ROBUST SIMPLE ADAPTIVE H_∞ MODEL FOLLOWING CONTROL DESIGN BY LMIS

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Abstract: An output-feedback direct adaptive control problem is considered for MIMO linear systems with polytopic-type parameter uncertainties and disturbances. The objective is to make the system output follow the output of a system model and to attain guaranteed H_∞ performance of the proposed adaptive control scheme. Sufficient conditions for closed-loop stability, model following performance, and achieving a prescribed bound on the H_∞ disturbance attenuation level are derived, in terms of linear matrix inequalities. A numerical example, taken from the field of flight control, demonstrates the proposed method.

1 INTRODUCTION

A class of direct adaptive controller schemes for continuous-time systems, known as Simple Adaptive Control (SAC), has received considerable attention in the literature (Kaufman et al., 1998)-(Peaucelle and Fradkov, 2008). Robustness of SAC controllers facing polytopic uncertainties has been established (Kaufman et al., 1998)-(Yaesh and Shaked, 2006) allowing application to real engineering problems (see e.g. reference (Barkana, 2005)). The stability of continuous-time SAC is related to the Strictly Positive Real (SPR) property of the controlled plant. The stability of closed-loop SAC is related to the Almost Strictly Positive Real (ASPR) property of the controlled plant. Namely, if a plant is ASPR there exists a static output-feedback gain (possibly parameter-dependent) which stabilizes the plant and makes it SPR. In such a case, SAC stabilizes the closed-loop dynamics and consequently leads to zero tracking errors.

The existing SPR or ASPR results are developed for systems with equal number of inputs and outputs (square systems). The concepts of passivity and passifiability (feedback passivity) are introduced in (Fradkov, 2003) to non-square systems. The latter passification results will be used in this paper.

In (Ben-Yamin et al., 2008), a framework for the combination of optimal H_∞ control and SAC model following has been developed. The idea is to use SAC while satisfying some H_∞ -norm bound on the disturbance attenuation level, and sufficient conditions have been derived for the stability of the closed-loop dynamics of the SAC scheme with a prescribed disturbance attenuation level γ. These sufficient conditions are expressed in terms of Bilinear Matrix Inequalities (BMI), which in many cases are difficult to solve.

A breakthrough achieved in (Peaucelle and Fradkov, 2008) is the formulation of a solution to the regulation problem, for robust adaptive L_2-gain control of polytopic MIMO LTI systems by LMIs rather then by BMIs. Note that in (Peaucelle and Fradkov, 2008) measurement noise was not considered.

The present paper applies and extends the method of (Peaucelle and Fradkov, 2008) in order to solve the problems considered in (Ben-Yamin et al., 2008) by LMI's, including the MIMO case which was not solved in (Ben-Yamin et al., 2008). As in (Ben-Yamin

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et al., 2008), a combination of SAC model following and optimal H_∞ control is applied. The objective is to use SAC while satisfying some H_∞ -norm bound γ. Sufficient conditions are derived for the stability and model following of the closed-loop dynamics of the SAC scheme with disturbance attenuation level γ. These sufficient conditions are expressed in terms of Linear Matrix Inequalities (LMI), which can be solved using Matlab's LMI Toolbox (Gahinet et al., 1995). A numerical flight control example is given which illustrates the method.

1.1 Notation

Throughout the paper the superscript 'T' stands for matrix transposition, $\mathcal{R}^n$ denotes the n dimensional Euclidean space, $\mathcal{R}^{n\times m}$ is the set of all $n \times m$ real matrices, and the notation $P > 0$, for $P \in \mathcal{R}^{n\times n}$, means that P is symmetric and positive definite. T_{zw} denotes the transference from the exogenous disturbance w to the objective function z, $\|T_{zw}\|_\infty$ is its H_∞ -norm and $\|T_{zw}\|_2$ is its H_2-norm. $col\{a,\, b\}$ stands for $[a^T\ b^T]^T$ and $tr\{H\}$ denotes the trace of the matrix H.

