TWO ELITIST VARIANTS OF DIFFERENTIAL ANT-STIGMERGY ALGORITHM

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Abstract: This paper deals with the analysis for two types of elitist variant proposed for the DASA algorithm. It is usual for the genetic algorithms to keep the best solution found in the population used from next generation. Another way to insert elitist behaviour in algorithms that construct solution is to use the most attractive components in order to obtain god quality solution, and may be the optimal ones. Based on particularities of ant colony based metaheuristics these two types of elitist behaviour were successfully applied to DASA algorithm. In this paper the efficiency of the proposed elitist variants of DASA algorithms is analyzed using experimental results. The analysis is applied to six benchmark functions from the class of high-dimensional real-parameter optimization problems.

1 INTRODUCTION

When a metaheuristic proves to be effective for a class of optimisation problems, one of the logical steps forward is to be adapted to solve other class of problems. Ant Colony Optimization (ACO) is a metaheuristic which has passed such a process. ACO was originally developed for combinatorial optimization problems such as Travel Salesman Problem (TSP) (M. Dorigo et al., 1996) or Single Machine Total Weighted Tardiness problem (Bauer et al. 1999). After that, several variants of algorithms which use a pheromone mediated communication have been proposed to solve real parameter optimization problems. The continuous ant colony optimization (CACO) (Bilchev and Parmee, 1995) was the first proposed adaptation of Ant System metaheuristic to continuous search space. CACO initialize the population of ants with the same solution (nest) and generate random directions which will be followed by ants in their search. If an ant improves the fitness function, the used direction is updated. The API algorithm was proposed in 2000(Monmarche et al, 2000). Here, all ants start from the nest and each of them search independently for solution. This algorithm also uses a recruitment strategy to refine the search. In 2004, Socha proposes ACOℝ (Socha, 2004) that uses a population including the n best solutions found so far by ants, to probabilistically sample the search space. Finally, Korosec proposed in 2006 the differential ant-stigmergy algorithm (DASA) (Korosec, 2006). This algorithm uses one solution which is improved iteratively. In DASA, the ants do not operate on the search space, but on the space of differences that will modify the current solution. This algorithm was successfully applied to high-dimensional benchmark functions.

In this paper, two elitist variants of DASA algorithm are proposed. Keeping the best found so far solution in the population is usual in genetic algorithms implementations. Using the most attractive components in construction of the solution is another way to insert elitist behaviour in algorithms. This type of elitist behaviour was successfully applied in ant based metaheuristics. The motivation for appealing to elitism is, as usual, the desire to increase the speed of convergence towards promising areas of search space. Based on the special properties of the DASA algorithm, both approaches were investigated.

The second section of this paper presents the two elitist proposed variants of the DASA algorithm. The presentation starting point is the basic form of the DASA algorithm and the improvement of the optimisation strategies are formulated as two elitist derivate behaviours.

The third section of the paper named Experimental results is structured in four subsections

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presenting the experimental environment and the benchmark functions, the algorithm parameter settings, the testing procedure and the obtained results. The last section is dedicated to conclusion and future work.

2 PROPOSED VARIANTS OF DIFFERENTIAL ANT-STIGMERGY ALGORITHM

2.1 Basic Form of DASA

Differential ant-stigmergy algorithm (DASA) uses a fine-grained discrete form of the continuous spaces (Korosec, 2006). For each direction of the search space, the difference, which can be applied to the current solution, may take a value from a finite set of discrete values. Each discrete value of the difference is attached to a node in a graph. In metaheuristics based on ant metaphor, nodes in the graph are associated with pheromone values, which will measure their attractiveness. In DASA algorithm, the nodes form level *j* of the construction graph corresponds to *j* direction (dimension) of the search space. To each level in the graph is associated a pheromone distribution function, which correspond to a Cauchy Probability Density Function (PDF)

$$C(x) = \frac{1}{s \cdot \pi \cdot \left(1 + \left(\frac{x-l}{s}\right)^2\right)} \qquad (1)$$

where l is the location offset and $s=s_{\text{global}}- s_{\text{local}}$ is a scale factor.

