AN OPEN-LOOP SOLUTION FOR A STOCHASTIC PRODUCTION-REMANUFACTURING PLANNING PROBLEM

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Abstract: A stochastic linear production planning problem with chance-constraints is introduced in order to provide a production plan that optimizes a reverse logistics system. Such a system is composed of two channels: in the forward channel, new and remanufactured products are produced and stored into a serviceable inventory unit. On the other hands, in the reverse channel, used and defected products are stored into the remanufacturable inventory unit. The uncertainties about the fluctuation of demand and the amount of returnable are the reason of the stochastic nature of the problem. Since global optimal solution is not easy to be achieved, an equivalent-deterministic suboptimal problem is proposed. An example of applicability of this deterministic problem is presented. In this example, two situations are proposed: the first considers that 50% of used-products are returnable; while the second considers 100%. It is assumed that 5% of new products are rejected during the quality inspection process. As a result, the example shows that, under certain circumstances, it is possible to improve the performance of the system by increasing the return rate for used-products.

1 INTRODUCTION

Planning and control the flow of used-products throughout the reverse channel has become increasingly crucial for the success of any supply chain. A major reason that explains this fact is the pressure that governments and society have imposed to companies for that they preserve the environment. In this sense, collect and recovery used products can reduce the necessity for extraction of raw materials, and, as a consequence, helps the preservation of the environment. Thus, reverse logistic issue has become an important topic of the supply chain area.

Conceptually, reverse logistics term can be understood as a process of dealing with different activities that involve, for instance: the collecting of used-products from the market for remanufacturing, recycling or disposing them. More generically, the objective of reverse logistics is to move used-products from the market to their final destination with the aim of capturing value, or proper disposal.

In the literature, there are several papers related to logistic reverses issues. Most of them are based on quantitative models that are used to represent remanufacturing and recycling activities in the reverse channel. Fleischmann et al (1997) provide a typology of quantitative models for reverse logistics, which is based on three classes of problems, namely: (i) reverse distribution problems; (ii) inventory control problems in systems with return flows; and (iii) production planning problem with reuse of parts and materials. In short, the first class of this typology is concerned with the collection and transportation of used-products and packages. According the authors: “the reverse distribution can take place through the original forward channel, through a separate reverse channel, or through combinations of the forward and the reverse channel”. The second class is related to appropriate control mechanism that allows returning the used products into the market; and, the third class is associated with the planning of the reuse of items, parts and products without any additional process of remanufacturing. At last, it is worth mentioning that there are many different approaches to deal with each one of these problems.

The paper considers a sequential stochastic production planning problem with chance constraints on decision variables. Such a problem is formulated in order to deal with a stochastic production-inventory system with a special structure for collecting used-products from the market, and

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remanufacturing or disposing them. The stochastic nature of this system is due to the fact of demand fluctuation for serviceable products and return rate of used-products are random variables.

Stochastic production-remanufacturable systems are very commons in the reverse logistics field. In fact, return rates are usually estimated with base on demand level for serviceable products. This means that they are directly dependent on the stochastic fluctuation of demand over periods of the planning horizon. Fleischmann et al. (1997) consider that traditional techniques used to deal with stochastic inventory balance systems can be reapplied for reverse systems. Based on this, the stochastic linear programming technique is taking into account here to model and solve a combined stochastic production/remanufacturing problem with chance constraints on inventory variables.

Without loss of generality, it is assumed here that the demand fluctuation for serviceable products and the return rate of used-products are described by uncorrelated normal stationary processes; see Graves (1999). Thus, the production/remanufacturing problem with constraints is stochastically well-defined. This means that its random variables have probability distributions well-known along periods of the planning-horizon.

Since a global optimal solution for a sequential, constrained stochastic problem is not easy to obtain, nearly-optimal (heuristics) should be considered. In this paper, the certainty-equivalence principle is used to transform the stochastic problem into an equivalent-deterministic, see Bertsekas (2007). An open-loop solution is provided for this equivalent problem. This sub-optimal solution can be used by managers to improve their decision-making skills. For instance, considering that the object of this paper is to solve a production/remanufacturing problem, the open-loop solution provided enables managers to decide about a proper return rate of used-products that improve the productivity of the company. In fact, production/remanufacturing plans provided by the equivalent problem can be used to create production/remanufacturing scenarios. These scenarios are built from the variation of some parameters of the original stochastic problem, as for instance: percentage of the return rate of used products, time-delays for collecting used-products, customer satisfaction level due to safety-stock of serviceable products, and so on.

