TRACKING CONTROL FOR
TWO-DIMENSIONAL OVERHEAD CRANE
Feedback Linearization with Linear Observer

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Abstract: A possible way to control non-linear systems is the use of exact linearization and the application of a tracking controller to ensure exponential decay of the error along the reference trajectory. In case of overhead cranes, it can be used if the load coordinates (or alternatively the rope angles) are known which is not the case in real applications, where the motor axis displacements are usually measured. This paper applies the linearization techniques such that the calculations of unmeasured states are realized with an observer, which is constructed for the linear approximation of the dynamics along the reference trajectory. Simulation results are provided to prove the applicability of the concept.

1 INTRODUCTION

Cranes and other types of weight handling equipment are used to carry heavy loads (Gustafsson, 1996; D. Buccieri and Bonvin, 2005; Kiss and Mullhaupt, 1999). In many cases, the load is attached to the mechanical structure with a rope, so the load position cannot be directly actuated and the resulting oscillatory behavior may present serious difficulties to inexperienced human operators. This oscillatory nature of crane-like underactuated mechanical systems makes them a popular benchmark application in control engineering. Several tracking and sway elimination algorithms are proposed in the literature (Lévine et al., 1997; Marttinen et al., 1990; Neupert and Schneider, 2006; Overton, 1996; Hong et al., 1998), but many of them are based on the knowledge of the load coordinates (or alternatively the rope angles) which are generally difficult to robustly measure in real applications.

The flatness property (Lévine, 2009) of the crane models implies their feedback linearizability and it can also be exploited for motion planning purposes. We propose to apply a time-varying linear observer to the linearized system dynamics along the reference trajectory to determine the value of the unmeasured state variables which need to be injected in the tracking feedback. The calculations and the simulation re-

sults will be presented on a simple, two-dimensional overhead crane and they can be generalized for more complex structures.

The remaining part of the paper is organized as follows. Section 2 introduces some notations and presents the dynamics of the two-dimensional overhead crane. The tracking controller and the observer design is presented in Section 3. Simulation results of the trajectory behavior are shown in Section 4, and our results are summarized in Section 5.

2 SYSTEM DYNAMICS

The two-dimensional overhead crane is illustrated in Figure 1. The horizontal displacement of the cart with mass M is denoted by R. The cart is actuated by a motor delivering the force F which is considered to be one of the input of the dynamics. The load with mass m, having the coordinates (x_m, z_m) is accelerated through a rope with the length of L, winched on a drum of inertia J and radius ρ with a motor delivering the torque T. The angle between the rope and the vertical is denoted by θ. We assume no friction and massless rope and due to the small value of ρ, we also assume that the rope always connects to the winch at point $(R+\rho, 0)$. The resulting mechanical system has
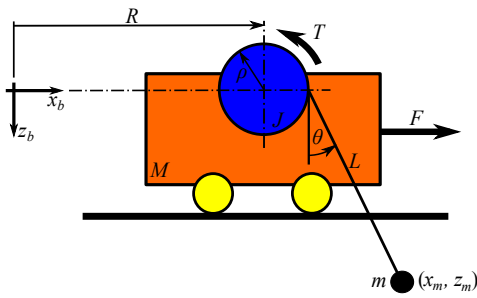
Figure 1: The overhead crane.

three degrees of freedom so the dimension of the state vector x is six. Several possible choices exist for x.

The non-linear system dynamics can be written in the classical form (Isidori, 1995):

$$\dot{x} = f(x) + g(x)u \qquad (1)$$

where x and u denote the state and input vectors, respectively. Let us define

$$x = \begin{bmatrix} R & \dot{R} & \theta & \dot{\theta} & L & \dot{L} \end{bmatrix}^T \qquad (2)$$

$$u = \begin{bmatrix} F & T \end{bmatrix}^T \qquad (3)$$

such that f and g read

$$f(x) = \begin{bmatrix} \dot{R} \\ \dfrac{mJL\dot{\theta}^2 \sin\theta + mgJ\sin\theta\cos\theta}{D} \\ \dot{\theta} \\ -\dfrac{\sin\theta(Dg + mJL\dot{\theta}^2\cos\theta) + mgJ\cos^2\theta + 2D\dot{L}\dot{\theta}}{LD} \\ \dot{L} \\ \dfrac{mM\rho^2(L\dot{\theta}^2 + g\cos\theta)}{D} \end{bmatrix} \qquad (4)$$