An ant constructs a path in the graph by sampling the PDFs. The constructed path corresponds to a difference vector

$$\Delta = \{\delta_1, ..., \delta_j, ...\delta_D\}$$

which specify the amplitude of the move in the search space. Adding to temporary solution the vector Δ with the values corresponding to path constructed by an ant weighted with a random values, generates the solution $X=\{x_1, x_j, ... x_D\}$ of that ant with

$$x_j = x'_j + \omega_j \delta_j \qquad (2)$$

where - x_j is the *j* component of ant solution;

- x'_j is the *j* component of temporary best solution;
- the weight ω_j is a random integer number draw from {1, 2,…, (*b*-1)};
- δ_j is the sampled offset step.

Table 1: Percent of optimum selection.

Number of choice	10.000	100.000
function	optimum selection [%]	
f1	11.34	16.37
f2	11.75	4.37
f3	11.26	4.21
f4	10.98	9.79
f5	11.76	6.77
f6	12.06	11.80
mean	11.52	8.88

After an improvement of the current solution, the pheromone is redistributed by centering the Cauchy PDFs on the differences that generated the improvement. In each of algorithm iteration, the parameters s_{global} and s_{local} are updated with the aim to balance between exploration of the search space and exploitation of a promising area.

Experimental results with the percent in which the ants chose the node corresponding to the maximum of pheromone are presented in table 1. It may be noted that ants, by sampling the Cauchy distribution, select the node corresponding to the value in that is centered the PDF on average only in 10% of the choices made. This means that ants do not effectively use the information memorized in pheromone trails. So the next two variants of elitist behavior were inserted in DASA Algorithm.

2.2 Pure Elitist DASA

In this variant of DASA algorithm, one of the *m* ants will use the same path/differences from the previous iteration, if that iteration has generated an improvement. In DASA, the differences corresponding to the path constructed by an ant, are weighted with a random value. The variant of algorithm, that use in (2) for elitist ant a weight random generated is named DASA-*elitist-A* algorithm. In the case that the elitist ant use the same weight that generate the improvement, at the previous iteration, the variants of algorithm is named DASA-*elitist-B*.

2.2 Probabilistic Elitist DASA

The probabilistic elitist DASA approach directly controls the percent in which an ant chose the node corresponding to the maximum value of pheromone. This type of elitism was successfully applied in Ant Colony System for Travelling Salesman Problem. In this variant of DASA algorithm, every ant chose with probability α∈(0, 1) the node in which the Cauchy probability density function is centered.

With complementary probability (1 - α), an ant chose a node by sampling the Cauchy PDF. This variant of algorithm is named DASA-*elitist-C*.

The parameter α controls the importance of the information given by ant pheromones. If α is small, the ants are able to achieve more choices different from the optimal value on which is centered Cauchy distribution. The choices made by ants, however, are not purely random but are also based on the Cauchy distribution, thus achieving a oriented search.

This type of elitism increases the local search. In the case of DASA-*elitist-C* algorithm, the local search action on the difference vector. So, this local search tries to keep the same speed in improving the current solution and not to search around the current solution.

3 EXPERIMENTAL RESULTS

3.1 The Experimental Environment and the Benchmark Suite

The computer platform used to perform the experiments was based on Intel dual core 2.13-GHz processor, 2 GB of RAM. The DASA was implemented in VisualC.

The proposed variants of DASA algorithm was tested on a set of six benchmark functions defined for CEC 2008 Special Session on Large Scale Global Optimization. The six functions are sphere, Schwefel, Rosenbrock, Rastrigin, Griewank and Ackley. To prevent exploitation of the symmetry of the search space and of the typical zero value associated with the global optimum, local optima of these functions are shifted to values different from zero, and the function values in the global optima are non-zero. A definition of them can be found in (Tang et al, 2007). The six functions are defined on a search space with D=100 dimensions (number of parameters) and the minima is searched.