An example is introduced to show the applicability of the proposed problem. It compares two distinct production/remanufacturing policies that are provided by solving the equivalent problem. The first policy is the result of a production situation where only 50% of used-products return from the market. On the other hands, the second policy considers the situation where 100% is assumed to return. It is also considered for both situations that 5% of new products present some kind of defect, which is detected during quality inspection. At last, from analyzes of the two situations, it is possible to conclude that 100% of the return rate can improves the productivity of the company. However, it is important to mention that such a conclusion depends on production/inventory costs and operational conditions specified to the problem. These aspects will be discussed ahead.

The paper is stated as follows: initially, section 2 introduces comments on stochastic remanufacturing literature (a brief review). In the section 3, the reverse system is described. In sequel, the section 4 formulates the stochastic problem. Section 5 presents the transformation process form the original stochastic problem to equivalent-deterministic one. Each step of this conversion process is brief considered. Section 6 illustrates the applicability of this equivalent-problem in a company that deal with a single product production/remanufacturing system.

2 LITERATURE REVIEW

In this section, it is introduced a brief review of the literature. It highlights aspects of planning and inventory control of dynamic models that include remanufacturing processes.

In recent years, companies have begun to seek for solutions that increase the life cycle of their products. Many reasons can explain such an interest, but a special is the scarcity of material resources (i.e., the environment is finite). A way considered by companies is to recovery returnable products by, for instance, using remanufacturing systems. Thus, the planning and inventory control of remanufacturing systems have become an essential task. All modeling and solving techniques used to plan and control the production process in the forward channel can be reapplied to reverse channel.

Glancing at the literature, we find that the remanufacturing problems can be classified as deterministic and stochastic. This classification will depend on the hierarchical decision level where the problem is being formulated. For example, at the operational level (short term decisions) a good part of the problem is deterministic, while at the strategic level (long term decisions), they are stochastic. The planning problem proposed in this paper belongs to a class of stochastic problems.

Some papers that are directly or indirectly related to the problem proposed here are due to: Shi et al. (2011) that discuss a stochastic production planning problem for a multi-product closed-loop system and solve it using a Lagrangian relaxation based approach; Wei et al. (2010) that propose an inventory control model for the return and remanufacturing processes under uncertains of the demand and returns. They propose a linear programming approach to deal with the uncertainty of the problem; Ahiska and King (2010) that use a Markov decision process to model and solve an inventory problem of single product recoverable manufacturing system; Inderfurth (2005) that considers a multi-period stochastic closed-loop supply chain with remanufacturing. A heuristic is proposed that allows evaluating this environment; Nakashima et al. (2004) that study a stochastic control problem of a remanufacturing system, which is modeled and solved via Markov decision process approach; and Dobos (2003) that formulates a quadratic production planning problem to deal with a reverse logistic system, and uses control theory to solve it. However, it is important to add that the processes of modeling and solution, adopted in this article, are different from those found in the papers mentioned above.

3 THE REVERSE SYSTEM

Figure 1 illustrates a schematic view of the reverse logistics system. Two channels define such a system: the forward channel pushes serviceable products (i.e. new and remanufacturable) to the market. In the backward (or reverse) channel, used-products are collected from the market. Part of them will be remanufactured, and others disposed.

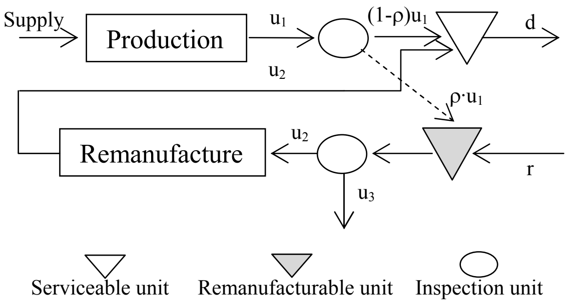

Figure 1: The production/remanufacturing system.

Note also that new products are previously inspected and all that fails, during this process, are sent to remanufacturable inventory unit (see dashed lines in the Figure 1).

Typical examples of organizations that follow the scheme of Figure 1 are companies that remanufacture bottles, cans, containers, etc.