2 PRELIMINARIES

2.1 Ideal Strictly Proper System and Ideal Control

Consider the following continuous-time linear system:

$$\begin{aligned} \dot{x}^*(t) &= Ax^*(t) + B_2u^*(t), \quad x^*(0) = 0 \\ y^*(t) &= C_2x^*(t) \end{aligned} \qquad \text{(1a,b)}$$

where $x^*(t) \in \mathcal{R}^n$ is the system state, $y^*(t) \in \mathcal{R}^l$ is the plant output which can be measured and $u^*(t) \in \mathcal{R}^m$ is the control input. A, B_2 and C_2 are constant matrices of appropriate dimensions.

The output of the plant (1) is required to follow the output of the asymptotically stable model:

$$\begin{aligned} \dot{x}_m(t) &= A_mx_m(t) + B_mu_m(t), \quad x_m(0) = 0 \\ y_m(t) &= C_mx_m(t) \end{aligned} \qquad \text{(2a,b)}$$

where $x_m(t) \in \mathcal{R}^q$ is the system state, $y_m(t) \in \mathcal{R}^l$ is the plant output, $u_m(t) \in \mathcal{R}^m$ is the control input and A_m, B_m and C_m are constant matrices of appropriate dimensions. The reference model (2) is used to define the desired input-output behavior of the plant. It is important to note that the dimension of the reference model state may be less than the dimension of the plant state. However, since $y^*(t)$ is to track $y_m(t)$, the number of the model outputs must be equal to number of the plant outputs.

2.2 Hyper Minimum Phase Systems

Following (Fradkov, 2003), we introduce the following notation:

$$\delta(s) = det(sI_n - A), \quad G(s) = C_2(sI_n - A)^{-1}B_2.$$

Let $T \in \mathcal{R}^{m\times l}$ be off full row-rank and define $\Psi(s) = \delta(s)det(TG(s))$, $\Lambda = TC_2B_2$.

Definition 1. *(Fradkov, 2003) The system (1) is called minimum phase if the polynomial $\Psi(s)$ is Hurwitz (its zeros belong to the open left half-plane). It is called Strictly Minimum Phase (SMP) if it is minimum phase and $det\Lambda \neq 0$, and Hyper Minimum Phase (HMP) if it is minimum phase and $\Lambda > 0$.*

Remark 1. *HMP is closely related to the ASPR for square systems. For strictly proper square systems, the conditions which define the ASPR property are similar, and in such cases any minimum phase system of m inputs and m outputs satisfying also $C_2B_2 > 0$ is ASPR (Kaufman et al., 1998).*

Suppose that (1) closed with the feedback

$$u^*(t) = Ky^*(t) + v(t) \qquad (3)$$

where $K \in \mathcal{R}^{m\times l}$ and $v(t)$ is an auxiliary input. The proof of the following Theorem can be found in (Fradkov, 2003).

Theorem 1. *The system (1) is strictly passifiable by the output feedback (3) with fixed matrix K iff the system (1) is HMP.*

We will see in the sequel, as in (Kaufman et al., 1998), that the HMP property allows applying a class of direct adaptive controllers referred to as "simple adaptive controllers". In the sequel we assume that the system (1) is HMP.

2.3 Perfect Following

Perfect Following (PF) is defined as following with zero tracking error, namely

$$y^*(t) = y_m(t)$$

The next lemma determines the relation that exists between the plant's and the model's state vectors.

