or impact should have high priority. We can use a typical 5x5 risk matrix to represent this pattern (see Fig. 1). From Fig. 1, we find that the risk with high probability or high impact should have a higher priority than those risks with relatively low probability and impact. For example, a risk in (Frequently, Insignificant) region and a risk in (Seldom, Catastrophic) region should have higher priority than any risks in the region formed by (Possible, Unlikely, Seldom) and (Moderate, Minor, Insignificant).

Frequently					
Likely					
Possible					
Unlikely					
Seldom					
Probability Impact	Insignific ant	Minor	Moderate	Major	Catastrop hic

Figure 1: A risk matrix.

2.2 Assessment of Risk Impact

One way to assess the risk impact is approximate it without working out the impacts in different dimensions, such as time, budget, quality, and scope (Mcmanus, 2004; Boehm, 1991). For example, we can assess the impacts on a relative scale of $(0, 10]$. This is commonly used in practice because of its simplicity. However, this kind of method is inaccurate.

Very few studies assess the risk impact with due consideration of the impact of risk in different dimensions of IT projects. The method proposed by Ferguson (2004) for assessing risk impact does not integrate the impact in different dimensions properly. The basic idea of his method is first divide the risk impact into 5 levels. Each level associates with a benchmark and an impact score. The benchmark of the 5th level is established according to the project. Then, the benchmarks of lower levels are 1/3 of its immediate upper level. The impact score is calculated as

$$\text{Impact score} = 3^{(\text{level}-1)} \quad (1)$$

Each risk can be classified into one impact level based on practitioners’ judgment and assigned the impact score of that level. For example, assume a project, “Project A”, will take 18 months with a project cost of \$3 million, and expect to achieve \$1 million revenue in the first year. The benchmark and the impact score of each level is shown in Table 1. Then, a risk is assigned an impact score of 9 if it is classified into level 3.

Table 1: Benchmark and impact score of “Project A”.

Impact Level	Benchmark	Impact Score
5	<ul style="list-style-type: none"> ● Overrun by 18 months. ● Overspend by \$3M. ● Lose \$1M in revenue. 	81
4	<ul style="list-style-type: none"> ● Overrun by 6 months. ● Overspend by \$1M. ● Lose \$333K in revenue. 	27
3	<ul style="list-style-type: none"> ● Overrun by 2 months. ● Overspend by 333K. ● Lose \$111K in revenue. 	9
2	<ul style="list-style-type: none"> ● Overrun by 3 weeks. ● Overspend by \$111K. ● Lose \$37K in revenue. 	3
1	<ul style="list-style-type: none"> ● Overrun by 1 week. ● Overspend by \$37K. ● Lose \$12K in revenue. 	1

This method has two major problems. First, the granularity is too big to accurately assess the risk impact. For example, assume that two risks have impacts in overrun by 4 months and 8 months respectively. Although they have significant difference in overrun, they are assigned the same impact value of 27. Second, it does not consider the impacts in different impact dimensions. For example, assume that one risk has impact in overrun by 6 months and overspend by 500K, and the other has impact in overrun by 6 months and overspend by 100K. Although their overspendings are significantly different, they may be classified into the same impact level of 3 according to their major impact in time dimension and then are assigned the same impact value of 27.

2.3 Risk Exposure (RE)

RE was introduced by Boehm (1989), and defined as the multiplication of probability and impact of the risk.

$$RE = P \times I \tag{2}$$

Although RE is widely used and accepted by most practitioners, we find that RE can not properly prioritize the risks. Fig. 2 shows RE contours as a function of probability and impact, where both probability and impact range from 0 to 1.

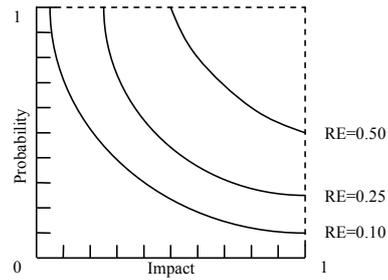


Figure 2: Contours of RE.