$$g(x) = \begin{bmatrix} 0 & 0 \\ \dfrac{m\rho^2 + J}{D} & \dfrac{m\rho\sin\theta}{D} \\ 0 & 0 \\ -\dfrac{\cos\theta(m\rho^2 + J)}{LD} & -\dfrac{m\rho\sin\theta\cos\theta}{LD} \\ 0 & 0 \\ -\dfrac{m\rho^2\sin\theta}{D} & -\dfrac{\rho(m\sin^2\theta + M)}{D} \end{bmatrix} \qquad (5)$$

with

$$D = D(\theta) = mM\rho^2 + mJ\sin^2\theta + MJ. \qquad (6)$$

3 TRACKING CONTROL

In this section we present the linearizing and the tracking controller first, then we give the details of the observer design steps.

3.1 Stabilizing Feedback

It has been shown in (Fliess et al., 1993; Lévine et al., 1997; Kiss and Mullhaupt, 1999) that the system (1)–(5) with output

$$y = \begin{bmatrix} h_1(x) \\ h_2(x) \end{bmatrix} = \begin{bmatrix} R + \rho + L\sin\theta \\ L\cos\theta \end{bmatrix} \qquad (7)$$

is differentially flat, which implies that the model is feedback linearizable. The conception is shown in Figure 2. Notice that the elements of the flat output have been chosen as the Cartesian coordinates (x_m, z_m) of the load.
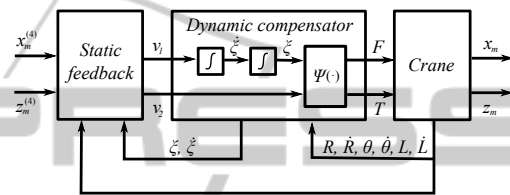

Figure 2: Block diagram of the linearizing feedback.

Based on (Boustany and d'Andréa Novel, 1992), the equations of the dynamic compensator are

$$\begin{bmatrix} F \\ T \end{bmatrix} = \alpha(x) + \beta(x)\begin{bmatrix} \xi \\ v_2 \end{bmatrix} \qquad (8)$$

$$\alpha(x) = \begin{bmatrix} -mg\sin\theta\cos\theta \\ mg\rho\cos\theta - \dfrac{J}{\rho}L\dot{\theta}^2 \end{bmatrix} \qquad (9)$$

$$\beta(x) = \begin{bmatrix} m\sin\theta & M \\ -\dfrac{m\rho^2 + J}{\rho} & \dfrac{J}{\rho}\sin\theta \end{bmatrix} \qquad (10)$$

$$\ddot{\xi} = v_1 \qquad (11)$$

Notice that ξ is the tension in the rope, divided by the load mass.

Combining (1)–(6) and (8)–(11), the extended state-space model can be written in the form

$$\dot{\tilde{x}} = \tilde{f}(\tilde{x}) + \begin{bmatrix} \tilde{g}_1(\tilde{x}) & \tilde{g}_2(\tilde{x}) \end{bmatrix}\tilde{u} \qquad (12)$$

with the new state and input vectors

$$\tilde{x} = \begin{bmatrix} R & \dot{R} & \theta & \dot{\theta} & L & \dot{L} & \xi & \dot{\xi} \end{bmatrix}^T \qquad (13)$$

$$\tilde{u} = \begin{bmatrix} v_1 & v_2 \end{bmatrix}^T \qquad (14)$$

In fact, the new vector fields $\tilde{f}$, $\tilde{g}_1$, $\tilde{g}_2$ read

$$\tilde{f}(\tilde{x}) = \begin{pmatrix} \dot{R} \\ 0 \\ \dot{\theta} \\ \frac{-2\dot{L}\dot{\theta}-g\sin\theta}{L} \\ \dot{L} \\ L\dot{\theta}^2 + \xi \\ \dot{\xi} \\ 0 \end{pmatrix} \quad (15)$$

$$\tilde{g}_1(\tilde{x}) = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \quad \tilde{g}_2(\tilde{x}) = \begin{pmatrix} 0 \\ 1 \\ 0 \\ -\frac{\cos\theta}{L} \\ 0 \\ -\sin\theta \\ 0 \\ 0 \end{pmatrix} \quad (16)$$