Lemma 1. *There exist $F(t) \in \mathcal{R}^{n\times q}$ and $G(t) \in \mathcal{R}^{n\times m}$ such that the trajectories of (1) are of the form:*

$$x^*(t) = F(t)x_m(t) + G(t)u_m(t) \qquad (4)$$

Proof: Equation (4) describes n equations with $n \times (q+m)$ variables, thus the existence of $F(t)$ and $G(t)$ is guaranteed for all $0 \leq t < \infty$. QED

Remark 2. *Note that $F(t)$ and $G(t)$ are not actually used; only their existence is required.*

Since the system (1) is HMP, $\Lambda = TC_2B_2 > 0$ so that Λ^{-1} exists. Define

$$\begin{array}{rcl} K_x^*(t) & \equiv & \Lambda^{-1}(TC_mA_m - TC_2AF(t)) \\ K_u^*(t) & \equiv & \Lambda^{-1}(TC_mB_m - TC_2AG(t)). \end{array} \quad (5a,b)$$

If the ideal control $u^*(t)$, defined as:

$$u^*(t) = K_x^*(t)x_m(t) + K_u^*(t)u_m(t), \quad (6)$$

is substituted in (1), we obtain that $y^*(t) = y_m(t)$. The ideal control signal $u^*(t)$ thus achieves PF.

3 PROBLEM FORMULATION

Consider the following continuous-time linear system:

$$\begin{array}{rcl} \dot{x}(t) & = & Ax(t) + B_1w(t) + B_2u(t), \quad x(0) = x_0 \\ y(t) & = & C_2x(t) + D_{21}w(t) \end{array} \quad (7a,b)$$

where $x(t) \in \mathcal{R}^n$ is the system state, $y(t) \in \mathcal{R}^l$ is the plant output which can be measured, $w(t) \in \mathcal{R}^m$ is the exogenous disturbance which is energy bounded and $w(t) \in \mathcal{L}_2$ and $u(t) \in \mathcal{R}^m$ is the control input. A, B_1, B_2, C_2 and D_{21} are constant matrices of appropriate dimensions.

The output of plant (7) is required to follow the output of the asymptotically stable model (2). We define the following objective vector:

$$z(t) = \overline{C}_1 e_y(t) + D_{12}e_u(t) \quad (8)$$

where following (Kaufman et al., 1998), we define

$$e_y(t) = y_m(t) - y(t) = y^*(t) - y(t) \quad (9)$$

$$e_u(t) = u^*(t) - u(t) \quad (10)$$

The matrices $\overline{C}_1$ and D_{12} are weights used to shape the control objective (8). It is required to assure that the plant (7) follows the output of the asymptotically stable model (2) so that the standard H_∞ cost J satisfies

$$J \triangleq ||z||_2^2 - \gamma^2||w||_2^2 < 0 \quad (11)$$

for any $w(t) \neq 0$ and $w(t) \in \mathcal{L}_2$, by employing a SAC controller.

4 SOLUTION

4.1 Control Law

We consider a controller of the form (Kaufman et al., 1998),(Ben-Yamin et al., 2008):

$$u(t) = K^*(t)r(t) - \widetilde{u}(t) \quad (12)$$

where:

$$K^*(t) = \left[\begin{array}{ccc} K_e(t) & K_x^*(t) & K_u^*(t) \end{array} \right] \quad (13)$$

$$r(t) = col\{e_y(t), x_m(t), u_m(t)\} \quad (14)$$

and where $K_e(t) \in \mathcal{R}^m$ is a stabilizing gain which is calculated in the sequel, $K_x^*(t) \in \mathcal{R}^{m\times q}$ and $K_u^*(t) \in \mathcal{R}^m$ are defined in (5), and where $\widetilde{u}(t)$ is an auxiliary input signal which will be defined later. Note that when $e_y(t) = 0$, the controller (12-14) reduces to (6), for $\widetilde{u}(t) = 0$. This control, however, requires calculation of $F(t)$ and $G(t)$ for all $0 \leq t < \infty$ and explicit knowledge of the system dynamics.

Instead, we use the direct adaptive control scheme known as the Simplified Adaptive Control (SAC) (Kaufman et al., 1998) to calculate the gains which lead, in the steady state, to the same control signal that would have been achieved by $K_e(t)$, $K_x^*(t)$ and $K_u^*(t)$. The application of SAC requires neither explicit knowledge of the gains matrix nor exact knowledge of the system dynamics or the exogenous disturbance $w(t)$.

