

OPTIMIZATION LEARNING METHOD FOR DISCRETE PROCESS CONTROL

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Abstract: The aim of the paper is to present a novel conception of the optimization method for discrete manufacturing processes control. This method uses gathering information during the search process and a sophisticated structure of local optimization task. It is a learning method of a special type. A general formal model of a vast class of discrete manufacturing processes (DMP) is given. The model is a basis for learning algorithms. To illustrate the presented ideas, the scheduling algorithm for a special NP-hard problem is given.

1 INTRODUCTION

The control of Discrete Manufacturing Process (DMP) lies in determining the manner of performing a certain set of jobs under restrictions referring to machines/devices, resources, energy, time, transportation possibilities, order of operation performing and others. Most of control algorithms are approximate (heuristic) due to NP-hardness of the optimization problems. Within the frame of artificial intelligence, one attempts at both formal elucidation of heuristic algorithm ideas and giving some rules for creating them (metaheuristics) (Dudek-Dyduch and Fuchs-Seliger, 1993), (Dudek-Dyduch and Dyduch, 1988), (Pearl, 1988), and (Rajedran, 1994). The paper ties in with this direction of research. It deals with formal modeling of discrete manufacturing/production processes and its applications for control/planning algorithms. It presents the development of ideas given in (Dudek-Dyduch, 2000). Its aim is twofold:

- To present a novel heuristic method that uses a sophisticated local optimization and gathering information during consecutive search iterations (learning method);
- To present an intelligent search algorithm based on the method for a certain NP-hard scheduling problem, namely a scheduling problem with state depended retooling.

The paper uses the formal model based on the special type of the multistage decision process given below.

2 FORMAL MODEL OF DMP

Simulation aimed at scheduling any DMP consists in determining a sequence of process states and the related time instances. The new state and its time instant depend on the previous state and the decision that has been realized (taken) then. The decision determines the job to be performed, resources, transport unit, etc. Manufacturing processes belong to the larger class of discrete processes, namely discrete deterministic processes (DDP). The formal model of DDP given in (Dudek-Dyduch, 1990), (Dudek-Dyduch, 1992), and (Dudek-Dyduch, 1993) will be adopted here for DMP.

Definition 1. A discrete manufacturing/production process (DMP) is a process that is defined by the sextuple $DMP=(U, S, s_0, f, S_N, S_G)$ where U is a set of control decisions or control signals, $S=X \times T$ is a set named a set of generalized states, X is a set of proper states, $T \subset \mathcal{R}^+ \cup \{0\}$ is a subset of non negative real numbers representing the time instants, $f:U \times S \rightarrow S$ is a partial function called a transition function, (it does not have to be determined for all elements of the set $U \times S$), $s_0=(x_0, t_0)$, $S_N \subset S$, $S_G \subset S$ are respectively: an initial generalized state, a set of not admissible generalized states, and a set of goal generalized states, i.e. the states in which we want the process to take place at the end.

It can be noticed, that DMP corresponds to some multistage decision processes.

The transition function is defined by means of two functions, $f=(f_x, f_t)$ where $f_x: U \times X \times T \rightarrow X$ determines the next state, $f_t: U \times X \times T \rightarrow T$ determines the next time instant. It is assumed that the difference $\Delta t = f_t(u, x, t) - t$ has a value that is both finite and positive.

Thus, as a result of the decision u that is taken or realized at the proper state x and the moment t , the state of the process changes for $x' = f_x(u, x, t)$ that is observed at the moment $t' = f_t(u, x, t) = t + \Delta t$.

Because not all decisions defined formally make sense in certain situations, the transition function f is defined as a partial one. As a result, all limitations concerning the control decisions in a given state s can be defined in a convenient way by means of so-called sets of possible decisions $U_p(s)$, and defined as: $U_p(s) = \{u \in U: (u, s) \in \text{Dom } f\}$.

At the same time, a DMP is represented by a set of its trajectories that starts from the initial state s_0 . It is assumed that no state of a trajectory, apart from the last one, may belong to the set S_N or has an empty set of possible decisions. Only a trajectory that ends in the set of goal states is admissible. The control sequence determining an admissible trajectory is an admissible control sequence (decision sequence). The task of optimization lies in the fact of finding such an admissible decision sequence \tilde{u} that would minimize a certain criterion Q .