Some features and properties of the system exhibited by Figure 1 are the following: (a) demand for serviceable products should be met by the combination of both manufactured and remanufactured products; (b) the fluctuation of demand and return rate are no correlate stationary stochastic processes that follow the normal distribution functions, whose first and second statistics moments are perfectly known; (c) there are upper limits for both manufacturing and remanufacturing capacities, but not for inventory units; and (d) used-products may be discarded after being collected from the market. There are two main reasons to discard used-products: the first has a technical justification, which is related to the quality of used-product, which may be inappropriate for remanufacturing. The second reason has a financial justification. It is motivated by the idea in which remanufacturing all products that return, can increase unnecessarily holding inventory costs of serviceable products. In short, the last strategy brings the possibility of increasing the overall cost for running the system.

4 THE STOCHASTIC PROBLEM

The combination of production/remanufacturing stochastic planning problem with chance-constraints can be formulated by a classical linear programming model as follows:

$$\begin{aligned}
\underset{u_1,u_2,u_3}{\text{Min}}\ Z = {} & h_1 E\{x_N^1\} + h_2 E\{x_N^2\} + \sum_{k=0}^{N-1} \{ h_1 E\{x_k^1\} + h_2 E\{x_k^2\} \\
& + c_1 u_1 + c_2 u_2 + c_3 u_3 \ \} && \text{(a)}\\
\text{s.t.} \quad & \\
& x_k^1 = x_0^1 + \sum_{i=1}^{k} \left[(1-\rho) u_i^1 + u_i^2 \right] - D_k && \text{(b)} \quad (1)\\
& x_k^2 = x_0^2 + \sum_{i=1}^{k} \left(\rho u_i^1 - u_i^2 - u_i^3 \right) + R_k && \text{(c)}\\
& \text{Prob.}(x_k^1 \geq 0) \geq \alpha && \text{(d)}\\
& \text{Prob.}(x_k^2 \geq 0) \geq \beta && \text{(e)}\\
& 0 \leq u_k^1 \leq \bar{u}_1;\ 0 \leq u_k^2 \leq \bar{u}_2;\ u_k^3 \geq 0 && \text{(f)}
\end{aligned}$$

Problem (1) minimizes the sum of linear of expected holding costs, at each store units, production costs for manufacturing and remanufacturing products, and costs with disposal. As a result, it provides an optimal global sequential plan, which encompasses optimal inventory goals for serviceable and remanufacturable units and, optimal production rates goals for manufacturing and remanufacturing processing units, as well.

For each period k of the planning horizon N, variables and parameters that compose (1) have the following notation:

x_k^1 - Serviceable inventory level at period k.

x_k^2 - Remanufacturable inventory level at period k.

u_k^1 - Production rate at period k.

u_k^2 - Remanufacturing rate at period k;

u_k^3 - Disposal rate at period k;

d_k - Demand level at period k.

r_k - Return rate at period k.

D_k - Cumulative level of demand at period k

R_k - Cumulative return rate at period k

h_1 - Serviceable holding cost

h2 - Remanufacturable holding cost

c_1 - Production cost

c_2 - Remanufacturing cost

c_3 - Disposal cost

$\rho \in [0, 1]$ – Historical index of new product that are rejected during the control quality process.

α and β – Probabilistic indexes that assure the validity of the probabilistic constraints (1.b)-(1.c).

Both $D_k = \sum_{i=1}^{k} d_i$ and $R_k = \sum_{i=1}^{k} r_i$ are random variable that follow cumulative normal distributions of probability. The first variable is related to each demand level d_i (i.e. orders placed at period i) that is accumulated until reaching the period k. The second variable refers to the cumulative number of returns of products collected during each period i. They are assumed stationary and not correlated variables with first and second statistics moments given, respectively, by:

$$\begin{cases} \hat{D}_k = \sum_{i=1}^{k} \hat{d}_i\,; \quad \Theta_k^D = \sqrt{k}\,\sigma_d \\ \hat{R}_k = \sum_{i=1}^{k} \hat{r}_i\,; \quad \Theta_k^R = \sqrt{k}\,\sigma_r \end{cases} \tag{2}$$

where $\hat{d}_i$ and $\hat{r}_i$ denote mean values of demand level and return rate during period i. The pair (σ_d, σ_r) denotes their finite standard deviations, respectively.

