HIGH-FREQUENCY ANALYSIS OF PHASE-LOCKED LOOP AND PHASE DETECTOR CHARACTERISTIC COMPUTATION

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Abstract: Problems of rigorous mathematical analysis of PLL are discussed. An analytical method for phase detector characteristics computation is suggested and new classes of phase detector characteristics are computed. Effective methods for nonlinear analysis of PLL are discussed.

1 INTRODUCTION

Phase-locked loop (PLL) systems were invented in the 1930s-1940s (De Bellescize, 1932; Wendt & Fredentall, 1943) and were widely used in radio and television (demodulation and recovery, synchronization and frequency synthesis). Nowadays PLL can be produced in the form of single integrated circuit and various modifications of PLL are used in a great amount of modern electronic applications (radio, telecommunications, computers, and others).

At present there are several types of PLL (classical PLL, ADPLL, DPLL, and others), intended for the operation with different types of signals (sinusoidal, impulse, and so on). In addition, it is also used different realizations of PLL, which are distinct from each other according to the principles of operation and realization of main blocks.

For the sake of convenience of description, in PLL the following main functional blocks are considered: phase detector (PD), low-pass filter (LPF), and voltage-controlled oscillator (VCO). Note that such a partition into functional blocks often turns out to be conditional, since in many cases in particular physical realization it is impossible to point out the strict boundaries between these blocks. However these blocks can be found in each PLL.

The general PLL operation consists in the generation of an electrical signal (voltage), a phase of which is automatically tuned to the phase of input (reference) signal, i.e. PLL eliminates misphasing (clock skew) between two signals. For this purpose the reference signal and the tunable signal of voltage-controlled oscillator are passed through a special nonlinear element — phase detector (PD). The phase detector produces an error correction signal, corresponding to phase difference of two input signals. For the discrimination of error correction signal, a signal at the output of phase detector is passed through low-pass filter (LPF). The error correction signal, obtained at the output of filter, is used for the frequency control of tunable oscillator, the output of which enters a phase detector, providing thus negative feedback.

The most important performance measure of PLL is the capture range (i.e. a maximal mistuning range of VCO, in which a closed contour of PLL stabilizes a frequency of VCO) and a locking speed (speed of frequency adjustment).

Thus, when designing PLL systems, an important task is to determine characteristics of system (involving parameters of main blocks) providing required characteristics of operation of PLL.

To solve this problem, it is used real experiments with concrete realization of PLL as well as the analytical and numerical methods of analysis of mathematical models of PLL. These tools are used for the obtaining of stability of required operating modes, the estimates of attraction domain of such modes, and the time estimates of transient processes.

Remark, however, that for the strict mathematical analysis of PLL it should be taken into account the fact that the above principles of operation of PLL result in the substantial requirements:

✓ *construction of adequate nonlinear mathemat-*

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ical models (since PLL contains nonlinear elements) in signal space and phase-frequency space

and

✓ *justification of the passage between these models (since PLL translates the problem from signal response to phase response and back again).*

Despite this, as noted by well-known PLL expert Danny Abramovitch in his keynote talk at American Control Conference ACC'2002 (Abramovitch, 2002), the main tendency in a modern literature (see, e.g., (Egan, 2000; Best, 2003; Kroupa, 2003; Razavi, 2003)) on analysis of stability and design of PLL is the use of simplified linearized models and the application of the methods of linear analysis, a rule of thumb, and simulation.

However it is known that the application of linearization methods and linear analysis for control systems can lead to untrue results (e.g., Perron effects of Lyapunov exponent sign inversion (Leonov & Kuznetsov , 2007), counterexamples to Aizerman's conjuncture and Kalman's conjuncture on absolute stability, harmonic linearization and filter hypothesis (Leonov et al., 2010[2])) and requires special justifications. Also simple numerical analysis can not reveal nontrivial regimes (e.g., semi-stable or nested limit cycles, hidden oscillations and attractors (Gubar, 1961; Kuznetsov & Leonov, 2008; Leonov et al., 2010[2]; Leonov et. al., 2010[1]; Leonov et. al., 2011)).

2 NONLINEAR MATHEMATICAL MODELS OF PLL

Various methods for analysis of phase-locked loops are well developed by engineers and considered in many publications (see, e.g., (Viterbi, 1966; Gardner, 1966; Lindsey, 1972; Shakhgildyan & Lyakhovkin, 1972)), but the problems of construction of adequate nonlinear models and nonlinear analysis of such models are still far from being resolved turn out to be difficult. and require to use special methods of qualitative theory of differential, difference, integral, and integro-differential equations (Leonov et al., 1996; Suarez & Quere, 2003; Margaris, 2004; Leonov, 2006; Kudrewicz & Wasowicz, 2007; Leonov et al., 2009).