In the most general case, sets U and X may be presented as a Cartesian product $U = U^1 \times U^2 \times \dots \times U^m$, $X = X^1 \times X^2 \times \dots \times X^n$ i.e. $u = (u^1, u^2, \dots, u^m)$, $x = (x^1, x^2, \dots, x^n)$. There are no limitations imposed on the sets; in particular they do not have to be numerical. Thus values of particular co-ordinates of a state may be names of elements (symbols) as well as some objects (e.g. finite set, sequence etc.). Particular u^i represent separate decisions that must or may be taken at the same time. The sets S_N , S_F , and U_p are formally defined with the use of logical formulae. Therefore, the complete model constitutes a specialized form of a knowledge-based model (logic-algebraic model). According to its structure, the knowledge on DMP is represented by coded information on U , S , s_0 , f , S_N , S_G . Function f may be defined by means of a procedure or by means of *IF..THEN* rules. The basic structure of DMP (def.1) is usually created on the basis of process technology description. Based on additional expert knowledge (or analysis of DMP) subsets of states can be differentiated, for which best decisions or some decision choice rules R (control rules) are known.

3 OPTIMIZATION LEARNING METHOD

The method of solution search with information gathering is a significant development of the method presented in papers (Dudek-Dyduch, 1990), (Dudek-Dyduch, 2000), which generates subsequent process trajectories with the use of previously obtained and analyzed solutions (admissible and non-admissible trajectories) so as to generate improved solutions.

The method uses local optimization tasks. The task lies in the choice of such a decision among the set of possibilities in the given state $U_p(s)$, for which the value of a specially constructed local criterion is the lowest. The form of the local criterion and its parameters are modified in the process of solution search.

The local criterion consists of three parts and is created in the following way. The first part concerns the value of the global index of quality for the generated trajectory. It consists of the increase of the quality index resulting from the realization of the considered decision and the value related to the estimation of the quality index for the final trajectory section, which follows the possible realization of the considered decision. This part of the criterion is suitable for problems, whose quality criterion is additively separable and monotonically ascending along the trajectory (Dudek-Dyduch, 1990).

The second part consists of components related to additional limitations or requirements. The components estimate the distance in the state space between the state in which the considered decision has been taken and the states belonging to the set of non-admissible states S_N , as well as unfavorable states or distinguished favorable states. Since the results of the decision are known no further than for one step ahead, it is necessary to introduce the "measure of distance" in the set of states, which will aid to define this distance. For that purpose, any semimetrics can be applied. As we know, semimetrics, represented here as ψ , differs from metrics in that it does not have to fulfill the condition $\psi(a, b) = 0 \Leftrightarrow a = b$.

The third part includes components responsible for the preference of certain types of decisions resulting from problem analysis. The basic form of the criterion $q(u, x, t)$ can be then represented as follows:

$$q(u, x, t) = AQ(u, x, t) + Q(u, x, t) + a_1 \cdot \varphi_1(u, x, t) + \dots + a_i \cdot \varphi_i(u, x, t) + \dots + a_n \cdot \varphi_n(u, x, t) + b_1 \cdot \rho_1(u, x, t) + \dots + b_j \cdot \rho_j(u, x, t) + \dots + b_n \cdot \rho_n(u, x, t) \quad (1)$$

where:

- $\Delta Q(u,x,t)$ - increase of the quality index value as a result of decision u , undertaken in the state $s=(x,t)$;
- $\hat{Q}(u,x,t)$ - estimation of the quality index value for the final trajectory section after the decision u has been realized;
- $\varphi_i(u,x,t)$ - component reflecting additional limitations or additional requirements in the space of states, $i=1,2,\dots,n$;
- a_i - coefficient, which defines the weight of i -th component $\varphi_i(u,x,t)$ in the criterion $q(u,x,t)$;
- $\rho_j(u,x,t)$ - component responsible for the preference of certain types of decisions, $j=1,2,\dots,m$;
- b_j - coefficient, which defines the weight of j -th component responsible for the preference of particular decision types.