This extended model can be exactly linearized with static feedback which has the form (Isidori, 1995):

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = A^{-1}(\tilde{x}) \cdot \left(\begin{bmatrix} x_m^{(4)} \\ z_m^{(4)} \end{bmatrix} - b(\tilde{x}) \right) \quad (17)$$

where A and b are calculated via Lie derivatives:

$$A(\tilde{x}) = \begin{bmatrix} L_{\tilde{g}_1} L_{\tilde{f}}^3 h_1 & L_{\tilde{g}_2} L_{\tilde{f}}^3 h_1 \\ L_{\tilde{g}_1} L_{\tilde{f}}^3 h_2 & L_{\tilde{g}_2} L_{\tilde{f}}^3 h_2 \end{bmatrix} \quad (18)$$

$$b(\tilde{x}) = \begin{bmatrix} L_{\tilde{f}}^4 h_1 \\ L_{\tilde{f}}^4 h_2 \end{bmatrix} \quad (19)$$

The resulting system is a decoupled linear system

$$\dot{x}' = A' x' + B' u' \quad (20)$$

with

$$x' = \begin{bmatrix} x_m & \dot{x}_m & \ddot{x}_m & x_m^{(3)} & z_m & \dot{z}_m & \ddot{z}_m & z_m^{(3)} \end{bmatrix}^T \quad (21)$$

$$u' = \begin{bmatrix} x_m^{(4)} & z_m^{(4)} \end{bmatrix}^T \quad (22)$$

The elements of the new state vector x' can be expressed in coordinates of $\tilde{x}$ in (13). Note that the diffeomorphism is singular if

$$\cos\theta(g\cos\theta - \xi) = 0 \quad (23)$$

One can easily design an additional feedback law for the linear dynamics (20) which ensures the exponential decay of the tracking error, so that

$$e_x^{(4)} + k_{x,3} e_x^{(3)} + k_{x,2} \ddot{e}_x + k_{x,1} \dot{e}_x + k_{x,0} e_x = 0 \quad (24)$$

$$e_z^{(4)} + k_{z,3} e_z^{(3)} + k_{z,2} \ddot{e}_z + k_{z,1} \dot{e}_z + k_{z,0} e_z = 0 \quad (25)$$

are satisfied where

$$e_x = x_{m,ref} - x_m \quad (26)$$

$$e_z = z_{m,ref} - z_m \quad (27)$$

To determine the coefficients $k_{x,i}$, $k_{z,i}$, $i = 0, 1, 2, 3$, several methods can be used, e.g., LQR and pole placement techniques (Brogan, 1990).

3.2 State Observer

The previously proposed method assumes that the state vector x in (2) is known. However, in real applications only motor positions can be easily measured, i.e., R and L, but θ is difficult to determine.

We apply a linear state observer (Brogan, 1990) for this purpose, calculating the first-order Taylor approximation of the non-linear model (1)–(5) for every timestep along track. Simulation results show that this technique gives satisfactory result (see Section 4).

Introducing

$$\bar{x} = x - x_0 \quad (28)$$

$$\bar{u} = u - u_0 \quad (29)$$

$$\bar{y} = y - y_0 \quad (30)$$

the linear approximation of (1)–(5) at (x_0, u_0) can be written as follows:

$$\dot{\bar{x}} = \bar{A}(x_0, u_0)\bar{x} + \bar{B}(x_0, u_0)\bar{u} \quad (31)$$

The matrices $\bar{A}$ and $\bar{B}$ are calculated via symbolic derivation:

$$\bar{A} = \left.\frac{\partial f(x,u)}{\partial x}\right|_{(x_0, u_0)} \quad (32)$$

$$\bar{B} = \left.\frac{\partial f(x,u)}{\partial u}\right|_{(x_0, u_0)} \quad (33)$$

One can design a linear observer in the form

$$\dot{\hat{x}} = \bar{F}(x_0, u_0)\hat{x} + \bar{G}(x_0, u_0)\bar{y} + \bar{H}(x_0, u_0)\bar{u} \quad (34)$$

where $\hat{x}$ denotes the estimation of $\bar{x}$, and $\bar{F}, \bar{G}, \bar{H}$ can be chosen such that the estimation error $\bar{x} - \hat{x}$ decays exponentially.