In the present paper some approaches to the nonlinear analysis of PLL are described. Nonlinear mathematical models of high-frequency oscillations are presented.

To construct an adequate nonlinear mathematical model of PLL in phase space it is necessary to find the characteristic of phase detector. The inputs of PD are high-frequency signals of reference and tunable oscillators and the output contains a low-frequency error correction signal, corresponding to a phase difference of input signals. For the suppression of high-frequency component of the output of PD (if such component exists) the low-pass filters are applied. The dependence of the signal at the output of PD (in phase space) on phase difference of signals at the input of PD is the characteristic of PD. This characteristic depends on the realization of PD and the types of signals at the input. Characteristics of the phase detector for standard types of signal are well-known to engineers (Viterbi, 1966; Shakhgildyan & Lyakhovkin, 1972; Abramovitch, 2002).

Further, on the examples of classical PLL with a phase detector in the form of multiplier, we consider general principles of computing phase detector characteristics for different types of signals based on a rigorous mathematical analysis of high-frequency oscillations (Leonov & Seledzhi , 2005a; Leonov, 2008; Kuznetsov et al., 2008; Kuznetsov et al., 2009[1]; Kuznetsov et al., 2009[2]; Leonov et al., 2010[3]).

2.1 Description of Classical PLL in the Signal Space

Consider classical PLL at the level of electronic realization (Fig. 1)

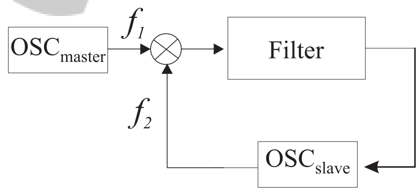

Figure 1: Block diagram of PLL at the level of electronic realization.

Here $\mathrm{OSC}_{\mathrm{master}}$ is a master oscillator, $\mathrm{OSC}_{\mathrm{slave}}$ is a slave (tunable voltage-controlled) oscillator, which generates oscillations $f_j(t)$ with high-frequencies $\omega_j(t)$.

Block $\otimes$ is a multiplier of oscillations of $f_1(t)$ and $f_2(t)$ and the signal $f_1(t)f_2(t)$ is its output. The relation between the input $\xi(t)$ and the output $\sigma(t)$ of linear filter has the form

$$\sigma(t) = \alpha_0(t) + \int\limits_0^t \gamma(t-\tau)\xi(\tau)\,d\tau. \tag{1}$$

Here $\gamma(t)$ is an impulse transient function of filter, $\alpha_0(t)$ is an exponentially damped function, depending on the initial data of filter at moment $t = 0$.

In the simplest ideal case, when

$$f_1 = \sin(\omega_1),\, f_2 = \cos(\omega_2)$$
$$f_1 f_2 = [\sin(\omega_1 + \omega_2) + \sin(\omega_1 - \omega_2)]/2,$$

standard engineering assumption is that the filter removes the upper sideband with frequency from the input but leaves the lower sideband without change. Thus it is assumed that the filter output is

$$\frac{1}{2}\sin(\omega_1 - \omega_2).$$

Here to avoid these non-rigorous arguments we consider mathematical properties of high-frequency oscillations.

2.2 Computation of Phase Detector Characteristic

A high-frequency property of signals can be reformulated as the following condition. Consider a large fixed time interval $[0,T]$, which can be partitioned into small intervals of the form

$$[\tau,\tau+\delta],\ \tau \in [0,T],$$

where the following relations

$$|\gamma(t)-\gamma(\tau)| \le C\delta,\ |\omega_j(t)-\omega_j(\tau)| \le C\delta, \quad \forall t \in [\tau,\tau+\delta],\ \ \forall\tau \in [0,T], \tag{2}$$

$$|\omega_1(\tau)-\omega_2(\tau)| \le C_1,\ \forall \tau \in [0,T], \tag{3}$$

$$\omega_j(t) \ge R,\ \forall t \in [0,T] \tag{4}$$

are satisfied.

We shall assume that δ is small enough relative to the fixed numbers T, C, C_1 and R is sufficiently large relative to the number δ : $R^{-1} = O(\delta^2)$.