The significance of particular local criterion components may vary. The more significant a given component is, the higher value is of its coefficient. It is difficult to define optimal weights a priori. They depend both on the considered optimization problem as well as the input data for the particular optimization task (instance). The knowledge collected in the course of experiments may be used to verify these coefficients. On the other hand, coefficient values established for the best trajectory represent aggregated knowledge obtained in the course of experiments.

The presented method consists in the consecutive construction of whole trajectories, whilst their generation always begins from the initial state $s_0=(x_0,t_0)$. For each generated trajectory, both admissible and non-admissible, its final characteristics is remembered and then used in further calculations. The method is characterized by the following features:

- A trajectory sequence is generated; each trajectory is analyzed, which provides information about the DMP taken control;
- Based on the analysis of so far generated whole trajectories, it is possible to modify coefficients used in local optimization or change the form of local optimization criterion when generating a new trajectory;
- In the course of trajectory creation, the subsequent state of the process is being analyzed and it is possible to modify the form or/and parameters used in local optimization.

4 SCHEDULING PROBLEM WITH STATE DEPENDENT RETOOLING

To illustrate the application of the presented method, let us consider the following real life scheduling problem that takes place during scheduling preparatory works in mines. The set of headings in the mine must be driven in order to render the exploitation field accessible. The headings form a net formally, represented by a nonoriented multigraph $G=(W,C,P)$ where the set of branches C and the set of nodes W represent the set of headings and the set of heading crossings respectively, and relation $P \subset (W \times C \times W)$ determines connections between the headings (a partial order between the headings).

There are two kinds of driving machines that differ in efficiency, cost of driving and necessity of transport. Machines of the first kind (set $M1$) are more effective but the cost of driving by means of them is much higher than for the second kind (set $M2$). Additionally, the first kind of machines must be transported when driving starts from another heading crossing than the one in which the machine is, while the second type of machines need no transport. Driving a heading cannot be interrupted before its completion and can be done only by one machine at a time.

There are given due dates for some of the headings. They result from the formerly prepared plan of field exploitation. One must determine the order of heading driving and the machine by means of which each heading should be driven so that the total cost of driving is minimal and each heading complete before its due date.

There are given: lengths of the headings $dl(c)$, efficiency of both kinds of machines $V_{Dr(m)}$ (driving length per time unit), cost of a length unit driven for both kinds of machines, cost of the time unit waiting for both kinds of machines, speed of machine transport $V_{Tr(m)}$ and transport cost per a length unit.

The problem is NP-hard (Kucharska, 2006). NP-hardness of the problem justifies the application of approximate (heuristic) algorithms. A role of a machine transport corresponds to retooling during a manufacturing process, but the time needed for a transport of a machine depends on the process state while retooling does not.

4.1 Formal Model of Problem

The process state at any instant t is defined as a vector $x=(x^0, x^1, x^2, \dots, x^{|M|})$, where $M=M1 \cup M2$. A coordinate x^0 describes a set of heading (branch) that has been driven to the moment t . The other coordinates x^m describes state of the m -th machine, where $m=1, 2, \dots, |M|$.

A structure of the machine state is as follows:

$$x^m=(p, \omega, \lambda) \quad (2)$$

where:

- $p \in C \cup \{0\}$ - represents the number of the heading assigned to the m -th machine to drive for $p \in C$ or no assignment for the machine for $p=0$ (if the machine is not assigned to a heading, i.e. the machine remains idle at the crossing w);
- $\omega \in W$ - the number of the crossing (node), where the machine is located (if it is not assigned to any heading, i.e. $p=0$) or the number of the node, in which it finishes driving the assigned heading c (information about movement direction of the machine);
- $\lambda \in [0, \infty)$ - the length of the route that remains to reach the node $\omega=w$ by the m -th machine, whilst $\lambda=0$ means, that the machine is in node w , $\lambda \in (0, dl(c))$ means that the machine is driving a heading, and the value of λ is the length that remains for the given heading c to be finished, whilst $\lambda > dl(c)$ means that the machine is being transported to the heading, the value λ is the sum of the length of heading c and the length of the route until the transportation is finished.

