MODELING AND SIMULATING A NARROW TILTING CAR

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Abstract: The use of an electrical narrow tilting car instead of a large gasoline car should significantly decrease traffic congestion, pollution and parking problem. The aim of this paper is to give an approach to develop a dynamic model for narrow cars. This model can be used to simulate their behaviours and evaluate tilt control systems. The approach is based on considering the vehicle as a multi-body poly-articulated system and the modelling is carried out using the robotics formalism based on the modified Denavit-Hartenberg geometric description.

1 INTRODUCTION

The idea behind narrow tilting car research is to develop a vehicle used in urban transportation having the advantages of motorcycle and passenger car. This will reduce the size of the vehicle such that it can be operated on reduced size lanes thereby increasing the effective capacity of highways. In order to maintain its stability, the vehicle should tilt while cornering, to compensate the effect of lateral acceleration and remain in its trajectory. Moreover the use of electric motors with a group of batteries is the most earth friendly technology.

In the literature, many works have been published on the topic of tilting narrow vehicle. Karnopp and Fang (Karnopp, 1992) were the first to suggest a leaning into the turn similar like motorcyclist's one. Karnopp, Hibbard and So (Hibbard, 1992. So, 1997) studied the tilt angle required and the dynamics of such a vehicle. But few people talked about the global dynamic model of a four wheel tilting car. Rajamani, Gohl and Alexander (Gohl, 2006) developed a dynamic model of a three wheel vehicle which has four degrees of freedom including lateral and tilt dynamics. All these models don't take into account the dynamics of the suspensions, the vertical dynamic and the study was on a simplified model called bicycle model. Therefore to model a complex system in 3D motion, many methods can be used. Kiencke described his model with 4 individual co-ordinate systems (Kiencke, 2000) while Rajamani with 6 co-ordinate system (Rajamani, 2006). We claim that it is preferable to proceed in a systematic method of geometrical description, based on the modified Denavit-Hartenberg parameterization (Khalil, 1986). The last was applied on a two wheeled vehicle model with suspensions (Maakaroun, 2011). This description allows to automatically calculate the symbolic expression of the geometric, kinematic and dynamic models by using a symbolic software package SYMORO+ (Symbolic Modelling of Robots) (Khalil, 1997). Moreover, the dynamical model can be calculated numerically using programming software as Matlab, C++. This formulation leads to a minimum set of equations where the constraint equations for the mechanical system are automatically eliminated.

This paper concentrates on developing a global dynamic model for a narrow tilting car "Lumeneo Smera " (Lumeneo, 2003) by applying methods used in robotics. Since the structure of the Smera is complex and contains loops, this approach can elaborate systematically the symbolic equations of motion and makes the implementation of the dynamic model very easy. This method is described and applied on the car in section 2. Then Kinematics and dynamics models are given in section 3 and 4. Finally, simulation results are illustrated and commented and conclusions are done.

2 GEOMETRIC DESCRIPTION OF THE CAR

2.1 Robotic Representation of a Multi-body System

The car is considered as a mobile tree-structured

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multi-body system composed of n bodies (links) where the chassis is the mobile base and the wheels are the terminal links. The links are numbered consecutively from the base to the terminal links. Each body C_j is connected to its antecedent C_i (i=a(j)) with a joint that represents an elastic or rigid translational or rotational degree of freedom. The symbol a(j) denotes the link antecedent to link j, and consequently a(j) < j .A body can be virtual or real; the virtual bodies are introduced to describe joints with multiple degrees of freedom like ball joint or intermediate fixed frames. The frame R_i (O_i, x_i, y_i, z_i) which is attached to the body C_i is defined as following:

The z_i axis is along the axis of joint i, the u_j axis is defined as the common normal between z_i and z_j. The x_i axis is along the common normal between z_i and one of the succeeding z axis, where link i is the antecedent of link j and the origin O_i is the intersection of z_i and x_i.
The homogeneous transformation matrix ${}^iT_j$ between two consecutive frames R_i and R_j is expressed as a function of the following six parameters (Figure 1):