The latter means that on small intervals $[\tau,\tau+\delta]$ the functions $\gamma(t)$ and $\omega_j(t)$ are "almost constant" and the functions $f_j(t)$ on them are rapidly oscillating. Obviously, such a condition occurs for high-frequency oscillations.

Consider now harmonic oscillations

$$f_j(t) = A_j \sin(\omega_j(t)t+\psi_j), \quad j=1,2, \tag{5}$$

where A_j and ψ_j are certain numbers, $\omega_j(t)$ are differentiable functions.

Consider two block diagrams shown in Fig. 2 and Fig. 3.

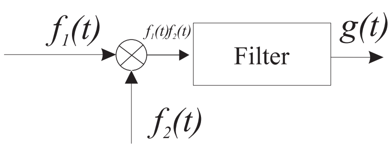

Figure 2: Multiplier and filter.

In Fig. 3 $\theta_j(t) = \omega_j(t)t+\psi_j$ are phases of oscillations $f_j(t)$, PD is a nonlinear block with the characteristic $\varphi(\theta)$. The phases $\theta_j(t)$ are the inputs of PD block and the output is the function $\varphi(\theta_1(t)-\theta_2(t))$.

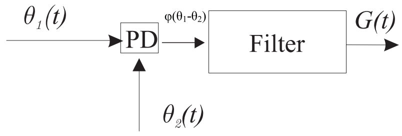

Figure 3: Phase detector and filter.

A shape of phase detector characteristic is based on a shape of input signals.

The signals $f_1(t)f_2(t)$ and $\varphi(\theta_1(t)-\theta_2(t))$ are inputs of the same filters with the same impulse transient function $\gamma(t)$. The filter outputs are the functions $g(t)$ and $G(t)$, respectively.

A classical PLL synthesis for the sinusoidal signals is based on the following result (Viterbi, 1966):

If conditions (2)–(4) are satisfied and

$$\varphi(\theta) = \frac{1}{2}A_1A_2\cos\theta,$$

then for the same initial data of filter, the following relation

$$|G(t)-g(t)| \le C_2\delta,\ \forall t \in [0,T]$$

is satisfied. Here C_2 is a certain number being independent of δ.

But what could be done for other types of signal?

Consider now signals in the following form of Fourier series

$$f_1(t) = \sum_{i=1}^{\infty} a_i \sin(i\theta_1(t)),\ f_2(t) = \sum_{j=1}^{\infty} b_j \sin(j\theta_2(t)), \tag{6}$$

where

$$a_k = O\left(\frac{1}{k}\right), \quad b_k = O\left(\frac{1}{k}\right), \qquad k = 1,2,\ldots.$$

Let functions $f_1(t)$ and $f_2(t)$ are integrable and bounded on each of the intervals of length δ.

Then the following assertion is valid

Theorem 1 . *If conditions (2)–(4) are satisfied and*

$$\varphi(\theta_1-\theta_2) = \sum_{l=1}^{\infty} \frac{a_l b_l}{2}\cos(l(\theta_1-\theta_2)), \tag{7}$$

then for the same initial states of filter the following relation

$$|G(t)-g(t)| \le C_3\delta, \quad \forall t \in [0,T] \tag{8}$$

is valid.

Proof. Consider a decomposition of the interval $[0,T]$ into the δ length time intervals. Then using (2) we obtain

$$g(t) - G(t) = \sum_{k=0}^{m} \gamma(t-k\delta) \int_{k\delta}^{(k+1)\delta} \Big[f^1\big(\theta^1(s)\big) f^2\big(\theta^2(s)\big) - \varphi\big(\theta^1(s) - \theta^2(s)\big) \Big] ds + O(\delta). \tag{9}$$

Because the frequencies are almost constant in the δ-intervals (3), we could introduce $\theta_k^p(s)$

$$\theta_k^p(s) = \omega^p(k\delta)s + \psi^p, \ p \in \{1,2\}. \tag{10}$$

Lemma 1. *Assuming conditions (2)–(4) the phases* $\theta^p(t)$ *could be replaced with* $\theta_k^p(t)$

$$\begin{aligned} &\int_{k\delta}^{(k+1)\delta} \varphi\big(\theta^1(s) - \theta^2(s)\big) = \\ &\int_{k\delta}^{(k+1)\delta} \varphi\big(\theta_k^1(s) - \theta_k^2(s)\big) ds + O(\delta), \\ &\int_{k\delta}^{(k+1)\delta} f^1\big(\theta^1(s)\big) f^2\big(\theta^2(s)\big) = \\ &\int_{k\delta}^{(k+1)\delta} f^1\big(\theta_k^1(s)\big) f^2\big(\theta_k^2(s)\big) ds + O(\delta), \end{aligned} \tag{11}$$