3.2 Algorithm Parameter Settings

The DASA has six parameters: the number of ants, m; the discrete base, b; the pheromone dispersion factor, ρ; the global scale-increasing factor, s+; the global scale-decreasing factor, s-; and the maximum parameter precision, ε. For the basic form of DASA and for its elitist variants it was used the default parameter settings: m = 10, b = 10, ρ= 0.2, s+= 0.02, s-= 0.01, and ε = 1.0E-15. This values was selected based on recommended values (Korosec, 2006).

3.3 Testing Procedure

The experimental results are recorded over 25 trials on each pair, benchmark function and algorithm. Every trial used different seed for random number generator.

The function error, Error=f(x)-f(x*), where x* is the optimum, is recorded after 50xD, 500xD, and 5000xD function evaluations (FEs). The Error is collected for n= 25 runs and then the trials are ordered from best to worst. The results of the 1st (Best) and 25th (Worst) trial, as well as the trial mean (Mean), standard deviation (StDev) and root relative squared error (RRSE) are presented in tables 3 and 5. Here, the RRSE is defined as:

$$RRSE = \sqrt{\frac{\sum_{i=1}^{n} Error_i^2}{\sum_{i=1}^{n} (Error_i - Mean)^2}}$$

3.4 Results

To evaluate the quality of elitist ants in pure elitist versions of DASA, elitist ant's performance was compared with those of a normal ant. In tests performed, it was counted the number of iteration in which the elitist ant is the best, and number of improves of temporary solution generated by elitist ant. In table 2 is presented as a percentage the efficiency of the two types of pure elitist ants.

If we consider the basic form of DASA algorithm, all ants are equals and they have the same chance to be the best of the iteration. Therefore the chance of one from the 10 ants, used in tests, to be the best of the iteration should be around 10%.

Table 2 shows that *elitist*-A ant is the best of the iteration in 35.01% of iterations. This result is repeated in the case of the percentage of temporary solution improvements generated by *elitist-A* ant reported to the total number of improvements. The average percentage for the 6 functions is 35.49%.

The *elitist-B* ant got better results. Thus, in 41.83% of iterations the *elitist-B* ant is the best of iteration. If a normal ant is the best of the iteration on average in a percentage (100% -41.83%) / (m-1) = 6.46% iterations, that means the *elitist-B* ant is better by 41.83/6.46 = 6.47 times than a normal ant.

The error evolution presented in table 3, prove that the elitist-A and elitist-B maintain the convergence of DASA for the six functions considered in test. The recommended number function evaluations, 50xD, 500xD and 500xD, to be used in paper for CEC 2008 Special Session on Large Scale Global Optimization, do not permit to rank the DASA variants. For all 3 variants of DASA

Table 2: The quality of elitist ant.

Alg.	Measure	Function						
		*f*1	*f*2	*f*3	*f*4	*f*5	*f*6	Mean
DASA-*elitist-A*	(1) number of iteration in which the elitist ant is the best of iteration [%]	33.93	43.84	37.88	30.86	33.29	30.25	35.01
	(2) number of iteration in which the elitist ant improves the temporary best solution [%]	21.39	31.80	24.60	19.71	20.89	18.97	22.89
	(3) number of iteration in which the temporary best solution is improved [%]	62.47	66.97	66.91	63.08	62.02	64.28	64.29
	(4) percentage of improvements generated by elitist ant 2/3 [%]	34.24	47.48	36.77	31.25	33.68	29.51	35.49
DASA-*elitist-B*	(1) number of iteration in which the elitist ant is the best of iteration [%]	39.90	51.60	44.54	37.56	40.19	37.19	41.83
	(2) number of iteration in which the elitist ant improves the temporary best solution [%]	26.42	39.10	30.50	24.76	26.58	24.38	28.62
	(3) number of iteration in which the temporary best solution is improved [%]	62.67	66.99	66.92	63.19	62.62	64.39	64.46
	(4) percentage of improvements generated by elitist ant (2)/(3) [%]	42.16	58.37	45.58	39.18	42.45	37.86	44.27