- γ_j: angle between x_i and u_j about z_i
- b_j: distance between x_i and u_j along z_i
- α_j: angle between z_i and z_j about u_j
- d_j: distance between z_i and z_j along u_j
- θ_j: angle between u_j and x_j about z_j
- r_j: distance between u_j and x_j along z_j

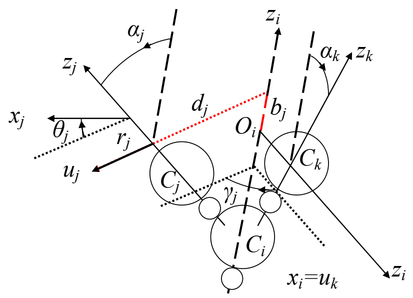

Figure 1: Geometric parameters.

The generalized coordinate of joint j is denoted by q_j, it is equal to r_j if j is translational and θ_j if j is rotational. In (Figure 1), since x_i is taken along u_k, the parameters γ_k and b_k are equal to zero. We define the parameter $\sigma_j = 1$ if joint j is translational and $\sigma_j = 0$ if joint j is rotational. If there is no degree of freedom between two frames that are fixed with respect to each other, we take $\sigma_j = 2$. In this case, the time derivative of q_j is zero.

2.1 Application for the Tilting Car

The model of the Smera (Figure 2): is composed of 19 real bodies (Lumeneo, 2003) connected by 24 joints. Thus it contains 6 closed Kinematic loops:

- C_1 is the chassis
- C_2 and C_{13} are two mechanical parts called "lyre" which have a rotational movement around the longitudinal axis of the chassis. C_2 is actuated by an electrical motor which controls the roll of the vehicle.
- C_5 and C_8 are the rear driving wheels.
- C_3, C_6 and C_{14}, C_{15} are respectively the rear and the front suspensions of the vehicle.
- C_4 and C_7 are the rear arms that connect the chassis to the rear wheels.
- C_{11} and C_{18} are the front steering wheels.
- C_9, C_{10} and C_{12} and the chassis constitute a parallelogram which carries the hub of the left front wheel.
- C_{16}, C_{17} and C_{19} constitute with the chassis a parallelogram which carries the hub of the right front wheel;

The Kinematic closed loops are defined as follows:

- LP1 is composed of C_1, C_2, C_3 and C_4
- LP2 is composed of C_1, C_2, C_6 and C_7
- LP3 is the left parallelogram; it is composed of C_1, C_9, C_{10} and C_{12}
- LP4 is composed of C_1, C_9, C_{13} and C_{14}
- LP5 is the right parallelogram composed by C_1, C_{16}, C_{17} and C_{19}
- LP6 is composed of C_1, C_{13}, C_{15} and C_{16}

Let R_f be a fixed reference frame attached to the ground. In robots manipulator, C_0 is fixed with respect to R_f. In case of mobile system C_0 is taken fixed with the chassis frame.

So according to MDH description and SYMORO+, the structure is defined as a robot with a mobile base by considering C_1 attached to C_0 via a blocked joint. The inertial parameters of this base are those of C_1 and the speed and the acceleration are then the ones of the chassis described in his own frame. The chassis motion is described with Euler coordinates (Cartesian) while all the other links are described with the generalized Lagrangien coordinates (joints). The body C_1 with a location ζ (i.e. position & orientation) gives the system posture in the frame R_f. The movement of the chassis in this mixed Euler-Lagrangien model is given by:

$$ {}^1\dot{V}_1 = \begin{bmatrix} {}^1\dot{V}_{x1} \\ {}^1\dot{V}_{y1} \\ {}^1\dot{V}_{z1} \end{bmatrix}, {}^1\omega_1 = \begin{bmatrix} {}^1\omega_{x1} \\ {}^1\omega_{y1} \\ {}^1\omega_{z1} \end{bmatrix}, {}^1\dot{\omega}_1 = \begin{bmatrix} {}^1\dot{\omega}_{x1} \\ {}^1\dot{\omega}_{y1} \\ {}^1\dot{\omega}_{z1} \end{bmatrix} $$

It is to be noted that: ${}^1\dot{V}_1 = \frac{d}{dt}{}^1V_1 + {}^1\omega_1 \wedge {}^1V_1$

Where ${}^1V_1 = [{}^1V_{x1}\ {}^1V_{y1}\ {}^1V_{z1}]^T$ are respectively the longitudinal, lateral and vertical translational speed of the chassis.