A state $s=(x, t)$ belongs to the set of non-admissible states if there is a heading whose driving is not complete yet and its due date is earlier than t . The definition S_N is as follows:

$$S_N = \{s=(x, t) : (\exists c \in C, c \notin x^0) \wedge d(c) < t\} \quad (3)$$

where $d(c)$ denotes the due date for the heading c .

A state $s=(x, t)$ is a goal if all the headings have been driven. The definition of the set of goal states S_G is as follows:

$$S_G = \{s=(x, t) : s \notin S_N \wedge (\forall c \in C, c \in x^0)\} \quad (4)$$

A decision determines the headings that should be started at the moment t , machines which drive,

machines that should be transported, headings along which machines are to be transported and machines that should wait. Thus, the decision $u=(u^1, u^2, \dots, u^{|M|})$ where the co-ordinate u^m refers to the m -th machine and $u^m \in C \cup \{0\}$. $u^m=0$ denotes continuation of the previous machine operations (continuation of driving with possible transport or further stopover). $u^m=c$ denotes the number of heading c that is assigned to be driven by machine m . As a result of this decision, the machine starts driving the heading c or is transported from the current location to the node of the heading c , to which the transportation route defined in the state s is the shortest. This route is computed by the Ford's algorithm (a polynomial one).

Obviously, not all decisions can be taken in the state (x, t) . The decision $u(x, t)$ must belong to the set of possible (reasonable) decisions $U_p(x, t)$. For example, a decision $u^m=c$ is possible only when the c -th heading is neither being driven nor complete and is available, i.e. there is a way to transport machine to the one of the heading crossing adjacent to the c -th heading or machine is standing in the one of the heading crossings adjacent to the c -th heading.

Moreover, in the given state $s=(x, t)$, to each machine waiting in the node w , (it has not assigned a heading to perform), we can assign an available heading or it can be decided that it should continue to wait. However, each machine which has been previously assigned a heading and is currently driving it or it is being transported to that heading, can be only assigned to continue the current activity. Also, none of the headings can be assigned to more than one machine. The complete definition of the set of the possible decision $U_p(x, t)$ will be omitted here because it is not necessary to explain the idea of the learning method.

Based on the current state $s=(x, t)$ and the decision u taken in this state, the subsequent state $(x', t')=f(u, x, t)$ is generated by means of the transition function f . The transition function is defined for each possible decision $u(s) \in U_p(s)$.

Firstly, it is necessary to determine the moment t' when the subsequent state occurs, that is the nearest moment in which at least one machine will finish driving a heading. For that purpose, t_m time of completion of the realized task needs to be calculated for each machine. The subsequent state will occur in the moment $t'=t+\Delta t$, where Δt equals the lowest value of the established set of t_m .

Once the moment t' is known, it is possible to determine the proper state of the process at the time. The first coordinate x^0 of the proper state, that is the set of completed headings, is increased by

Table 1: Particular parameters of the coordinate of the new machine state.

for the decision to continue the activity of the machine $u^m = 0$:	
$p' = \begin{cases} p & \text{for } t_m > \Delta t \\ 0 & \text{for } t_m = \Delta t \end{cases}$	
$\omega' = \omega$	
$\lambda' = \begin{cases} \lambda - V_{Tr(m)} \cdot \min\left(\frac{\max(\lambda - dl(c), 0)}{V_{Tr(m)}}, \Delta t\right) - V_{Dr(m)} \cdot \max\left(\Delta t - \frac{\max(\lambda - dl(c), 0)}{V_{Tr(m)}}, 0\right) & \text{for } m \in M1 \\ \lambda - V_{Dr(m)} \cdot \Delta t & \text{for } m \in M2 \end{cases}$	
for the decision to assign a new task to the machine $u^m = c$:	
$p' = \begin{cases} c & \text{for } t_m > \Delta t \\ 0 & \text{for } t_m = \Delta t \end{cases}$	
$\omega' = w_k(c)$	
$\lambda' = \begin{cases} r_{\min}(m, c) + dl(c) - V_{Tr(m)} \cdot \min\left(\frac{ r_{\min}(m, c) }{V_{Tr(m)}}, \Delta t\right) - V_{Dr(m)} \cdot \max\left(\Delta t - \frac{ r_{\min}(m, c) }{V_{Tr(m)}}, 0\right) & \text{for } m \in M1 \\ dl(c) - V_{Dr(m)} \cdot \Delta t & \text{for } m \in M2 \end{cases}$	

the number of headings whose driving has been finished in the moment t' .