From Fig. 2, we can find that risks with high probability and low impact, or high impact and low probability, have the same RE value as those risks with relatively low probability and risk. For example, let’s consider four risks as shown in Table 2. Three of them, risk *a*, *b*, and *c*, have the same RE value 0.18. Then, they will have the same priority in risk response process if we use RE to prioritize risks. Further, as risk *d* has the highest RE value among all four risks, it has the highest priority. However, risk *a* and *b* should have a higher priority than risk *c* and *d*, because the risks with high probability or high impact should have a higher priority than those risks with relatively low probability and impact (Boehm, 1989; Mcmanus, 2004; Pandian, 2007). As demonstrated in this example, we find that RE can not properly prioritize the risks for certain cases.

Table 2: Four risks and their RE.

Risk	Probability	Impact	RE
a	0.9	0.2	0.18
b	0.2	0.9	0.18
c	0.4	0.45	0.18
d	0.45	0.45	0.20

2.4 Analytic Hierarchy Process (AHP)

Analytic Hierarchy Process (AHP) has been extensively studied and refined since it was proposed twenty years ago (Saaty, 1994; Lipovetsky, 1996; Lipovetsky and Tishler, 1994; Forman and Gass, 2001). It is a structured technique for decision making. The AHP is most useful when people work on complex problems which include different stakes and involve human perceptions and judgment (Bhushan and Rai, 2004). It calculates a numerical value, global rating, for alternatives which can be processed and compared over the entire range of the problem. The global rating can also be used to evaluate objects with multiple dimensions (Bhushan and Rai, 2004; Forman and Gass, 2001). Note that, although there are many versions of AHP (Bhushan

and Rai, 2004), we will use the standard approach in this paper for its universality.

The users of AHP need to decompose the problem into a hierarchy of goal, criteria, sub-criteria and alternatives first. Then, the weighted-sum method (WSM) is used for evaluating each alternative. We can get the weighted rating in each criterion by multiplying the rating of alternatives in each criterion and the importance of the criterion. This product is summed over all the criteria to generate the global ratings of the alternative. Mathematically,

$$Rating_i = \sum_{j=1}^n r_{ij} w_j \quad (3)$$

where $Rating_i$ is the rating of the i^{th} alternative, r_{ij} is the rating of the i^{th} alternative in the j^{th} criterion, and w_j is the weight or importance of the j^{th} criterion.

An important issue in using AHP is how to weight different criteria. One way to do this is compare all pairs of combinations. The result of the comparisons can be represented with a square decision matrix, A , which is shown in (4). Each element in the matrix, a_{ij} , represents the relative importance of the i^{th} criterion compared with the j^{th} criterion. The measurement scale used by the AHP is one of 1 – 9 in absolute numbers. The higher value means higher comparative importance while 1 means equal importance. For example, if the i^{th} criterion is moderately more important than j^{th} criterion, then a_{ij} can be assigned 3. On the contrary, a_{ij} can be assigned 1/3. Based on decision matrix, A , we can calculate the weights of different criteria (Saaty, 1994; Bhushan and Rai, 2004).

$$A = \begin{pmatrix} 1 & a_{12} & \cdots & a_{1n} \\ 1/a_{12} & 1 & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ 1/a_{1n} & 1/a_{2n} & \cdots & 1 \end{pmatrix} \quad (4)$$

3 A METHOD TO ASSESS RISK IMPACT

A risk may have potential impacts on different dimensions of a project. For example, turnover is a common risk of many projects. If this risk happens, it may require some additional schedule time and additional resource cost to hire and train staff. Besides that, the quality of product/service may downgrade since the new team member is not familiar with the project.

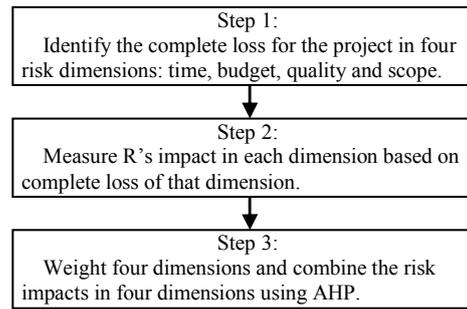


Figure 3: Procedure for assessing risk impact.

A problem of existing methods for risk impact assessment is they do not combine the impacts in different dimensions. In order to assess the risk impact accurately, practitioners can follow our proposed procedures as shown in Fig. 3.