Alg.	Measure	Function						
		*f*1	*f*2	*f*3	*f*4	*f*5	*f*6	Mean
DASA-*elitist-A*	(1) number of iteration in which the elitist ant is the best of iteration [%]	33.93	43.84	37.88	30.86	33.29	30.25	35.01
	(2) number of iteration in which the elitist ant improves the temporary best solution [%]	21.39	31.80	24.60	19.71	20.89	18.97	22.89
	(3) number of iteration in which the temporary best solution is improved [%]	62.47	66.97	66.91	63.08	62.02	64.28	64.29
	(4) percentage of improvements generated by elitist ant 2/3 [%]	34.24	47.48	36.77	31.25	33.68	29.51	35.49
DASA-*elitist-B*	(1) number of iteration in which the elitist ant is the best of iteration [%]	39.90	51.60	44.54	37.56	40.19	37.19	41.83
	(2) number of iteration in which the elitist ant improves the temporary best solution [%]	26.42	39.10	30.50	24.76	26.58	24.38	28.62
	(3) number of iteration in which the temporary best solution is improved [%]	62.67	66.99	66.92	63.19	62.62	64.39	64.46
	(4) percentage of improvements generated by elitist ant (2)/(3) [%]	42.16	58.37	45.58	39.18	42.45	37.86	44.27

analyzed in table 3, the mean of error over 25 trials are under 1E-10 for function f1, f4, f5 and f6, after 500000 FEs. Usually, it is considered that for an error under 1.E-9 the searched optima is founded, so the 3 DASA variants from table 3 are equivalent for function f1, f4, f5 and f6. For this 4 function, a supplementary test, record the number of iteration needed to obtain an error under 1.E-9. The minimum number of iterations, the maximum number of iterations and the mean number of iterations over 25 trials are presented in table 4. If we analyze the mean number of iteration over the 25 trails, than we can observe that the standard variant of DASA perform better like elitist variants for function f1 and f6. For function f4 and f5 the DASA *elitist-A* variant performs better.

For function f2 and f3, that have non-separable parameters, the performance of standard DASA and elitist-A DASA are equivalent.

The least performing, between the three variant of DASA, is the *elitist*-B. However, the performance difference is not significant. The errors obtained by DASA *elitist*-B have the same order of magnitude with those of the other two variants of DASA.

The experimental results for DASA-*elitist-C* are presented in table 5 and 6. Evolution of the average error obtained after 500,000 FEs shows for all six functions that a higher value of alpha increases the algorithm convergence. Analysis of the results table 5 recommends for α a value of 0.8. The performance of DASA *elitist-C* is equivalent to those of DASA elitist-B, the error after 500,000 Fes having the same order of magnitude.

The maximum, minimum and average numbers of iterations required by DASA-*elitist-C* algorithm to obtain an error less than 1E-9 are given in table 6. The associated execution time is also recorded. The time needed by DASA *elitist-C*, to obtain an error less than 1E-9, decrease when parameter α increases. This is happening because of the time needed to sample Cauchy PDF, that is non negligible if it is compared with time needed to evaluate the optimized function. The results from table 6 recommend also a value of α=0.8

4 CONCLUSIONS

The tests results prove that using elitism in DASA algorithm can improve for same function the convergence of algorithm. The expected results for the elitist strategies are confirmed through the experimental results, presented in the tables above. The number of iterations necessary to reach the optima is smaller for the DASA-elitist-A, and DASA-elitist-B. The future work is to analyze the use of booth type of elitism in parallel.