Afterwards, the values of subsequent coordinates in the new state are determined $x^m = (p', \omega', \lambda')$, for $m=1, 2, \dots, |M|$, which represent the states of particular machines.

Particular parameters of the coordinate of the new machine state are determined in the way described in Table 1, where $w_k(c)$ is the node adjacent to the heading c , in which the machine will finish driving, and $|r_{\min}(m, c)|$ is the length of the shortest transportation route to the heading c for the machine m .

4.2 Learning Algorithm

The algorithm based on the learning method consists in generating consecutive trajectories. Each of them is generated with the use of the specially designed local optimization task and then is analyzed. The information gained as a result of the analysis is used in order to modify the local optimization task for the next trajectory, i.e. for the next simulation experiment. This approach is treated as a learning without a teacher.

In the course of trajectory generation in each state of the process, a decision is taken for which

the value of the local criterion is the lowest. The local criterion takes into account a component connected with cost of work, a component connected with necessity for trajectory to omit the states of set S_N and a component for preferring some decisions.

The first components is a sum of $\Delta Q(u, x, t)$ and $\mathcal{Q}(u, x, t)$ where $\Delta Q(u, x, t)$ denotes the increase of work cost as a result of realizing decision u and $\mathcal{Q}(u, x, t)$ the estimate of the cost of finishing the set of headings matching the final section of the trajectory after the decision u has been realized.

The second component $\varphi_1(u, x, t) = E(u, x, t)$, connected with the necessity for the trajectory to omit the states of set S_N , is defined by means of a semimetrics.

The third component is aimed at reduction of machine idleness time. Since the model considers the possibility that the machines will stand idle in certain cases, it seems purposeful to prefer decisions which will engage all machines to for most of the time. It is therefore necessary to reduce the probability of selecting the decision about machine stopover when headings are available for driving and machines could be used for work. This may be realized by using an additional auxiliary

criterion $\rho_1(u, x, t) = F(u, x, t)$, which takes into consideration penalty for a decision about a stopover in the case of a machine which could have started work.

Thus, the local criterion is of the form:

$$q(u, x, t) = \Delta Q(u, x, t) + \mathcal{Q}(u, x, t) + a_1 \cdot E(u, x, t) + b_1 \cdot F(u, x, t) \quad (5)$$

where a_1, b_1 are weights of particular components.

In the course of trajectory generation, the local optimization task may be changed. Problem analysis reveals that the moment all headings with due dates are already finished, it is advisable to use only cheaper machines. Formally, this corresponds to the limitation of the set of possible decisions $Up(s)$. Moreover, it is no longer necessary to apply the component $E(u, x, t)$ in the local criterion. The modified form of the criterion can be then represented as follows:

$$q(u, x, t) = \Delta Q(u, x, t) + \mathcal{Q}(u, x, t) + b_1 \cdot F(u, x, t) \quad (6)$$

In order to select a decision in the given state s , it is necessary to generate and verify the entire set of possible decisions in the considered state $U_p(s)$. For each decision u_k , it is necessary to determine the state the system would reach after realizing it. Such a potentially consecutive state of the process will be represented as $s_{p,k} = (x_{p,k}, t_{p,k})$.

Afterwards, the criterion components are calculated. The increase of cost $\Delta Q(u_k, x, t)$ is the sum of costs resulting from the activities of particular machines in the period of time $t_{p,k} - t$. The estimate of the cost of the final trajectory section $\mathcal{Q}(u_k, x, t)$ can be determined in a number of ways. One of these is to establish the summary cost of finishing previously undertaken decisions, whose realization has not been completed yet, and the cost of a certain relaxed task, realized in the cheapest way. Taking into consideration that the estimate should take place with the lowest number of calculations, relaxation has been proposed which would include omitting temporal limitations and the assuming the least expensive procedure for finishing the remaining headings; this would involve using the least expensive machines.