4.2 Simple Adaptive Control Law

Consider the following SAC scheme (Kaufman et al., 1998):

$$u(t) = K(t)r(t) \quad (15)$$

where:

$$\begin{array}{rcl} K(t) & = & \left[\begin{array}{ccc} K_e(t) & K_x(t) & K_u(t) \end{array} \right] \\ \dot{K}_e(t) & = & T_e e_y(t)e_y^T(t) - \phi(t), \quad K_e(0) = 0 \\ \dot{K}_x(t) & = & T_x e_y(t)x_m^T(t), \quad K_x(0) = 0 \\ \dot{K}_u(t) & = & T_u e_y(t)u_m^T(t), \quad K_u(0) = 0 \end{array} \quad (16a\text{-}d)$$

where T_e, T_x and T_u are constant weighting matrices and where

$$\phi(t) = \sigma(tr\{K_e(t)K_e(t)^T\})K_e(t). \quad (17)$$

σ is a scalar function such that:

$$\sigma(\mu) = \left\{ \begin{array}{ll} \frac{\mu-\alpha}{\alpha\beta-\mu} & if \quad \alpha < \mu < \alpha\beta \\ 0 & if \quad otherwise \end{array} \right. \quad (18)$$

where $\alpha > 0$ and $\beta > 1$.

The next two Lemma, which will be required to assure model following of (7) with a disturbance attenuation level γ, are proved in (Peaucelle and Fradkov, 2008).

Lemma 2. *(Peaucelle and Fradkov, 2008) $tr\{K_e(t)K_e(t)^T\} < \alpha\beta$ if $e_y(t)$ is bounded for all $t \geq 0$.*

Lemma 3. *(Peaucelle and Fradkov, 2008) For all F, $K_e(t)$ satisfying $tr\{F^TF\} \leq \alpha$ and $tr\{K_e(t)K_e(t)^T\} < \alpha\beta$, the inequality $tr\{\phi(t)(K_e(t) - F)^T\} \geq 0$ holds.*

We define $\delta(t)=K^*(t)-K(t)$, that is the difference between the ideal gain $K^*(t)$ and the current SAC gain $K(t)$. The control law of (15) can now be expressed by the following choice of the auxiliary control signal $\widetilde{u}(t)$ of (12):

$$\widetilde{u}(t) \quad = \quad \delta(t)r(t). \tag{19}$$

We define the state errors:

$$e_x(t) = x^*(t) - x(t)$$

and using (10) , (12) and (6), we obtain that $e_u(t)$ of (10) is given by

$$e_u(t) \quad = \quad -K_e(t)e_y(t) + \widetilde{u}(t) \tag{20}$$

which, after simple algebraic manipulations, leads to:

$$\begin{aligned} \dot{e}_x(t) &= A\,e_x(t) + B_1\,w(t) + B_2 e_u(t) \\ e_y(t) &= C_2\,e_x(t) + D_{21}\,w(t) \\ z(t) &= C_1\,e_x(t) + D_{11}\,w(t) + D_{12}e_u(t) \end{aligned} \tag{21a-c}$$

where $C_1 \triangleq \overline{C}_1 C_2$, $D_{11} \triangleq \overline{C}_1 D_{21}$ and D_{12} are weights used to shape the control objective (8).

In order to establish the desired model following of (7) with a disturbance attenuation level γ when (11) is satisfied, the asymptotic stability of the error system (21a-b) with the objective vector (21c) should be proven. Stability will be proven here by applying the fact that passivity implies stability (Peaucelle and Fradkov, 2008). To this end, define the signal $y_p(t) = T_e e_y(t) + D e_u(t)$, where D will be defined below, and $\widehat{\beta} = \alpha\beta$. We are now in a position to state the main result of this section.