The first step is identifying what are the complete loss for a project in its most important four dimensions, time, budget, quality, and scope (PML, 2008; Kerzner, 2006). The practitioners should decide the complete loss in each dimension based on the constraints and assumptions of the project. As Jones (1996) reported that “the typical project is 100 percent over budget when it is cancelled”, in many cases, it is a good choice to set the complete loss of each dimension to the value that would cause the project to be cancelled. For example, the complete loss in the time dimension could be set to 12 months if the schedule of the project is 12 months. Practitioners should identify the complete loss according to specific characteristics of their project. We use CL_k , $k=1, \dots, 4$, to denote the complete loss in time, budget, quality, and scope respectively.

The second step is measuring the risk impact in each dimension based on the complete loss of that dimension. We use I_k , $k=1, \dots, 4$, to denote the impact in time, budget, quality, and scope respectively. Then, the impact of a risk can be represented as a 4-tuple, (I_1, I_2, I_3, I_4) . For any risk, we use PL_k to denote its potential loss in the k^{th} dimension. Then, the ratio of PL_k over CL_k , can be used to rank the impact in the k^{th} dimension. Note that practitioners should confine the PL_k in the range of $[0, CL_k]$. If the estimated PL_k is bigger than CL_k , practitioners can set PL_k equal to CL_k .

$$I_k = \frac{PL_k}{CL_k} \quad (5)$$

where, I_k ranges in $[0, 1]$. For example, assume a risk has a potential loss of 6 months in the time dimension, and the complete loss in this dimension is 18 months. That is $CL_t=18$ months and $PL_t=6$

months. Then, the risk impact in the time dimension, $I_1 = PL_1 / CL_1 = (6 \text{ months}) / (18 \text{ months}) = 0.33$.

The third step is weighting different impact dimensions and then calculating the risk impact using AHP. The practitioners can weight different impact dimensions based on the characteristics of the project, enterprise environment factors, past experiences, and so on. When we use AHP to assess risk impact, the risks of the project serve as the alternatives in AHP, the four impact dimensions serve as the criteria in AHP, and I_k serves as the rating in the k^{th} criterion. Then, the risk impact, I , of a risk is the global rating of the risk in AHP. If we use $W_k, k=1, \dots, 4$, to denote the weights of four impact dimensions, time, budget, quality, and scope respectively, the risk impact, I , is calculated with (6) in terms of global rating of AHP.

$$I = \sum_{k=1}^4 I_k W_k \quad (6)$$

where, $0 \leq I_k \leq 1, 0 < W_k < 1$ and $\sum W_k = 1$.

We next use an example and Table 3 to illustrate the procedures of assessing risk impact. Assume there is a project with 12 months schedule and budget of \$200K. The project has 30 quality metrics defined in requirement and has a size of 100 function points (FP). We also assume that practitioners found 4 risks in risk identification. To assess the risk impact of the risks, the practitioners should define the complete loss in each dimension based on specific characteristics of the project first. We assume that the complete loss in each dimension is overrun by 2 months, overspend by \$50K, failed in 5 quality metrics and missing 30 FP respectively. That is, $CL_1=2\text{months}, CL_2=\$50K, CL_3=5\text{metrics}, CL_4=30\text{FP}$. Second, practitioners should assess the potential loss of all risks of the project in each dimension. Assume that the potential losses of those 4 risks in our example were already assessed by practitioners. Then, we can compute their impact in each dimension. The results of this step are shown in the 5th row to 8th row of Table 3. Third, practitioners should work out the decision matrix based on the characteristics of the project. The decision matrix is shown in Table 4. This matrix shows that time and quality dimension have equal importance, both quality and time are moderately more important than budget and strongly more important than scope, and budget is moderately important over scope. Based on the decision matrix, we can use the method defined in AHP to compute the weight of each dimension. Practitioners can use some tools to calculate the weight of each dimension (For example, an online tool is available at <http://www.isc.senshu-u.ac.jp/~thc0456/EAHP/AHPweb.html>). The weight of each dimension in this example is W_1

$= 0.379, W_2 = 0.140, W_3 = 0.400, W_4 = 0.081$. After that, practitioners can calculate the risk impact of each risk with (6). The results of this step are shown in the last column of Table 3.

Note that practitioners can use other ways to measure the complete loss and potential loss in quality dimension and scope dimension. Our example assumes that all quality metrics and functions have the same importance, whereas, in real project some quality metrics and functions are key metrics and key functions we can not miss. If a risk has potential impact on those key metrics and functions, we should consider the potential loss as a complete loss.