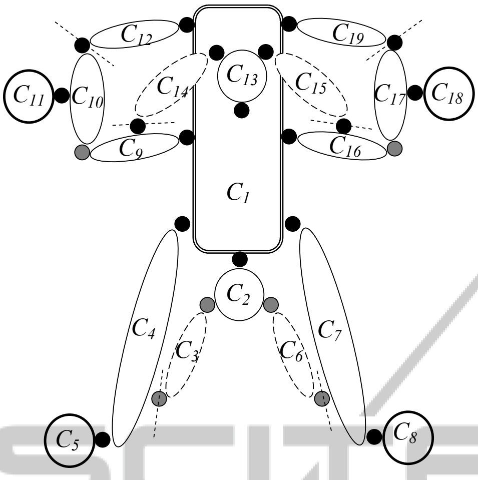

Figure 2: Multi-body description of the Smera.

All the joints which connect the various bodies are revolute joints except the joints on both sides of the rear suspensions and the joints below the front hubs are respectively spherical and cardan joints. The suspensions are represented by prismatic flexible joints.

The rear Lyre is motorized, and all the joints can be calculated in terms of this actuated joint, by resolving the geometric equations of each loop. The resolution of these equations was validated experimentally (Maakaroun, 2010).

The modelling of a complex structure which contains multiple loops is carried out by opening each loop in order to obtain, an equivalent tree structure system. The opened joints are chosen among the non-motorized (passive) joints.

Loops LP1, LP2, LP3, LP4, LP5 and LP6 are opened as shown in figure 3.

According to this description, the vehicle motion is completely described by the vector q of the 36 generalized coordinates:

$$q = \begin{bmatrix} q_a & q_p \end{bmatrix}^T ; q_a = \begin{bmatrix} \xi & \xi_a \end{bmatrix}^T ; \ q_p = \xi_p$$

ξ[1x6] is the posture of the chassis (position & orientation)

$$\xi_a = \begin{bmatrix} q_2 & r_3 & r_6 & r_{14} & q_{15} & q_{11} & q_{18} & q_5 & q_8 & q_{10} & q_{17} \end{bmatrix}$$

ξ_a is the vector of the actuated and independent joints.

$$\xi_p = \begin{bmatrix} \xi_{p1} & \xi_{p2} \end{bmatrix} \ \xi_{p1} = \begin{bmatrix} q_{2g'} & q_{2g''} & q_4 & q_{2d'} & q_{2d''} & q_7 & q_9 \end{bmatrix}$$

$$\xi_{p2} = \begin{bmatrix} q_{10} & q_{12} & q_{16} & q_{17} & q_{19} & q_{13} & q_{13g'} & q_{13d'} \end{bmatrix}$$

ξ_p is the vector of the passive joints angular position.

- $q_{2g'}$, $q_{2g''}$, $q_{2d''}$, $q_{2d''}$, $q_{13g'}$ and $q_{13d'}$ are the angular positions of the revolute joints linked by the suspensions,
- q_9, q_{10}, q_{12}, q_{16}, q_{17} and q_{19} are the angular positions of the parallelograms,
- q_4 and q_7 are the angular positions of the rear arms. The revolute axes of the two drive motors are coincident with the ones of these joints.
- q_5, q_8, q_{11} and q_{18} are the angular positions of the four wheels with respect to their revolute axis,
- $q_{10'}$ and $q_{17'}$ are the steering angles.
- r_3, r_6, r_{14} and r_{15} are the length of the suspensions. They are calculated from the dynamic model.
- q_2 is the angular position of the rear motorized lyre and q_{13} is the angular position of the front lyre.

It is to be noted that the frame i is the antecedent of frame i' which is the antecedent of frame i''.