Theorem 2. *If there exist two scalars $\alpha > 0$ and $\widehat{\beta} > \alpha$ and two matrices F and D, and three matrices $P > 0$, $R > 0$, $G > 0$ such that the following LMI conditions hold*

$$\begin{bmatrix} -R & C_2^T T_e^T - PB_2 \\ * & -I \end{bmatrix} \quad \leq \quad 0. \tag{22}$$

$$\begin{bmatrix} G & F^T \\ F & I \end{bmatrix} \quad \geq \quad 0 \;, \quad tr(G) \leq \alpha \tag{23}$$

$$\begin{bmatrix} L & PB_2 - C_2^T T_e^T & PB_1 + C_2^T \Gamma D_{21} & C_1^T \\ * & -D - D^T & -T_e D_{21} & D_{12}^T \\ * & * & -\gamma^2 I + D_{21}^T \Gamma D_{21} & D_{11}^T \\ * & * & * & -I \end{bmatrix} \leq 0 \tag{24}$$

where

$$L = A^T P + PA + C_2^T \Gamma C_2 + R,$$

$$\Gamma = (\widehat{\beta} I + T_e^T F + F^T T_e)$$

then the adaptive scheme consisting of the plant (7), the control law (15) and the gain adaptation formula (16) satisfy the following
i) It strictly passifies the system (21) with respect to signals $\widetilde{u}(t)$ and $y_p(t)$ in case of zero disturbance $w=0$.
ii) Achieve a disturbance attenuation level γ, for zero initial conditions when $\widetilde{u}(t) = 0$.
In such a case, the controller is given by (15)-(18), where $\beta = \frac{\widehat{\beta}}{\alpha}$.

Proof: We consider the radially-unbounded Lyapunov function candidate

$$\begin{aligned} V(e_x(t), K_e(t)) \quad = \quad & \tfrac{1}{2} e_x^T(t) P e_x(t) + \\ & \tfrac{1}{2} tr\{(K_e(t) - F)(K_e(t) - F)^T\}. \end{aligned} \tag{25}$$

Note that $V(0,F) = 0$ and $V(e_x(t),K_e(t)) > 0$ for all $\{e_x(t),K_e(t)\} \neq \{0,F\}$. Note also that $V(e_x(t),K_e(t)) \to \infty$ if $\|e_x(t)\| \to \infty$ or $\|K_e(t)\| \to \infty$. Using (20), the derivative of (25) along the trajectories of (21) is given by

$$\begin{aligned} \dot{V}(t) \quad = \quad & e_x^T(t) P(Ae_x(t) + B_1 w(t) - B_2 K_e(t) e_y(t) + \\ & B_2 \widetilde{u}(t)) + tr\{\dot{K}_e(t)(K_e(t) - F)^T\}. \end{aligned} \tag{26}$$

Using Schur complement argument, we can rewrite the inequalities of (24) as:

$$\begin{bmatrix} L + C_1^T C_1 & PB_2 - C_2^T T_e^T + C_1^T D_{12} & \Psi_1 \\ * & -D - D^T + D_{12}^T D_{12} & \Psi_2 \\ * & * & \Psi_3 \end{bmatrix} \leq 0$$

where

$$\Psi_1 = PB_1 + C_2^T \Gamma D_{21} + 2C_1^T D_{11}$$
$$\Psi_2 = -T_e D_{21} + 2D_{12}^T D_{11}$$
$$\Psi_3 = -\gamma^2 I + D_{11}^T D_{11} + D_{21}^T \Gamma D_{21}$$

Pre and post multiply this inequality by $\begin{bmatrix} e_x^T(t) & e_u^T(t) & w^T(t) \end{bmatrix}$ and its transpose respectively, to get

$$\begin{aligned} & 2e_x^T(t) P(Ae_x(t) + B_1 w(t) + B_2 \widetilde{u}(t)) \\ & \quad -\gamma^2 w(t)^T w(t) + z^T(t) z(t) \\ & +e_y^T(t) \Gamma e_y(t) - 2y_P^T(t) \widetilde{u}(t) + e_x^T(t) R e_x(t) \quad \leq \quad 0 \end{aligned}$$