Note that our method can also be extended to more dimensions. That is, practitioners can identify other impact dimensions rather than our proposed dimensions. To extend our method, practitioners should identify all impact dimensions first, and then follow the steps shown in Fig. 3 to assess the risk impact. The only difference is that practitioners should consider all identified risk dimensions, not just our proposed dimensions.

Table 3: Example for assessing risk impact.

Risks	Time		Budget		Quality		Scope		Impact $I = \sum_{k=1}^4 I_k W_k$
	$CL_1=2\text{mont hs}$		$CL_2=\$50K$		$CL_3=5\text{metric s}$		$CL_4=30\text{FP}$		
	$W_1=0.379$		$W_2=0.140$		$W_3=0.400$		$W_4=0.081$		
	PL_1 wee k	$I_1=$ $PL_1/$ CL_1	PL_2 K	$I_2=$ $PL_2/$ CL_2	PL_3 metri cs	$I_3=$ $PL_3/$ CL_3	PL_4 FP	$I_4=$ $PL_4/$ CL_4	
1	4	0.50	0	0.20	0	0	20	0.67	0.272
2	0	0	0	0.40	3	0.60	10	0.33	0.323
3	2	0.25	0	0.20	2	0.40	5	0.17	0.297
4	0	0	0	0	2	0.40	0	0.33	0.187

Table 4: Decision matrix for the example.

	Time	Budget	Quality	Scope
Time	1	3	1	4
Budget	1/3	1	1/3	2
Quality	1	3	1	5
Scope	1/4	1/2	1/5	1

We use AHP to weight different impact dimensions and combine the impacts of different dimensions into one single value because of following reasons. AHP has solid mathematical basis and “it has been applied literally to hundreds of examples both real and hypothetical” (Saaty, 2008). The power and validity of AHP have already been validated in practices. Compared with the simple weighted-sum method AHP provides a hierarchy structure for risk assessment, which is useful when

we want to divide impact dimensions into sub-dimensions. It also provides us a useful method for weighting different impact dimensions based on the decision matrix.

The proposed method for assessing risk impact is more accurate than other methods, such as approximate methods and the Ferguson method, since we not only measure the impacts of different dimensions but also integrate them together.

4 A NEW METRIC FOR RISK PRIORITIZATION

4.1 Risk Intensity

As mentioned earlier, RE can not properly address the priority of the risks. We propose a new metric, Risk Intensity (RI), to measure the priority of the risk.

We assume that a project includes a set of n risks at time t , $RSet(t) = \{R_1, R_2, \dots, R_n\}$. Both the probability and the impact of R_i , $R_i \in RSet(t)$, and the number of risks may change as time elapse. The new metric, RI, should satisfy following constrains:

1. $\forall R_i:(P_i, I_i), R_j:(P_j, I_j) \in RSet(t)$, if $P_i = P_j$ and $I_i = I_j$ then $Pri_i = Pri_j$;
2. $\forall R_i:(P_i, I_i), R_j:(P_j, I_j) \in RSet(t)$, if $P_i > P_j$ and $I_i > I_j$ then $Pri_i > Pri_j$;
3. $\forall R_i:(P_i, I_i), R_j:(P_j, I_j) \in RSet(t)$, if $P_i > P_j$ and $I_i = I_j$ then $Pri_i > Pri_j$;
4. $\forall R_i:(P_i, I_i), R_j:(P_j, I_j) \in RSet(t)$, if $P_i = P_j$ and $I_i > I_j$ then $Pri_i > Pri_j$;
5. $\forall R_i:(P_i, I_i), R_j:(P_j, I_j) \in RSet(t)$, if $(P_i > P_j$ and $I_i < I_j)$ or $(P_i < P_j$ and $I_i > I_j)$, the Pri_i and Pri_j should approximately match the pattern of the risk matrix in Fig. 1.

where, Pri_i and Pri_j are the priority of risks R_i and R_j , respectively.

A risk can be mapped to a point in the PI (probability and impact) area $(0, 1) \times (0, 1]$ if we only focus on risk rather than opportunity. Fig. 4 shows the mapping of 3 risks and their points in the 2-dimension PI space. Risks $R_A:(0.5, 0.5)$, $R_B:(0.1, 1)$, and $R_C:(0.9, 0.2)$, are mapped to points A, B, and C respectively.