3 KINEMATIC CONSTRAINTS

3.1 Rear Train

Since the loops LP1 and LP2 are opened in spherical joints, we can conclude respectively that the translational velocity of the frame R_5 and R_{11} are equal from both sides of the opened joints with respect to the base frame.

$$\begin{bmatrix} \dot{q}_{2g'} & \dot{q}_{2g''} & \dot{q}_4 & \dot{q}_{2d'} & \dot{q}_{2d''} & \dot{q}_7 \end{bmatrix}^T = J_1 \begin{bmatrix} \dot{q}_2 & \dot{q}_3 & \dot{q}_6 \end{bmatrix}^T \qquad (1)$$

The derivative of the above equation gives:

$$\begin{bmatrix} \ddot{q}_{2g'} & \ddot{q}_{2g''} & \ddot{q}_4 & \ddot{q}_{2d'} & \ddot{q}_{2d''} & \ddot{q}_7 \end{bmatrix}^T = J_1 \begin{bmatrix} \ddot{q}_2 & \ddot{q}_3 & \ddot{q}_6 \end{bmatrix}^T + Y_1$$

J_1 is the jacobian matrix (6x3) between the velocities of the rear train articulation.

- $Y_1 = \dot{J}_1 \begin{bmatrix} \dot{q}_2 & \dot{q}_3 & \dot{q}_6 \end{bmatrix}^T ; \ \dot{J}_1 = \dfrac{d}{dt} J_1$

3.2 Front Train

The first and second derivate of the geometric equations of Loops LP3, LP4, LP5 and LP6 gives: (Maakaroun, 2010)

$$\begin{bmatrix} \dot{q}_9 & \dot{q}_{10} & \dot{q}_{12} & \dot{q}_{16} & \dot{q}_{17} & \dot{q}_{19} & \dot{q}_{13} & \dot{q}_{13g'} & \dot{q}_{13d'} \end{bmatrix}^T = J_2 U$$

$$\begin{bmatrix} \ddot{q}_9 & \ddot{q}_{10} & \ddot{q}_{12} & \ddot{q}_{16} & \ddot{q}_{17} & \ddot{q}_{19} & \ddot{q}_{13} & \ddot{q}_{13g'} & \ddot{q}_{13d'} \end{bmatrix}^T = J_2 \dot{U} + Y_2 \qquad (2)$$

- J_2 is the jacobian matrix (9x5) between the velocities of the front train articulation and the angular velocities of the chassis.

- $U = \begin{bmatrix} {}^1\omega_1^T & \dot{q}_3 & \dot{q}_6 \end{bmatrix}^T$; $\dot{J}_2 = \frac{d}{dt} J_2$
- $\dot{U} = \begin{bmatrix} {}^1\dot{\omega}_1^T & \ddot{q}_3 & \ddot{q}_6 \end{bmatrix}^T$; $Y_2 = \dot{J}_2 U$;

By combining the equations obtained in section 3-A and 3-B, we can elaborate the relation between the velocities of the active, passive joints and the translational, angular velocities of the chassis.

$$\begin{aligned} \left[\dot{\xi}_p\right] &= J\left[{}^1V_1^T \quad {}^1\omega_1^T \quad \dot{\xi}_a^T\right] \Rightarrow \dot{q}_p = J\dot{q}_a \\ \left[\ddot{\xi}_p\right] &= J\left[{}^1\dot{V}_1^T \quad {}^1\dot{\omega}_1^T \quad \ddot{\xi}_a^T\right] + Y \Rightarrow \ddot{q}_p = J\ddot{q}_a + Y \end{aligned} \tag{3}$$

$$J = \begin{bmatrix} 0_{(6x6)} & J_1 & 0_{(6x8)} \\ 0_{(9x3)} \quad J_2(:,1:3) & 0_{(6x8)} & J_2(:,4:5) \quad 0_{(9x6)} \end{bmatrix}$$

$$Y = \begin{bmatrix} Y_1^T & Y_2^T \end{bmatrix}^T$$

4 DYNAMIC MODEL

4.1 Dynamic Parameters

For each real link there are 14 standard dynamic parameters (Gautier, 1990) composed of 10 standard inertial parameters:

- J_j = [XX_j XY_j XZ_j YY_j YZ_j ZZ_j]: The six components of the inertia matrix of link j given in the frame R_j ,
- MS_j = [MX_j MY_j MZ_j]: the three components of first moment of link j around the origin of the frame j,
- M_j: the mass of link j

For each actuated joint j, we introduce:

- I_{aj} as the total inertia of the rotor of motor and the drive transmission.
- F_{vj}, F_{sj} as the viscous and coulomb friction parameters.