Since the matrices $\bar{A}, \bar{B}, \bar{F}, \bar{G}, \bar{H}$ depend on the operating point, they are time-varying. Nevertheless, they can be calculated off-line for every reference trajectory which is important in real-time applications.

Note that this method also enables to design an additional load estimator which ensures robustness against unknown disturbances on the system input channels, i.e., F and T. Laboratory experiments show that this is an effective way to cancel friction forces.

3.3 Motion Planning

It is expected to carry the load with zero velocity and acceleration in the initial and final positions, so any oscillations in these two points are to be avoided. One can easily design a reference trajectory based on polynomial approximation, which satisfies these constraints (Lévine, 2009).

In the two-dimensional overhead crane example, the following expression can be used to calculate the reference trajectory in x direction:

$$x_m(t) = x_{m,I} + (x_{m,F} - x_{m,I}) \cdot \sum_{i=5}^{9} a_i \left(\frac{t - t_I}{t_F - t_I} \right)^i \quad (35)$$

where $t \in [t_I, t_F]$ and t_I, t_F denotes the time at the initial and final positions, respectively. The coefficients a_i are computed by solving a system of linear equations produced by the constraints mentioned above, namely $x_m(t_I) = x_{m,I}$, $\dot{x}_m(t_I) = \ddot{x}_m(t_I) = x_m^{(3)}(t_I) = x_m^{(4)}(t_I) = 0$ and $x_m(t_F) = x_{m,F}$, $\dot{x}_m(t_F) = \ddot{x}_m(t_F) = x_m^{(3)}(t_F) = x_m^{(4)}(t_F) = 0$ In fact, the numerical values of the coefficients are

$$
\begin{aligned}
a_5 &= 126 & a_6 &= -420 \\
a_7 &= 540 & a_8 &= -315 \\
a_9 &= 70
\end{aligned}
\quad (36)
$$

The derivatives can be easily calculated as well. The method in z direction is exactly the same as presented for x.

There are some situations, for example if an obstacle is present in the crane's workspace, when other type of path is required. According to (Lévine, 2009), the geometry of the trajectory can be given by the function

$$z_m = z_m(x_m) \quad (37)$$

where $x_m = x_m(t)$ can be calculated using the polynomial interpolation described by (35). Moreover, the geometry can also be specified in the following form:

$$x_m = x_m(\lambda) \quad (38)$$
$$z_m = z_m(\lambda) \quad (39)$$

Here $\lambda = \lambda(t)$ denotes the path parameter which can also be calculated using (35).

4 SIMULATIONS

This section shows simulation results based on the methods described above. The parameters are given as follows: $m = 1\,\mathrm{kg}$, $M = 3\,\mathrm{kg}$, $J = 0.1\,\mathrm{kg\,m^2}$, $\rho = 1.5\,\mathrm{cm}$, $x_{m,I} = 0.1\,\mathrm{m}$, $z_{m,I} = 1.1\,\mathrm{m}$, $x_{m,F} = 1.1\,\mathrm{m}$, $z_{m,F} = 0.1\,\mathrm{m}$, $T = 2\,\mathrm{s}$; sample time is set to $0.001\,\mathrm{s}$ and
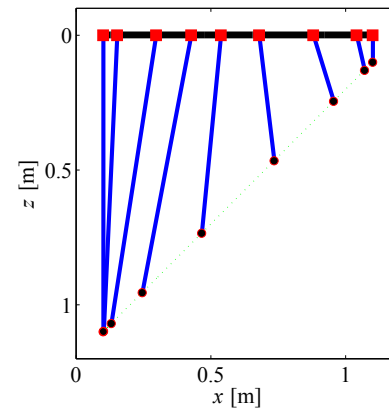

Figure 3: Illustration of the desired behavior.

the reference path is a straight line with a length of 1.41m. It is illustrated in Figure 3. Using pole placement technique, the observer's poles have been placed equidistantly between -5 and -15, while the closed-loop poles have been placed with the outer-loop controller to -5.