Table 3: Error values produced with DASA standard, DASA *elitist*-A and DASA *elitist*-B for function f1-f6.

FEs	Error	Algorithm	Functions *f1*	*f2*	*f3*	*f4*	*f5*	*f6*
5,000	Best	standard	**2,21E+003**	6,03E+001	**7,70E+007**	**1,35E+002**	**7,40E+000**	**9,25E+000**
		elitist-A	3,00E+003	6,62E+001	1,17E+008	1,47E+002	9,41E+000	9,38E+000
		elitist-B	2,57E+003	**5,99E+001**	1,20E+008	1,43E+002	1,09E+001	1,02E+001
	Worst	standard	**5,40E+003**	**7,80E+001**	**3,25E+008**	**2,06E+002**	**2,00E+001**	1,75E+001
		elitist-A	6,87E+003	8,91E+001	5,08E+008	2,47E+002	2,73E+001	1,75E+001
		elitist-B	6,00E+003	8,91E+001	5,99E+008	2,54E+002	2,85E+001	**1,63E+001**
	Mean	standard	**3,51E+003**	**6,96E+001**	**1,70E+008**	**1,80E+002**	**1,37E+001**	1,34E+001
		elitist-A	4,38E+003	7,47E+001	2,56E+008	1,91E+002	1,60E+001	**1,31E+001**
		elitist-B	4,40E+003	7,62E+001	3,59E+008	1,99E+002	1,73E+001	1,35E+001
	StDev	standard	**8,24E+002**	**4,14E+000**	**6,65E+007**	**1,73E+001**	**3,12E+000**	1,97E+000
		elitist-A	1,02E+003	5,99E+000	9,00E+007	2,44E+001	4,51E+000	2,19E+000
		elitist-B	9,04E+002	7,37E+000	1,08E+008	2,75E+001	4,78E+000	**1,71E+000**
50,000	Best	standard	**7,49E-011**	**1,18E+001**	1,55E+002	**4,24E-009**	**1,27E-011**	**4,05E-006**
		elitist-A	3,81E-009	1,84E+001	**1,49E+002**	4,90E-009	4,53E-010	1,56E-005
		elitist-B	5,62E-008	2,03E+001	1,79E+002	1,93E-007	8,91E-009	4,36E-005
	Worst	standard	**2,95E-009**	**1,69E+001**	1,65E+004	1,99E+000	7,07E-002	**3,07E-005**
		elitist-A	1,10E-007	2,55E+001	1,61E+004	3,20E+000	**6,58E-002**	9,83E-005
		elitist-B	2,57E-006	2,71E+001	1,61E+004	1,99E+000	7,86E-002	3,08E-004
	Mean	standard	**7,33E-010**	**1,43E+001**	**3,81E+003**	**4,78E-001**	1,14E-002	**1,15E-005**
		elitist-A	2,53E-008	2,16E+001	5,44E+003	6,06E-001	**8,14E-003**	3,86E-005
		elitist-B	5,09E-007	2,41E+001	3,90E+003	7,96E-001	1,11E-002	1,36E-004
	StDev	standard	**5,95E-010**	**1,17E+000**	**4,99E+003**	**5,72E-001**	1,79E-002	**6,53E-006**
		elitist-A	2,43E-008	1,57E+000	6,78E+003	8,70E-001	**1,45E-002**	2,03E-005
		elitist-B	5,16E-007	1,74E+000	5,94E+003	7,96E-001	1,95E-002	6,80E-005
	RRSE	standard	1,59E+000	1,23E+001	1,26E+000	1,30E+000	1,19E+000	2,03E+000
		elitist-A	1,44E+000	1,38E+001	1,28E+000	1,22E+000	1,15E+000	2,15E+000
		elitist-B	1,40E+000	1,39E+001	1,20E+000	1,41E+000	1,15E+000	2,23E+000
500,000	Best	standard	**3,52E-012**	**1,67E-002**	2,21E-001	**5,80E-012**	3,75E-012	6,11E-012
		elitist-A	5,29E-012	7,19E-001	**6,30E-002**	6,76E-012	**3,64E-012**	7,30E-012
		elitist-B	5,17E-012	1,55E+000	2,49E-001	6,42E-012	3,67E-012	**6,05E-012**
	Worst	standard	1,42E-011	**3,54E-002**	1,41E+003	2,73E-011	**1,17E-011**	1,15E-011
		elitist-A	1,46E-011	1,10E+000	1,30E+003	1,99E-011	1,43E-011	1,19E-011
		elitist-B	**1,24E-011**	2,11E+000	**5,43E+002**	**1,92E-011**	2,15E-011	**1,13E-011**
	Mean	standard	9,51E-012	**2,44E-002**	**1,66E+002**	1,19E-011	6,55E-012	**8,17E-012**
		elitist-A	9,47E-012	8,47E-001	2,52E+002	1,22E-011	6,74E-012	9,07E-012
		elitist-B	**8,54E-012**	1,86E+000	1,77E+002	**1,10E-011**	**6,06E-012**	8,19E-012
	StDev	standard	2,49E-012	**4,39E-003**	2,77E+002	4,96E-012	**1,84E-012**	**1,07E-012**
		elitist-A	2,34E-012	8,59E-002	3,24E+002	3,76E-012	2,54E-012	1,31E-012
		elitist-B	**2,26E-012**	1,35E-001	**1,37E+002**	**3,66E-012**	3,51E-012	1,37E-012
	RRSE	standard	3,95E+000	5,66E+000	1,17E+000	2,60E+000	3,70E+000	7,71E+000
		elitist-A	4,17E+000	9,91E+000	1,27E+000	3,39E+000	2,84E+000	6,98E+000
		elitist-B	3,92E+000	1,39E+001	1,63E+000	3,16E+000	1,99E+000	6,05E+000