For a flexible joint, we define:

- K_j as the stiffness of the joint j

4.2 External Forces

The external forces applied to the car, which have the most significant impact on vehicle dynamics, are the contact forces between the ground and the tires. These external forces can be modelled using the magic formula of Pacejka (Pacejka, 2002), estimated or measured at the center of the wheels by using dynamometric wheels. Aerodynamic forces also have an effect on the vehicle behaviours, particularly at high speed (> 90 Km/h).

4.3 Euler-Lagrange Dynamic Model

The mixed Euler-Lagrange model is obtained from two recursive equations using recursive calculations of Newton-Euler algorithm in the following way (Khalil, 2002):
In the first (forward) recursive, we calculate the total forces and moments on each link. Then in the second (backward) recursive equations, we calculate the forces f_j and moments m_j exerted on body C_j by its antecedent C_i .
The inverse Dynamic model (IDM) of the tree structure with a mobile base can be written as:

$$\begin{aligned} \begin{bmatrix} \Gamma_a \\ \Gamma_p \end{bmatrix} &= A_{ar}(q)\ddot{q} + H_{ar}(q, \dot{q}, f_e, g, F_s, F_v, K) \\ &= f(q, \dot{q}, \ddot{q}, f_e, g, F_s, F_v, K) \end{aligned} \tag{4}$$

- A_{ar} is the inertial matrix of the system
- H_{ar} is the vector of centrifugal, Coriolis, gravity and generalized efforts terms.
- $\dot{q} = \begin{bmatrix} {}^1V_1^T & {}^1\omega_1^T & \dot{\xi}_a^T & \dot{\xi}_p^T \end{bmatrix}$
- $\ddot{q} = \begin{bmatrix} {}^1\dot{V}_1^T & {}^1\dot{\omega}_1^T & \ddot{\xi}_a^T & \ddot{\xi}_p^T \end{bmatrix}$;
- $A_{ar} = \begin{bmatrix} A_{aa} & A_{ap} \\ A_{pa} & A_{pp} \end{bmatrix}$; $H_{ar} = \begin{bmatrix} H_a \\ H_p \end{bmatrix}$

Since the joint velocities and accelerations are expressed in the terms of the independent actuated variables, and the torque of the passive joints is equal to zero, the IDM of the closed chain structure will be:

$$\begin{aligned} \Gamma_m &= \Gamma_a + J^T\Gamma_p \\ &= A_{aa}\ddot{q}_a + A_{ap}\ddot{q}_p + H_a + J^T(A_{pa}\ddot{q}_a + A_{pp}\ddot{q}_p + H_p) \end{aligned} \tag{5}$$

By using equations (2) and (3), the above equation can be written as:

$$\Gamma_m = A_m\ddot{q}_a + H_m \tag{6}$$

Where: $A_m = (A_{aa} + A_{ap}J + J^T A_{pa} + J^T A_{pp} J)$

$$H_m = (H_a + A_{ap}D + J^T A_{pp} D + J^T H_p)$$

The matrix A_{ar} can be calculated by the algorithm of Newton-Euler, by noting from the relation (4) that the ith column is equal to Γ:

$$A_{ar}(:, i) = f(q, 0, u_i, 0,0,0,0,0) \tag{7}$$

u_i is the unit (n x 1) vector, whose elements are zero except the ith element which is equal to 1.