Then, usign Lemma 1, equation (9) can be rewritten

$$\begin{aligned} &g(t) - G(t) = \\ &\sum_{k=0}^{m} \gamma(t-k\delta) \int_{[k\delta,(k+1)\delta)} \Big[f^1\big(\theta_k^1(s)\big) f^2\big(\theta_k^2(s)\big) \\ &- \varphi\big(\theta_k^1(s) - \theta_k^2(s)\big) \Big] ds + O(\delta) \end{aligned} \tag{12}$$

Lemma 2. *For the neighborhoods* $W_{\varepsilon,k}$ *of discontinuity points, there is a number* M*, such that*

$$\begin{aligned} &g(t) - G(t) = \sum_{k=0}^{m} \gamma(t-k\delta) \\ &\int_{[k\delta,(k+1)\delta]\setminus W_{\varepsilon,k}} \Big[\Big(\sum_{i=1}^{M} a_i^1 \sin\big(i\theta_k^1(s)\big)\Big) \\ &\Big(\sum_{j=1}^{M} a_j^2 \sin\big(j\theta_k^1(s)\big)\Big) - \varphi\big(\theta_k^1(s) - \theta_k^2(s)\big) \Big] ds + O(\delta). \end{aligned}$$

Lemma 2 implies

$$\begin{aligned} &g(t) - G(t) = \sum_{k=0}^{m} \gamma(t-k\delta) \int_{[k\delta,(k+1)\delta]\setminus W_{\varepsilon,k}} \Big[\\ &\sum_{i=1}^{M}\sum_{j=1}^{M} \Big(a_i^1 \sin\big(i\theta_k^1(s)\big)\Big) \Big(a_j^2 \sin\big(j\theta_k^2(s)\big)\Big) - \\ &- \varphi\big(\theta_k^1(s) - \theta_k^2(s)\big) \Big] ds + O(\delta). \end{aligned} \tag{13}$$

It's obvious, that

$$\begin{aligned} &\sin\big(i\theta_k^1(s)\big) \sin\big(j\theta_k^2(s)\big) = \\ &\frac{1}{2}\Big(\cos\big(i\theta_k^1(s) - j\theta_k^2(s)\big) - \cos\big(i\theta_k^1(s) + j\theta_k^2(s)\big)\Big) \end{aligned} \tag{14}$$

Lemma 3. *Assuming conditions (2)–(4) the following equations can be obtained*

$$\int_{k\delta}^{(k+1)\delta} \frac{1}{q} \cos\big(p(Rs + \psi)\big) ds = \frac{O(\delta^2)}{pq}, \tag{15}$$

Using (13),(4),(14) and Lemma 3, the theorem statement can be obtained

$$g(t) - G(t) = O(\delta)\ . \tag{16}$$

■

This result could be easily extended to the case of full Fourier series and allows one to calculate the phase detector characteristic in the following standard cases of signals (Kuznetsov et al., 2010).

Example 1. Two sign signals

$$\begin{aligned} &f_k(t) = A_k \mathrm{sign}\ \sin(\theta_k(t)) = \\ &= \frac{4A_k}{\pi} \sum_{n=0}^{\infty} \frac{1}{2n+1} \sin((2n+1)(\omega_k(t)t + \psi_k)), \quad k = 1,2 \end{aligned}$$

$$\varphi(\theta_1 - \theta_2) = \frac{8A_1A_2}{\pi^2} \sum_{n=0}^{\infty} \frac{1}{(2n+1)^2} \cos(\theta_1 - \theta_2)$$

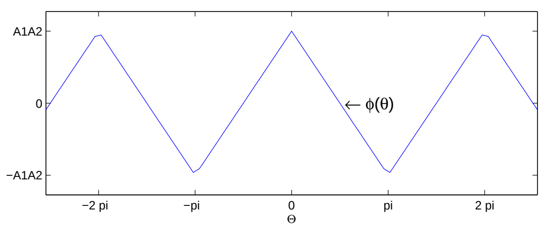

Figure 4: Phase detector characteristic $\varphi(\theta)$ for two sign signals.