Table 4: Number of iteration needed by DASA *elitist*-A and DASA *elitist*-A to obtain an error under 1E-9.

Number of iteration	Algorithm	Function *f1*	relativ[%]	*f4*	relativ[%]	*f5*	relativ[%]	*f6*	relativ[%]
Minimum	standard	**46.951**	0,00	**52.301**	0,00	**44.081**	0,00	**75.371**	0,00
	elitist-A	52.382	11,57	53.072	1,47	48.732	10,55	81.072	7,56
	elitist-B	57.211	21,85	57.111	9,20	55.381	25,63	88.701	17,69
Maximum	standard	51.781	0,00	124.222	0,00	263.975	14,01	82.981	0,00
	elitist-A	57.812	11,65	144.583	16,39	231.545	0,00	93.962	13,23
	elitist-B	**64.291**	24,16	**232.023**	86,78	**>500.000**	>115,94	**100.351**	20,93
Mean	standard	**49.195,80**	0,00	71.870,76	0,79	103.731,64	7,29	**79.688,60**	0,00
	elitist-A	55.054,40	11,91	**71.306,48**	0,00	**96.685,56**	0,00	86.699,20	8,80
	elitist-B	61.141,00	24,28	78.804,72	10,52	136.245,36	40,92	94.807,80	18,97

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Table 5: Error values produced with DASA standard and DASA *elitist-C* for function f1-f6.