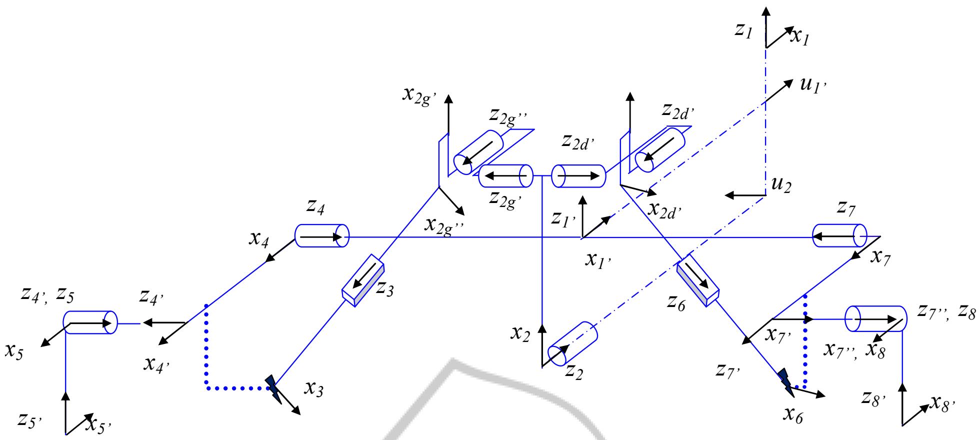

Figure 3: Geometric description of the rear train.

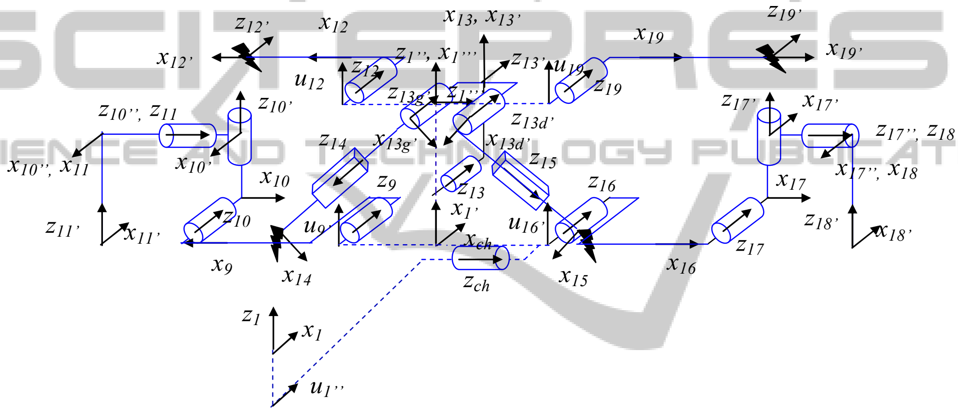

Figure 4: Geometric description of the front train.

The calculation of the vector H_{ar} can be obtained with the Newton-Euler method, by noting that $H = \Gamma$ if:

$$H_{ar} = f(q, \dot{q}, 0, f_e, g, F_s, F_v, K) \tag{8}$$

5 SIMULATOR

To predict the behaviours of the vehicle, we made a simulator by using the dynamic model obtained from the equation of Newton-Euler. The simulator architecture is described in figure 5.

5.1 Direct Dynamic Model with Constraints

To keep the tires in contact with the ground, we must add four constraints to the dynamic model. Therefore, the vertical velocity and acceleration of the contact tire/road with respect to the reference frame must be equal to zero.

$$\left[{}^{f}V_{5(3)} \;\; {}^{f}V_{8(3)} \;\; {}^{f}V_{11(3)} \;\; {}^{f}V_{18(3)}\right]^T = \Phi\dot{q}_a = 0 \tag{9}$$

$$\frac{d}{dt}\left[{}^{f}V_{5(3)} \;\; {}^{f}V_{8(3)} \;\; {}^{f}V_{11(3)} \;\; {}^{f}V_{18(3)}\right]^T = \Phi\ddot{q}_a + \dot{\Phi}\dot{q}_a = 0 \tag{10}$$

Equation (13) becomes:

$$\Gamma = A_m\ddot{q}_a + H_m + \Phi^T\lambda$$

Where:

- λ is the lagrangien multiplier vector
- $\Phi^T\lambda$ represents the vector of the efforts transmitted by joints to respect the constraints.