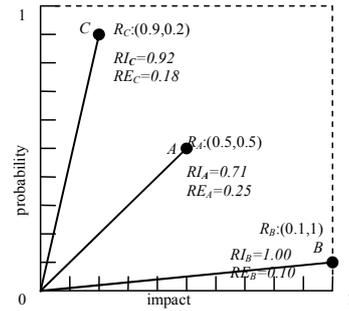


Figure 4: RI and RE of risks.

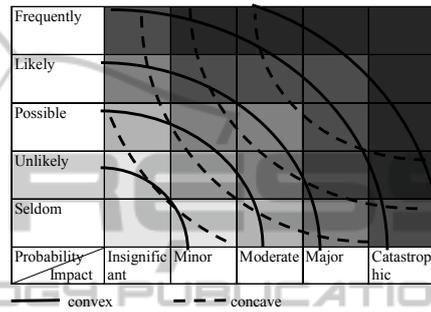


Figure 5: Contours and the risk matrix.

Most practitioners would accept that the risk with higher probability and higher impact should have higher priority. Thus, a risk that is further away from the original point in PI space should have a higher priority. Based on this idea and the geometric meaning of Euclidean distance, we define RI as the Euclidean distance between the risk point and the original point.

$$RI = \sqrt{P^2 + I^2} \quad (7)$$

where, P and I are the probability and risk impact of the risk. RI ranges in $(0, 1.41)$. As shown in Fig. 4, RI of risks $R_A:(0.5, 0.5)$, $R_B:(0.1, 1)$, and $R_C:(0.9, 0.2)$ are $RI_A=0.71$, $RI_B=1$, and $RI_C=0.92$ respectively. It is easy to verify that RI satisfies constrains 1-4. As shown in Fig. 6, the contours of RI are convex. Thus, RI also satisfies constraint 5.

Using the risks shown earlier in Table 2 as example, the RI and RE of these risks are shown in Table 5. In this example, we find that RI gives high priority to those risks which have high probability or high impact whereas RE does not.

Table 5: Decision matrix for the example.

Risk	P	I	RE	RI
a	0.9	0.2	0.18	0.92
b	0.2	0.9	0.18	0.92
c	0.4	0.45	0.18	0.60
d	0.45	0.45	0.20	0.64

The most important advantage of RI over RE is that RI matches the risk matrix well while RE not. Fig. 6 shows contours of RI and RE respectively. It is easy to find that the RI contours are convex while the RE contours are concave. RI is more accurate than RE in risk prioritization when the risk matrix used in the project shows a convex pattern.

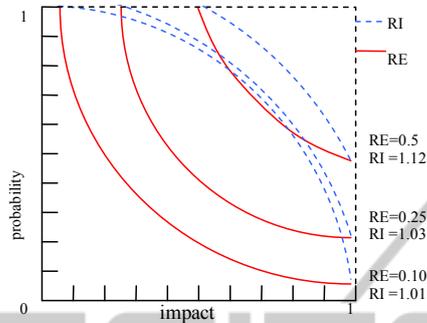


Figure 6: Contours of RI and RE.

In risk management, we apply different strategies for dealing with the risks according to the specific circumstances of the project. Sometimes, the practitioners may consider the risk impact slightly more important than probability. For other cases, the practitioners may consider the probability slightly more important than impact. However, both RI in (7) and RE give equal weight to probability and risk impact. For example, there are two risks, $R_1:(0.9, 0.2)$, and $R_2:(0.2, 0.9)$. If we want to give a higher weight for risk impact in prioritizing risks (that is, R_2 should be assigned a higher priority than R_1), both RI and RE can not distinguish the difference between R_1 and R_2 .

To solve this problem, we can weight probability and risk impact differently, and (7) can be improved as (8).

$$RI = \sqrt{P^2 + (w \cdot I)^2} \tag{8}$$

where, w is a positive real number and is used to weight probability and risk impact. Table 6 shows the functions of w .