FEs	Error	Algorithm	α	*f1*	*f2*	*f3*	*f4*	*f5*	*f6*
500,000	Best	standard		**3,52E-012**	**1,67E-002**	2,21E-001	5,80E-012	3,75E-012	6,11E-012
		elitist-C	0,5	6,54E-012	4,38E-002	1,85E-001	8,07E-012	**2,76E-012**	6,34E-012
			0,6	4,32E-012	5,29E-002	2,18E-001	7,05E-012	3,21E-012	6,51E-012
			0,7	5,46E-012	1,13E-001	**5,88E-003**	7,16E-012	3,38E-012	**6,00E-012**
			0,8	3,69E-012	3,20E-001	3,26E-001	7,05E-012	3,10E-012	6,48E-012
			0,9	4,49E-012	2,20E+000	1,40E+001	**5,06E-012**	3,67E-012	6,99E-012
	Worst	standard		1,42E-011	**3,54E-002**	1,41E+003	2,73E-011	1,17E-011	1,15E-011
		elitist-C	0,5	1,72E-011	9,09E-002	1,33E+003	2,25E-011	**8,92E-012**	**1,10E-011**
			0,6	**1,27E-011**	1,17E-001	1,29E+003	2,43E-011	1,34E-011	1,14E-011
			0,7	1,49E-011	2,27E-001	7,44E+002	2,16E-011	9,86E-003	**1,10E-011**
			0,8	1,35E-011	6,40E-001	1,22E+003	**2,02E-011**	1,21E-011	1,39E-011
			0,9	1,57E-011	3,83E+000	**5,66E+002**	2,12E-011	1,83E-011	1,42E-011
	Mean	standard		9,51E-012	**2,44E-002**	**1,66E+002**	1,19E-011	6,55E-012	**8,17E-012**
		elitist-C	0,5	1,05E-011	6,46E-002	3,00E+002	1,40E-011	**5,71E-012**	8,74E-012
			0,6	8,90E-012	9,30E-002	1,75E+002	1,25E-011	5,93E-012	8,46E-012
			0,7	9,61E-012	1,70E-001	1,85E+002	1,26E-011	6,90E-004	8,70E-012
			0,8	**8,72E-012**	4,47E-001	2,59E+002	1,18E-011	6,15E-012	9,28E-012
			0,9	9,41E-012	2,97E+000	2,01E+002	**1,15E-011**	6,49E-012	9,78E-012

Table 6: Number of iteration needed by DASA *elitist-C* to obtain an error under 1E-9.

Function		f1		f4		f5		f6	
	α	FEs	time[s]	FEs	time[s]	FEs	time[s]	FEs	time[s]
Minimum	0,5	48.211	1,9960	49.111	2,4960	44.251	2,6520	75.951	3,9630
	0,6	47.611	1,8560	52.931	2,5740	44.651	2,5430	78.321	3,8220
	0,7	48.261	1,7470	54.771	2,5270	44.641	2,4490	77.551	3,5880
	0,8	48.661	**1,6530**	55.451	2,4180	46.111	**2,4180**	80.711	**3,5410**
	0,9	53.331	1,6690	55.391	**2,2780**	51.341	2,5740	86.161	3,5720
Maximum	0,5	52.701	2,2000	119.482	6,2710	328.526	20,7480	84.241	4,5710
	0,6	52.921	2,0440	123.682	6,1460	392.797	23,5870	84.391	4,1180
	0,7	54.011	1,9500	141.902	6,5990	>500.000	>19,0630	86.801	4,0090
	0,8	54.981	2,4650	146.092	6,5050	350.256	18,9700	89.361	4,4150
	0,9	59.631	2,4500	214.523	9,1100	376.516	20,0770	96.571	4,1810
Mean	0,5	50.041,40	2,0802	63.460,24	3,2392	84.289,28	**5,1910**	80.400,20	4,2001
	0,6	50.349,00	1,9524	72.306,76	3,5294	103.546,80	6,0865	81.451,00	3,9786
	0,7	50.755,80	1,8377	67.517,44	3,1000	>139.199,08	7,5730	82.566,60	3,8201
	0,8	52.055,40	**1,7890**	68.731,88	3,0052	108.038,00	5,8419	83.873,40	**3,7091**
	0,9	56.746,60	1,8433	70.185,88	**2,9072**	132.655,44	6,8709	92.127,00	3,8401

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