The component of the local criterion $E(u_k, x, t)$ uses the value of the estimated "distance" between the state $s_{p,k}$ and the set of inadmissible states. The distance is estimated with the help of semimetrics $\psi(s, S_N) = \min\{\psi(s, s') : s' \in S_N\}$.

Assuming that the speed of transporting the "fastest" machine is significantly higher than its speed of performance, and this one in turn

significantly exceeds the speed of performance of the remaining machines, it is possible to omit the time of transporting the fastest machine. For the sake of simplicity, let us assume that there is one fastest machine.

One of the methods of determining the component $E(u_k, x, t)$ is to calculate, for each not realized and not assigned heading c with due date, the time reserve $rt_c(s_{p,k})$.

Taking into consideration these assumptions, the time reserve is defined by the following formula:

$$rt_c(s_{p,k}) = d(c) - t_{p,k} - \tau(c) - t_{end} \quad (7)$$

where:

- $d(c)$ - due date for the heading c ;
- $\tau(c)$ - time necessary to drive heading c and all the headings situated along the shortest route from the heading c to the so-called *realized area* in the given state, by the fastest machine;
- t_{end} - time necessary to finalize the current activity of the fastest machine.

The parameters t_{end} and $\tau(c)$ results for following situations:

a) "the fastest" machine may continue the previously assigned task to drive another heading.

b) heading c may be inaccessible and it might be necessary to drive the shortest route to this spot from the *realized area*, involving already excavated headings as well as those assigned for excavation together with relevant crossings. For the period of time when the fastest machine finishes the excavation of the previously assigned heading, a fragment of this distance may be excavated by the fastest of the remaining machines. When the heading is accessible the time equals to the time of excavating the length of the heading with the fastest machine.

Finally, the component estimating the influence of time limitations assumes the following form:

$$E(u_k, x, t) = \begin{cases} \infty & \text{for } \min rt_c(s_{p,k}) < 0 \\ \frac{1}{\min rt_c(s_{p,k})} & \text{for } \min rt_c(s_{p,k}) \geq 0 \end{cases} \quad (8)$$

As a result, the decision to be taken, from the set of considered decisions, is the one for which the subsequent state is most distant from the set of non-admissible states.

The shape of the local criterion component $F(u_k, x, t)$ in certain cases should make it purposeful to prefer decisions which will engage all machines

for most of the time. It is therefore necessary to reduce the probability of selecting the decision about machine stopover when headings are available for driving and machines could be used for work. For that purpose, a penalty may be imposed for the stopover of each machine that could potentially start driving an available heading. The proposed form of $F(u_b, x, t)$ is the following:

$$F(u_b, x, t) = P \cdot i_{waiting} \tag{9}$$

where:

- P - denotes the penalty for machine stopover, (calculated for the decision about stopover when there are still headings available for work);
- $i_{waiting}$ - the number of machines that are supposed to remain idle as a result of such a decision.

The values a_1 and b_1 are respectively coefficients defining the weight of particular components of the local criterion $q(u, x, t)$ and reflect current knowledge about controls, whilst their values change in the course of calculations. The higher the weight of a given parameter, the higher its value. The weights depend both on the considered optimization problem as well as input data for the particular optimization task (instance). Coefficient values, as well as their mutual proportions are not known nor can they be calculated a priori.

The knowledge gained in the course of experiments may be used to change weight values. If the generated trajectory is non-admissible, then for

the subsequent trajectory, the value of weight a_1 should be increased; which means the increase of the weight of the component estimating the distance from the set of non-admissible states and/or the increase of weight b_1 value, which would result in lower probability of machine stopover.

Whereas, if the generated trajectory is admissible, then for the subsequent trajectory the values of this coefficients may be decreased.

4.3 Experiments

The aim of conducted experiments was to verify the effectiveness of applying the components $E(u, x, t)$ and $F(u, x, t)$ in the local criterion.

The research was conducted for the set of 10 heading networks. Each network is represented by a planar graph, in which the vertex degrees equal from 1 to 4. The lengths of headings are numbers from the range [19, 120]. The number of headings with due dates is approximately 25% of all headings. The parameters of heading networks used in simulation experiments are given in Table 2.