Table 6: Functions of variable w in (8).

w	Function	Example		
		w	$R_1:(0.9, 0.2)$	$R_2:(0.2, 0.9)$
$w > 1$	I is more important than P	1.05	$RI_1=0.924$	$RI_2=0.966$
$w = 1$	Both P and I have the same weight	1.00	$RI = 0.922$	$RI_2=0.922$
$0 < w < 1$	P is more important than I	0.95	$RI_1=0.920$	$RI_2=0.878$

From Table 6, we can find that w can be used to weight probability and risk impact differently. In practice, an easy way can be used to determine the value of w . Practitioners first assume the weight of probability, w_p is 1, and then decide the comparative weight of risk impact, w_I , according to the circumstances of the project, and w can be derived from w_I/w_p .

It's easy to verify that (8) satisfies constraints 1-4. Since the introduction of w only change the range of I from $(0, 1]$ to $(0, w]$, and does not change the contours of RI from convex to concave, (8) also satisfies constraint 5.

According to above analysis, RI not only can match the risk matrix shown in Fig. 1 well but also can weight probability and risk impact differently, whereas RE can not.

To include risk events that offer opportunity, the definition of RI is enhanced as (9).

$$RI = \begin{cases} \sqrt{P^2 + (w \cdot I)^2} & \text{if } I > 0 \\ -\sqrt{P^2 + (w \cdot I)^2} & \text{if } I < 0 \end{cases} \tag{9}$$

4.2 A Case Study

To evaluate RI, we conduct an empirical study on applying it to a real project. Project A is a project which enhanced System X to allow customers to submit documents and related information electronically via Internet.

The practitioners identified 25 risks after the project planning and system design phase. Table 7 shows these risks and their probability, impact and risk level. Both the probability and impact of the risk are ranked from 0 to 5 (and can be normalized from 0 to 1 as shown in bracket), where a higher probability value represents a higher chance of occurrence and a higher impact value represents a higher negative effect. According to the risk matrix used by the organization (see Fig. 7), each risk was classified as Low Risk (L), Medium Low Risk (M-), Medium Risk (M), Medium High Risk (M+), or High Risk (H).

In risk prioritization, those risks with the same probability and impact have the same priority. For example, Risk 7, 19, 23 and 24 have the same priority. Thus, we focus on those risks with different probability and/or impact (Risk 1, 2, 3, 5, 7, 8, 10, 11 and 22) in the following prioritization exercise.

In this project, the practitioners use two rules to prioritize the risks.

Table 7: Functions of variable w in (8).

Risk No.	1	2	3	4	5	6	7
Probability	2 (0.4)	3 (0.6)	1 (0.2)	3 (0.6)	1 (0.2)	1 (0.2)	2 (0.4)
Impact	4 (0.8)	4 (0.8)	3 (0.6)	4 (0.8)	4 (0.8)	3 (0.6)	3 (0.6)
Level	M+	H	M-	H	M	M-	M
Risk No.	8	9	10	11	12	13	14
Probability	3 (0.6)	2 (0.4)	4 (0.8)	4 (0.8)	4 (0.8)	4 (0.8)	3 (0.6)
Impact	3 (0.6)	4 (0.8)	4 (0.8)	3 (0.6)	3 (0.6)	4 (0.8)	3 (0.6)
Level	M+	M+	H	H	H	H	M+
Risk No.	15	16	17	18	19	20	21
Probability	3 (0.6)	2 (0.4)	4 (0.8)	3 (0.6)	2 (0.4)	4 (0.8)	3 (0.6)
Impact	3 (0.6)	4 (0.8)	3 (0.6)	4 (0.8)	3 (0.6)	4 (0.8)	3 (0.6)
Level	M+	M+	H	H	M	H	M+
Risk No.	22	23	24	25			
Probability	2 (0.4)	2 (0.4)	2 (0.4)	3 (0.6)			
Impact	2 (0.4)	3 (0.6)	3 (0.6)	4 (0.8)			
Level	M-	M	M	H			

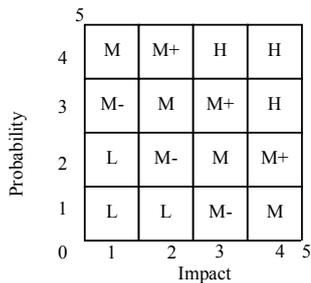


Figure 7: Risk matrix used by the organization.