In the first simulation the model was initialized with the values of the reference trajectory. In this case, Figure 4 shows the differences between the real and estimated positions. One can see that the absolute value of the highest error is 10^{-4} m, which is totally acceptable in most cases.
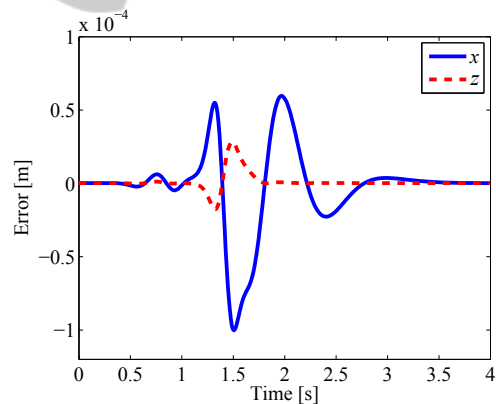

Figure 4: Estimation error in x and z directions.

On the other hand, if the initial states of the crane model differ from the reference states, the controller is still able to stabilize the system. Let us consider some initial oscillation, i.e., $\theta_I = \pi/20$, $\dot{\theta}_I = -0.5\,\mathrm{rad/s}$. The position errors are shown in Figure 5. The observer produces greater error here than in the previous case, but the system still reaches the equilibrium. Notice that initial error is also present since the estimator starts from the reference initial states.
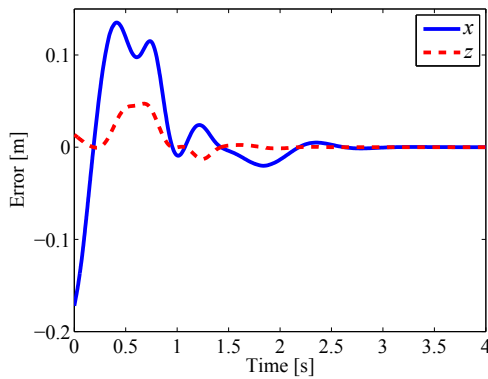
Finally, Figure 6 shows the case when the ini-

Figure 5: Estimation error in x and z directions if the crane model starts with an initial trajectory error.
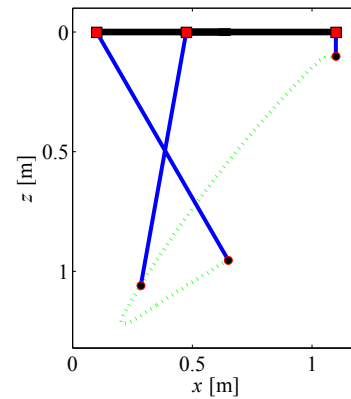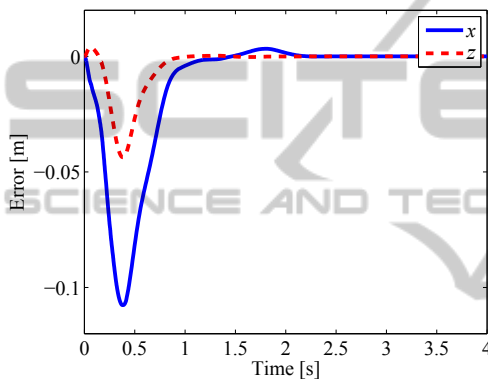

Figure 6: Estimation error in x and z directions if both the model and the observer start with the same initial state, which is not the initial reference state.

tial states of the crane model differ from the reference states, but the observer's states are also set to the model's initial states. It means that the initial error is known so the observer can be initialized properly. Thus, the initial estimation error is zero in this case.

The states have been modified to $\theta_I = \pi/6$, $\dot{\theta}_I = -1\,\mathrm{rad/s}$. Although the load starts far from the reference path (in fact, the distance is 0.57m), the controller stabilizes the system and the load is taken to the desired location. Figure 7 illustrates the trajectory in this situation.

5 CONCLUSIONS

We have presented the tracking control problem of the two-dimensional overhead crane. A general approach to solve it is using feedback linearization. It requires the exact knowledge of the system states, however, usually only two of the six states are measured. We proposed a simple estimation method to calculate the unmeasured states, which is based on the Taylor ap-

Figure 7: Trajectory in the case of high initial error.

proximation of the non-linear crane model. Simulation results show that this approach gives satisfactory result for small errors. Further research on this topic might include stability analysis with parameter uncertainties, and application to a laboratory equipment.

ACKNOWLEDGEMENTS

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