Thus, here phase detector characteristic $\phi(\theta)$ corresponds to 2π-periodic function

$$A_1A_2\left(1-\frac{2|\theta|}{\pi}\right), \quad \text{for} \quad \theta \in (-\pi,\pi]. \tag{17}$$

Example 2. Sin signal and sign signal

$$\begin{aligned} &f_1(t) = A_1 \sin(\theta_1(t)) \\ &f_2(t) = A_2 \text{sign} \sin(\theta_2(t)) = \\ &= \tfrac{4A_2}{\pi} \sum_{n=0}^{\infty} \tfrac{1}{2n+1} \sin((2n+1)(\omega_2(t)t+\psi_2)) \\ &\varphi(\theta_1-\theta_2) = \tfrac{2A_1A_2}{\pi}\cos(\theta_1-\theta_2) \end{aligned}$$

Example 3. Triangle wave signals.

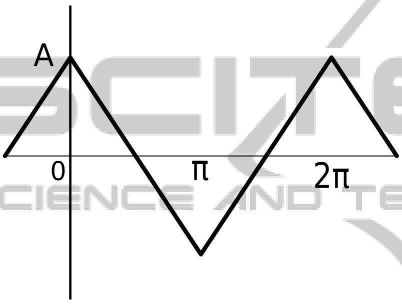

Figure 5: Triangle-wave signal.

$$f_k(t) = A_k \sum_{i=0}^{\infty} \frac{1}{(2i-1)^2} \sin\big((2i-1)\theta_k(t)\big) \tag{18}$$

$$\varphi(\theta_1-\theta_2) = A_1A_2 \sum_{l=1}^{\infty} \frac{1}{(2l-1)^4} \cos\big((2l-1)(\theta_1-\theta_2)\big) \tag{19}$$

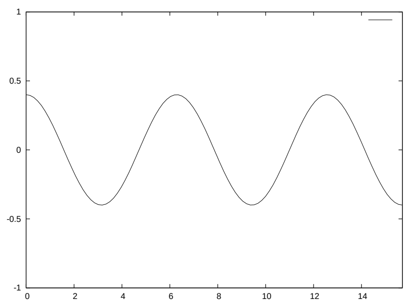

Figure 6: Phase detector characteristic $\varphi(\theta)$ for triangle signals.

2.3 PLL Equations in Phase-frequency Space

From Theorem 1 it follows that block-scheme of PLL in signal space (Fig. 1) can be asymptotically changed (for high-frequency generators) to a block-scheme at the level of frequency and phase relations (Fig. 7).

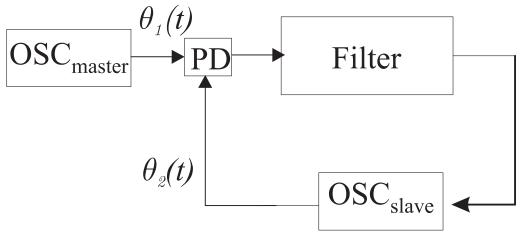

Figure 7: Phase-locked loop with phase detector.

Here PD is a phase detector with corresponding characteristics. Thus, here on basis of asymptotical analysis of high-frequency pulse oscillations characteristics of phase detector can be computed.

Characteristic $\varphi(\theta)$, computed in Examples 1 and 2, tends to zero if $\theta = (\theta_1-\theta_2)$ tends to $\pi/2$, so one can proceed to stability analysis (Leonov, 2006; Leonov et al., 2009) of differential (or difference) equations depend on misphasing θ.

Let us make a remark necessary for derivation of differential equations of PLL.

Consider a quantity

$$\dot{\theta}_j(t) = \omega_j(t) + \dot{\omega}_j(t)t.$$

For the well-synthesized PLL such that it possesses the property of global stability, we have exponential damping of the quantity $\dot{\omega}_j(t)$:

$$|\dot{\omega}_j(t)| \le Ce^{-\alpha t}.$$

Here C and α are certain positive numbers independent of t. Therefore, the quantity $\dot{\omega}_j(t)t$ is, as a rule, small enough with respect to the number R (see conditions (3)– (4)). From the above we can conclude that the following approximate relation $\dot{\theta}_j(t) \approx \omega_j(t)$ is valid. In deriving the differential equations of this PLL, we make use of a block diagram in Fig. 7 and exact equality

$$\dot{\theta}_j(t) = \omega_j(t). \tag{20}$$

Note that, by assumption, the control law of tunable oscillators is linear:

$$\omega_2(t) = \omega_2(0) + LG(t). \tag{21}$$

Here $\omega_2(0)$ is initial frequency of tunable oscillator, L is a certain number, and $G(t)$ is a control signal, which is a filter output (Fig. 3). Thus, the equation of PLL is as follows

$$\dot{\theta}_2(t) = \omega_2(0) + L\left(\alpha_0(t) + \int_0^t \gamma(t-\tau)\varphi\big(\theta_1(\tau)-\theta_2(\tau)\big)d\tau\right).$$