and hence

$$\begin{aligned} \dot{V}(t) \quad \leq \quad & e_x^T(t) PB_2 K_e(t) e_y(t) + y_P^T(t) \widetilde{u}(t) \\ & + \tfrac{1}{2}(\gamma^2 w(t)^T w(t) - z^T(t) z(t)) \\ & - \tfrac{1}{2} e_y^t(t) \Gamma e_y(t) - \tfrac{1}{2} e_x^T(t) R e_x(t)) \\ & + tr\{\dot{K}_e(t)(K_e(t) - F)^T\} \end{aligned}$$

Pre and post multiplying (22) by $\begin{bmatrix} e_x^T(t) & -e_y^T(t) K_e^T(t) \end{bmatrix}$ and its transpose, respectively, the following is obtained:

$$\begin{aligned} & e_x(t)^T PB_2 K_e(t) e_y(t) - \tfrac{1}{2} e_x(t)^T R e_x(t) \leq \\ & e_y(t)^T K_e(t)^T T_e C_2 e_x(t) + \tfrac{1}{2} e_y(t)^T K_e(t)^T K_e(t) e_y(t) \end{aligned}$$

Combining the last two inequalities, we find that the derivative of $V(t)$ satisfies the following:

$$\begin{aligned} \dot{V}(t) \quad \leq \quad & y_P^T(t) \widetilde{u}(t) + \tfrac{1}{2}(\gamma^2 w(t)^T w(t) - z^T(t) z(t)) \\ & + \tfrac{1}{2} e_y^T(t)(K_e(t)^T K_e(t) - \widehat{\beta} I) e_y(t) + \\ & e_y^T(t)(K_e(t) - F)^T T_e e_y(t) + \\ & tr\{\dot{K}_e(t)(K_e(t) - F)^T\} \end{aligned} \tag{27}$$

Since $tr\{M_1M_2\} = tr\{M_2M_1\}$, we obtain that: $e_y^T(t)(K_e(t)-F)^T T_e e_y(t) = tr\{T_e e_y(t) e_y^T(t)(K_e(t)-F)^T\}$. Therefore, using (16b), one obtains

$$e_y^T(t)(K_e(t)-F)^T T_e e_y(t) + tr\{\dot{K}_e(t)(K_e(t)-F)^T\} = -tr\{\phi(t)(K_e(t)-F)^T\}$$

which is negative due to Lemma 2. Moreover, Lemma 1 guarantees that $tr(K_e^T K_e) \leq \alpha\beta$, and using the fact that $\widehat{\beta} = \alpha\beta$, hence $K_e(t)^T K_e(t) - \widehat{\beta} I \leq 0$. The derivative of the Lyapunov function along the closed-loop trajectories is therefore, for all $t \geq 0$, bounded by:

$$\dot{V}(t) \leq y_P^T(t)\widetilde{u}(t) + \frac{1}{2}(\gamma^2 w(t)^T w(t) - z^T(t)z(t)). \tag{28}$$

For $w(t) = 0$, taking the integral of (28) over time proves strict passivity of the system. For $\widetilde{u}(t) = 0$ and zero initial conditions, taking the integral over time leads to the standard interpretation of γ as a bound on the H_∞ norm of the system. From the definition of $\widetilde{u}(t)$ it follows that $\widetilde{u}(t) = 0$ if

$$(K_x^*(t) - K_x(t))x_m(t) + (K_u^*(t) - K_u(t))u_m(t) = 0. \tag{29}$$

Note that model following does not require $K^*(t) = K(t)$; it suffices that the LHS of (29) vanishes. Note also that $K^*(t)$ may not be unique. QED

Remark 3. *The parameters γ, α and $\widehat{\beta} = \alpha\beta$ appear distinctly in LMIs (22-24). The H_∞ performance of the closed-loop is represented by γ, whereas $\widehat{\beta} - \alpha$ is the allowed dynamic range of the SAC gain. Since a larger gain range (intuitively) can cope with larger system performance variations, it may be said that, in principle, we are faced with a Pareto optimal performance versus robustness problem. The minimization of γ traded the maximization of $\widehat{\beta} - \alpha$. (In fact, separater minimization of α and maximization of $\widehat{\beta}$)*