Also, two examples of network are presented in Figure 1.

Two machines are used to perform the task during our experiments, one of the first type and one of the second type. Parameters for both types of machines are given in the Table 3.

The effectiveness of component $E(u, x, t)$ for each network was tested by constructing 40 trajectories with the changing value of coefficient a_1 and zero value of coefficient b_1 .

Table 2: The heading network parameters.

Parameter	GI-1a	GI-1b	GI-2	GI-3	GII-4	GII-5a	GII-5b	GII-6	GII-7	GII-8
Number of heading	20	20	20	20	24	27	27	29	67	68
Number of heading crossing	18	18	18	18	20	20	20	20	50	50
Length of the shortest heading	20	20	22	19	30	30	30	30	30	30
Length of the longest heading	109	109	106	120	81.59	107.52	107.52	94.75	114.65	111.95
Sum of the length of the headings	1008	1008	1010	1066	1364.45	1863.07	1863.07	1869.88	4579.11	4607.81
Number of heading with deadline	5	5	5	5	5	5	5	5	18	16
Minimum deadline	55	60	55	55	60	70	100	50	80	80
Maximum deadline	70	60	70	70	100	150	100	100	320	200

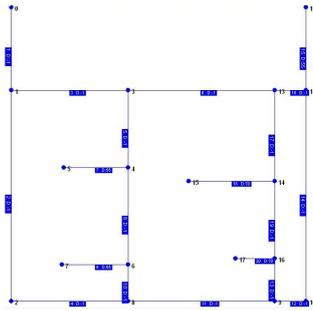


Figure 1a: Example of heading network - GI-1a.

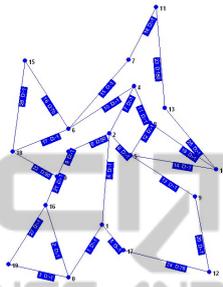


Figure 1b: Example of heading network - GII-5.

Table 4 presents results for one of the tested networks, i.e. for network GII-4.

The time reserve refers to the time that remains after a given heading has been driven until the due date, whilst the minus value means that the due date has been exceeded. The symbol ”*” means that heading driving has not commenced because the trajectory reached a non-admissible state. Based on the obtained results, it can be concluded that increasing the value of $E(u,x,t)$ increases the probability of obtaining an admissible solution.

Table 4: The effectiveness of applying parameter $E(u,x,t)$ for the network GII-4.

Coeff. a_1	Total cost	Time reserve of head. 4	Time reserve of head. 9	Time reserve of head. 12	Time reserve of head. 16	Time reserve of head. 22	Min. reserve	Average reserve
0	-	7.40	*	*	*	*	-	-
1	-	12.90	*	*	*	*	-	-
50	-	12.90	*	-0.62	*	*	-	-
5000	-	12.90	*	-2.84	*	*	-	-
7500	22887.64	12.90	4.02	7.68	5.15	11.64	4.02	8.28
25000	23153.10	13.18	32.10	48.01	1.46	2.67	1.46	19.48
50000	23359.93	12.90	21.73	48.01	38.90	30.34	12.90	30.38
75000	23204.35	25.18	19.42	48.01	20.54	25.59	19.42	27.75
500000	23027.78	27.47	20.13	19.71	40.06	29.18	19.71	27.31
2500000	23084.25	27.47	20.13	19.71	36.57	37.53	19.71	28.28

Table 3: Parameters of the machines.

Parameter	Machine of M1 type	Machine of M2 type
Efficiency [m/h]	10.0	5.0
Transport speed [m/h]	100.0	<i>unspecified</i>
Driving cost [\$/h]	200.0	50.0
Transport cost [\$/h]	100.0	0.0
Waiting cost [\$/h]	30.0	5.0

When the component $E(u,x,t)$ was omitted, an admissible solution was not found.

Table 5 presents in columns the best obtained total cost at changeable value of coefficient b_1 , responsible for the weight of component $F(u,x,t)$ in the local criterion. In most cases, the increase of weight of this component resulted in increased total costs, but at the same time the probability of finding an admissible solution was higher. Moreover, in some cases, the use of this component resulted in decreased total costs of performing work. A lot of this experiments have been conducted, they are presented in (Kucharska, 2006). Unfortunately, they must be omitted here because the limited length of the paper.