First, the risk with higher risk level has higher priority. Second, for any two risks, R_a and R_b , in the same risk level, if $(P_a > P_b \text{ and } I_a > I_b)$, or $(P_a > P_b \text{ and } I_a = I_b)$, or $(P_a = P_b \text{ and } I_a > I_b)$ then R_a has higher priority than R_b , otherwise the risk with higher impact value has higher priority as the practitioners pointed out that “impact is usually easier to be estimated than probability and people trend to put more efforts on those higher impact risks”. We prioritize Risk 1, 2, 3, 5, 7, 8, 10, 11 and 22 according to these two rules. The results are shown in column 5 of Table 8 and this prioritized list can serve as “standard list” in the evaluation of RI. Note that the lower priority value indicates higher priority.

We also compute RE, RI and RI ($w=1.05$) with normalized probability and impact value and

prioritize the risks respectively. The RE, RI and RI ($w=1.05$) values of the risks and the priority values which derived from RE, RI and RI ($w=1.05$) respectively are shown in column 6-8 of Table 8. Note that priority values are shown in bracket.

In this project, we can find that RE has more conflicts with the “standard list” than RI. From Table 8, it is easy to find that RI better match the “standard list” than RE, and RI ($w=1.05$) better match the “standard list” than RI. First, the RE does not correctly prioritize the risks in different risk levels for certain cases whereas RI does. For example, Risk 22 with M- risk level should have lower priority than Risk 5. RE wrongly give them the same priority, however RI and RI ($w=1.05$) prioritize them correctly. Second, for the risks in the same risk level, RE does not correctly prioritize them in some cases. For example, Risk 1 should have higher priority than Risk 8, and Risk 5 should also has higher priority than Risk 7. RE wrongly prioritize them, but RI and RI ($w=1.05$) could prioritize them correctly. Further, in this project, the practitioners view the impact more important than the probability. Then the RI with $w > 1$ could satisfy the need of the practitioners to a greater degree. We can find that RI ($w=1.05$) could distinguish Risk 2 and Risk 11, whereas both RE and RI can not. According to above discussion, we can draw a conclusion that, in this project, RI is better than RE in risk prioritization.

Table 8: Priority, RE and RI of 9 risks.

Risk No.	P	I	Level	Priority	RE	RI	RI ($w=1.05$)
10	4 (0.8)	4 (0.8)	H	1	0.64 (1)	1.13 (1)	1.16 (1)
2	3 (0.6)	4 (0.8)	H	2	0.48 (2)	1 (2)	1.03 (2)
11	4 (0.8)	3 (0.6)	H	3	0.48 (2)	1 (2)	1.02 (3)
1	2 (0.4)	4 (0.8)	M+	4	0.32 (5)	0.89 (4)	0.93 (4)
8	3 (0.6)	3 (0.6)	M+	5	0.36 (4)	0.85 (5)	0.87 (5)
5	1 (0.2)	4 (0.8)	M	6	0.16 (7)	0.82 (6)	0.86 (6)
7	2 (0.4)	3 (0.6)	M	7	0.24 (6)	0.72 (7)	0.75 (7)
3	1 (0.2)	3 (0.6)	M-	8	0.12 (9)	0.63 (8)	0.66 (8)
22	2 (0.4)	2 (0.4)	M-	9	0.16 (7)	0.57 (9)	0.58 (9)

Note that RI is not always better than RE. For example, if there is a Risk26:(4, 1) in the project. It should have lower priority than Risk5 and Risk7, that

is, Risk5 > Risk7 > Risk26. RE prioritize them as Risk7 > Risk5 = Risk26, RI prioritize them as Risk5 = Risk26 > Risk7, and RI ($w=1.05$) prioritize them as Risk5 > Risk26 > Risk7. In this case, none of the metrics can prioritize the risks correctly.

5 CONCLUSIONS

In this paper, we identified the research gap in measuring risk impact and some problems with the common indicator RE for prioritizing risks. To bridge this gap, we propose a method for measuring risk impact by using AHP. Then, we propose risk intensity (RI) to prioritize the risks of a project. According to our analysis in section III and IV, our method for assessing risk impact would be more accurate than other existing methods and RI is better than RE in prioritizing risks.

Our study has some limitations. First, although the proposed method for assessing risk impact would be more accurate than other methods according to our analysis, it has not been fully evaluated in practice. Second, we only evaluate RI in one project. The usefulness of RI needs to be confirmed by applying it to more projects in the future. We plan to evaluate the validity of our method for assessing risk impact and the practical usage of RI with more real-life projects in the near future.

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