Remark 4. *LMI's (22-24) are affine in the system matrices, therefore Theorem 1 can be used to derive a criterion that will guarantee the stability in the case where the system matrices are not exactly known and they reside within a given polytope. Denoting*

$$\Omega = \{\ A \quad B_1 \quad B_2\ \} \tag{30}$$

where $\Omega \in Co\{\Omega_j,\ j = 1, ...N\}$, namely,

$$\Omega = \sum_{j=1}^{N} f_j \Omega_j \quad \text{for some} \quad 0 \leq f_j \leq 1, \ \sum_{j=1}^{N} f_j = 1 \tag{31}$$

where the vertices of the polytope are described by

$$\Omega_j = \left\{\ A^{(j)} \quad B_1^{(j)} \quad B_2^{(j)}\ \right\}, \quad j = 1, 2..., N. \tag{32}$$

Multiplying (22-24) by f_j and summing over $j = 1, 2, ..., N$, it is readily obtained that the stability and performance conditions are satisfied over Ω.

5 NUMERICAL EXAMPLES - MIMO LATERAL CONTROL FOR A 747 JET TRANSPORT

In this section we present a numerical example to demonstrate the application of the theory developed above. Consider a modified version of a 747 aircraft using the classical control design features in the Control System Toolbox of MATLAB (Mathworks, 1995). The example is modified to include disturbances and deals with bank angle and yaw rate control (MIMO case) of the airplane.

The example describes the dutch roll mode of a 747 jet transport. A simplified trim model of the aircraft during cruise flight at $MACH = 0.8$ and $H = 40,000 ft$ has four states: sideslip angle $[rad]$, bank angle $[rad]$, yaw rate $[rad/sc]$, roll rate $[rad/sec]$. The plant inputs are rudder $[rad]$ and aileron $[rad]$ deflections. We assume that the bank angle, the yaw rate and the roll rate are measured. The plant of (7) is described by the following matrices:

$$A = \begin{bmatrix} -0.0558 & -0.9968 & 0.0802 & 0.0415 \\ 0.5980 & -0.1150 & -0.0318 & 0 \\ -3.0500 & 0.3880 & -0.4650 & 0 \\ 0 & 0.0805 & 1.0 & 0 \end{bmatrix}$$

$$B_1 = B_2 = \begin{bmatrix} 0.0073 & 0 \\ 0.4750 & 0.0077 \\ 0.1530 & 0.1430 \\ 0 & 0 \end{bmatrix},$$

$$C_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0.2 & 10 \end{bmatrix}, D_{21} = \begin{bmatrix} 0.06 & 0 \\ 0 & 0.06 \end{bmatrix},$$

The weights matrices of the control objective (8) are chosen as:

$$\overline{C}_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, D_{12} = \begin{bmatrix} 0.001 & 0 \\ 0 & 0.001 \end{bmatrix}.$$

Using Matlab's LMI Toolbox (Gahinet et al., 1995), we find that the LMI's (22-24) are feasible for:

$$D = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},$$

$$P = \begin{bmatrix} 1.52 & -0.05 & 0.03 & 0.01 \\ -0.05 & 1.62 & -0.01 & 0.01 \\ 0.03 & -0.01 & 0.05 & 0.02 \\ 0.01 & 0.01 & 0.02 & 0.02 \end{bmatrix}$$

$$R = 10^6 \begin{bmatrix} 5.44 & -0.01 & -0.001 & 0.002 \\ -0.004 & 5.450 & -0.001 & -0.003 \\ -0.001 & -0.001 & 5.456 & 0.001 \\ 0.002 & -0.003 & 0.001 & 5.445 \end{bmatrix}$$