Assuming that the master oscillator is such that $\omega_1(t) \equiv \omega_1(0)$, we obtain the following relations for

PLL

$$\left(\theta_1(t)-\theta_2(t)\right)'+L\left(\alpha_0(t)+\int_0^t\gamma(t-\tau)\varphi\left(\theta_1(\tau)-\theta_2(\tau)\right)\mathrm{d}\tau\right)$$
$$=\omega_1(0)-\omega_2(0). \tag{22}$$

This is an equation of standard PLL. Note, that if filter (1) is an integrating filter with the transfer function $(p+\alpha)^{-1}$

$$\dot{\sigma}+\alpha\sigma=\varphi(\theta)$$

then for $\phi(\theta)=\cos(\theta)$ in place of of equation (22) from (20) and (21) we have pendulum-like equation (Leonov & Smirnova, 1996; Leonov et al., 1996)

$$\ddot{\tilde{\theta}}+\alpha\dot{\tilde{\theta}}+L\sin\tilde{\theta}=\alpha\left(\omega_1(0)-\omega_2(0)\right) \tag{23}$$

with $\tilde{\theta}=\theta_1-\theta_2+\frac{\pi}{2}$. Thus, if here phases of the input and output signals mutually shifted by $\pi/2$, then the control signal $G(t)$ equals zero.

Arguing as above, we can conclude that in PLL it can be used the filters with transfer functions of more general form $K(p)=a+W(p)$, where a is a certain number, $W(p)$ is a proper fractional rational function. In this case in place of equation (22) we have

$$\left(\theta_1(t)-\theta_2(t)\right)'+L\Bigg[a\varphi\left(\theta_1(t)-\theta_2(t)\right)+\alpha_0(t)+$$
$$+\int_0^t\gamma(t-\tau)\varphi\left(\theta_1(\tau)-\theta_2(\tau)\right)\mathrm{d}\tau\Bigg]=\omega_1(0)-\omega_2(0). \tag{24}$$

In the case when the transfer function of the filter $a+W(p)$ is non-degenerate, i.e. its numerator and denominator do not have common roots, equation (24) is equivalent to the following system of differential equations

$$\dot{z}=Az+b\psi(\sigma),\quad \dot{\sigma}=c^*z+\rho\psi(\sigma). \tag{25}$$

Here $\sigma=\theta_1-\theta_2$, A is a constant $(n\times n)$-matrix, b and c are constant (n)-vectors, ρ is a number, and $\psi(\sigma)$ is 2π-periodic function, satisfying the relations:

$$\rho=-aL,\quad W(p)=L^{-1}c^*(A-pI)^{-1}b,$$
$$\psi(\sigma)=\varphi(\sigma)-\frac{\omega_1(0)-\omega_2(0)}{L(a+W(0))}.$$

The discrete phase-locked loops obey similar equations

$$\begin{aligned} z(t+1)&=Az(t)+b\psi(\sigma(t))\\ \sigma(t+1)&=\sigma(t)+c^*z(t)+\rho\psi(\sigma(t)), \end{aligned} \tag{26}$$

where $t\in Z$, Z is the set of integers. Equations (25) and (26) describe the so-called standard PLLs (Shakhgildyan & Lyakhovkin, 1972).

For analysis of the above mathematical models of PLL is applied in the theory of phase synchronization, which was developed in the second half of the last century on the basis of three applied theories: the theory of synchronous and induction electrical motors, the theory of auto-synchronization of the unbalanced rotors, and the theory of phase-locked loops. Modification of direct Lyapunov method with the construction of periodic Lyapunov-like functions, the method of positively invariant cone grids, and the method of nonlocal reduction turned out to be most effective (Leonov et al., 1996; Leonov, 2006; Leonov et al., 2009). The last method, which combines the elements of direct Lyapunov method and bifurcation theory, allows one to extend the classical results of F. Tricomi (Tricomi , 1933) and his progenies (Kudrewicz & Wasowicz, 2007) to the multidimensional dynamical systems.

3 CONCLUSIONS

Considered above methods for high-frequency analysis of PLL allow one to construct adequate nonlinear dynamical model of PLL and to apply special methods of qualitative theory of differential, difference, integral, and integro-differential equations for PLL design.

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