The experiments have confirmed effectiveness of the use of the component $F(u,x,t)$.

To evaluate the effectiveness of the proposed algorithm one has compared the obtained results with the optimal solution. A complete review algorithm for all considered heading network has been applied. To reduce calculations, the generation of each trajectory was interrupted when the lower estimate of its cost in a given state was greater than the best found solution.

Table 5: Effectiveness of applying component $F(u,x,t)$.

Network	Best found cost				
	$b_j=0$	$b_j=0,5$	$b_j=1$	$b_j=2$	$b_j=5$
GI-1a	17026.90	17026.90	17249.10	16922.00	16922.00
GI-2	*	*	*	17284.10	17284.10
GI-3	16359.90	16359.90	16359.90	16359.90	16359.90
GII-4	22665.17	22665.17	22667.89	22667.89	22667.89
GII-5a	30982.02	30946.33	31004.27	31004.27	31004.27
GII-5b	30888.34	30888.34	31139.72	31139.72	31139.72
GII-6	30662.32	30675.19	30675.19	30675.19	30675.19
GII-7	77428.92	77717.02	77288.44	77717.02	77717.02

Table 7: Comparison of results from the solution obtained by a complete review of the algorithm for which the calculations were interrupted.

Network	Review algorithm		Best cost founded by learning algorithm	Percentage difference of cost
	time to stop calculation	best found cost		
GI-1a	43h 48min	16480,00	16922.00	2,68%
GI-3	43h 41min	17448,90	16359.90	-6,24%
GII-4	44h 3min	23132,40	22665.17	-2,01%
GII-6	43h 38min	31142,54	30662.32	-1,54%

The lower bound was calculated as the sum of the cost of the current part of trajectory and a cost estimation of remaining part.

Optimal solution was found only for network GI-2. In other cases, the calculations were interrupted after more than 2 days. Only for four networks an admissible solution has been found, while for the other networks an acceptable solution has not been found during 2 days.

The comparison between the optimal cost with the best found cost is presented in Table 6. Error of found solution is also given. It can be noticed that the proposed learning algorithm has found very good solution (almost optimal). It should be point out also that the its calculation time was very short (a several seconds). While the exact algorithm needs very long time (over 40 hours).

Table 6: Comparison of results from the optimal solution.

Net-work	Time for the optimal solution	Optimal cost	The best cost found by the proposed algorithm	Error of found solution
GI-2	43h 43min 59s	16524.40	17284.10	4.59%

The comparison of the learning algorithm results with solution obtained by complete review algorithm after over 40 hour are presented in Table 7. Also, the time after which the calculation of the algorithm was stopped, and the cost of which could be determined at this time are given. Percentage difference of cost is calculated as $(LAcost-RAcost)/RAcost$ where LA, RA denote best cost calculated by learning algorithm and completed review algorithm respectively.

The result of experiments shows that the difference between sub-optimal cost and the best found by the learning algorithm is small and in the worst case is 4.59%. One can say that the learning algorithm finds a better solution in most cases.

Based on the obtained results, it can be concluded that the application of the proposed algorithm for the DMP problem yields very positive results.

5 CONCLUSIONS

The paper presents a conception of intelligent search method for optimization of *discrete manufacturing processes control (scheduling, planning)*. The method uses a sophisticated structure of local optimization task. The structure as well as parameters of the task are modified during search process. It is done on a basis of gathering information during previous iterations. Thus the method is a learning one. The method is based on a general formal model of discrete manufacturing processes (DMP), that is given in the paper.

A large number of difficult scheduling problems in manufacturing can be efficiently solved by means of the method. Moreover, the proposed method is very useful for another difficult scheduling problems, especially for problems with state depended resources. Managing projects, especially software projects belongs to this class.

To illustrate the conception, some NP-hard problem, namely a scheduling problem with state depended retooling is considered and the learning algorithm for it is presented. Results of computer experiments confirm the efficiency of the algorithm.

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