$$F = -3.5, G = 12.7, \alpha = 12.8, \beta = 1.4, \gamma = 0.6$$

The chosen reference system is:

$$A_m = \begin{bmatrix} -3 & -2.5 & 0 & 0 \\ 4 & 0 & 0 & 0 \\ 0 & 0 & -3 & -2.5 \\ 0 & 0 & 4 & 0 \end{bmatrix}, \; B_m = \begin{bmatrix} 2 & 0 \\ 0 & 0 \\ 0 & 2 \\ 0 & 0 \end{bmatrix},$$

$$C_m = \begin{bmatrix} 0 & 1.25 & 0 & 0 \\ 0 & 0 & 0 & 1.25 \end{bmatrix}$$

Our aim is to make the plant outputs track the reference model outputs for bank angle step response, say,

$$w(t) = \begin{bmatrix} 0.1 \\ 1 \end{bmatrix} \sin(10t) e^{-0.01t}.$$

Note that aircraft coordinated turns are accompanied by non-zero yaw rates and the model includes this feature. The relation between the yaw rate command (r_{com}) and the bank angle command (ϕ_{com}) is:

$$r_{com} = \frac{g \tan(\phi_{com})}{TAS}$$

where $g = 9.81$ $[m/sec^2]$ is a the earth's gravitational and $TAS = 235$ $[m/sec]$ is a true air speed of the aircraft. The simulation results are given in Fig 1-2. Fig. 1a describes the yaw rate and yaw rate command, Fig. 1b the bank angle and bank angel command, Fig 2.a the rudder command and Fig 2.b the aileron command. Evidently, the yaw rate and the bank angel successfully tracks their commands by the proposed control law (15) and the gain adaptation formula (16).

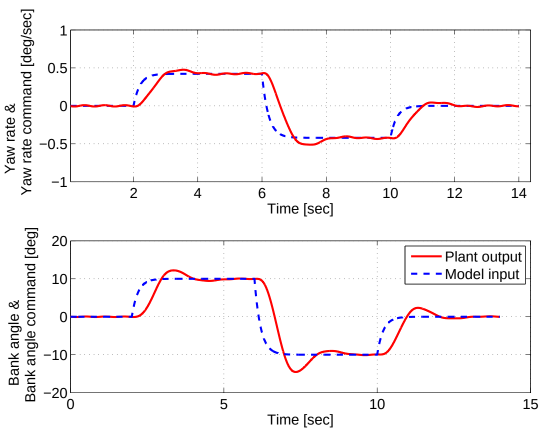

Figure 1: Simulation result: yaw rate and bank angle(command and measured).

6 CONCLUSIONS

In this note, the existing model following theory of simple adaptive control for continuous-time systems is generalized for MIMO systems. The results assure closed-loop stability and best disturbance attenuation level γ. The conditins are formulated in LMI (rather then BMI) form, and are shown to be valid also for systems with polytopic uncertainties.

The design method is simple and the results are most encouraging. The results are illustrated via a numerical example from the field of flight control and encourage further research of the effects of exogenous disturbances and measurement noise for measurements delayed MIMO systems.

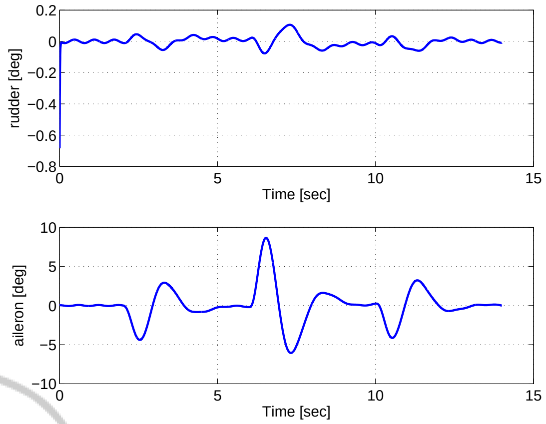

Figure 2: Simulation results: The rudder and the aileron [deg].

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