And the direct dynamical model that gives the joint accelerations as a function of joint positions, velocities torques, and external wrenches (forces and moments) will be:

$$\begin{bmatrix} \ddot{q}_a \\ \lambda \end{bmatrix} = \begin{bmatrix} A_m(q_t) & \Phi^T \\ \Phi & 0_{(4x4)} \end{bmatrix}^{-1} [\Gamma_m - H_m] \tag{11}$$

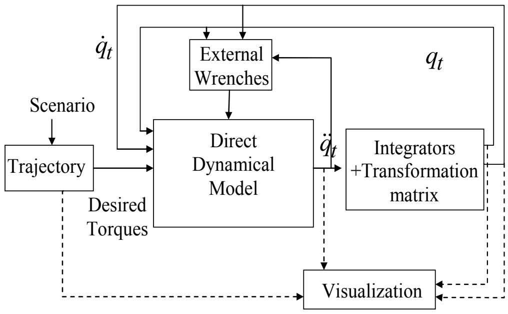

Figure 5: Simulator architecture.

6 SIMULATION

Two scenarios are considered. In the first, the vehicle is subject to a traction torque applied on the rear wheels which generate an acceleration phase then a decelerating one as shown in Figure 6. The trajectory is a straight line with initial velocity of 5 m/s.

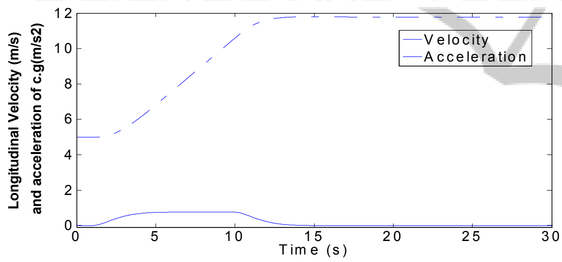

Figure 6: Longitudinal velocity and acceleration of c.g.

In the second the vehicle is subject to a desired steering torque to follow the trajectory of Figure 7. However, to maintain the stability, the bicycle must tilt into the corner such that the resultant force of the lateral acceleration and the weight of the vehicle is along the vertical axis of the vehicle (So et al 1997). The desired tilt angle will be the roll of the bicycle and it will be equal to:

$$\theta_{des} = \tan^{-1}({}^{f}V_{x1}\dot{\psi} \; / \; g) \qquad (12)$$

In order to get that, a simple PD controller is used to stabilize the roll dynamics to the desired tilt angle (Figure 8). The controller's output represents the required tilting torque applied on the rear lyre to stabilize the vehicle (Figure 9).

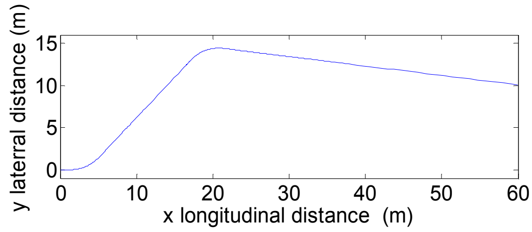

Figure 7: Trajectory of c.g in the horizontal plan.

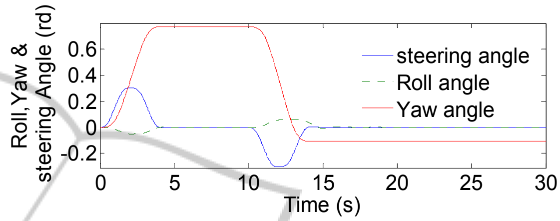

Figure 8: Trajectory of c.g in the horizontal plan.

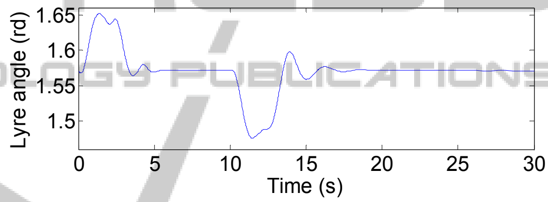

Figure 9: Rear Lyre motorized angle.

7 CONCLUSIONS

This paper presents the dynamical model of a narrow tilting car where the structure is complex. Future works consists on, simulating more scenarios as high speed, considering aerodynamic forces, elaborating robust control strategies to avoid external perturbation and maintain the stability of the vehicle.

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