It is important to detach that the normal distribution is usually a good approximation for a variety of other types of distributions. For example, the Poisson distribution is commonly used to represent events that involve arrivals. Thus, in a production environment where products are moved from inventory as soon as orders are placed, an interesting representation for these orders is the Poisson distribution. However, whenever the arrival flow becomes intense, the normal distribution can be a good approximation for the Poisson Process, see (DasGupta, 2010). Additionally, Graves (1999) justifies the use of normality hypothesis to describe demand fluctuation in a manufacturing environment.

It is important to emphasize here that the problem formulated in (1) reflects the economic aspects of the productive environment that intends to represent. This means that processes of collecting, storing and remanufacturing used-products are economically viable, when transport and inventory costs, in reverse channel, consume only a small portion of the budget of the organization. Additionally, if the cost of remanufacturing is less than the cost of manufacturing new-products, the chances of reverse logistics model to be economically viable are very high. Also, it is important to have in mind that costs with raw-material and components to manufacture new products can increase significantly the production cost of new products. In short, the production/ manufacturing policy, provided by problem (1), must be interpreted from cost viewpoint, in order to conclude something about its economic viability.

4.1 Solving Problem (1)

Find an optimal global solution to the problem (1) is not an easy task. The reason of this is the stochastic nature of the inventory balance system and constraints on main decision variables. As a consequence, heuristic techniques are viable alternatives that allow finding a near-optimal solution to problem (1). The normal distributions, assumed to random variables of problem (1), makes possible to use the certainty-equivalence principle to transform problem (1) into an equivalent, but deterministic, problem. This deterministic problem is entirely based on the first and second statistic moments of the random variables (Bertsekas, 2007).

5 AN EQUIVALENT PROBLEM

The certainty-equivalence principle is commonly employed to reduce the complexity of a stochastic programming problem. In fact, it allows transforming problems like (1) to a deterministic-equivalent form, which is easier to solve.

The deterministic-equivalent problem is derived by using the first and second statistic moment of the serviceable and remanufacturable inventory variables. Because of the normal assumption, the process of conversion is very simple. The main steps of this process are given in the sequel.

5.1 Deterministic Inventory-production Balance System

The stochastic inventory-production system, descry-bed in the Figure 1, is mathematically modeled by discrete-time balance equations given in (1.b) and (1.c). Note that these equations have five decision variables. Two of them are state variables, which describe inventory units for serviceable and returnable (i.e. remanufacturable) products, while three others are control variables describe rates of manufacturing, remanufacturing, and discarding. Thus, taking into account directly the expected value on (1.b) and (1.c), results that they can be rewritten as follows:

$$\begin{aligned} \hat{x}_k^1 &= \hat{x}_0^1 + \sum_{i=1}^{k}\left[(1-\rho)u_i^1 + u_i^2\right] - \hat{D}_k \\ \hat{x}_k^2 &= \hat{x}_0^2 + \sum_{i=1}^{k}\left(\rho u_i^1 - u_i^2 - u_i^3\right) + \hat{R}_k \end{aligned} \tag{3}$$

where $\hat{x}_k^1 = E\{x_k^1\}$ and $\hat{x}_k^2 = E\{x_k^2\}$ are the expected levels of serviceable and remanufacturable units. Note also that $\hat{x}_0^1 = x_0^1$ is provided a prior by manager.

5.2 Deterministic Criterion

The linearity of the criterion (1.a) associated to the *normal* random nature of decision variables allow that this criterion can be quickly transformed to an equivalent, but deterministic linear criterion, which preservers the first and second statistical moments of random variables.

Based on above, it is only necessary to compute the expected value of (1.a). Thus proceeding, the original criterion, given by:

$$\begin{aligned} Z = h_1 E\left\{x_N^1\right\} + h_2 E\left\{x_N^2\right\} + \sum_{k=0}^{N-1}\Big\{ & h_1 E\left\{x_k^1\right\} + h_2 E\left\{x_k^2\right\} \\ & + c_1 u_1 + c_2 u_2 + c_3 u_3 \Big\} \end{aligned} \tag{4}$$

is transformed int o anequivalente to

$$\begin{aligned} \hat{Z} = h_1 \hat{x}_N^1 + h_2 \hat{x}_N^2 + \sum_{k=0}^{N-1}\Big\{ & h_1 \hat{x}_k^1 + h_1 \hat{x}_k^2 + c_1 u_1 + c_2 u_2 \\ & + c_3 u_3 \end{aligned} \tag{5}$$

where $\hat{x}_k^1$ and $\hat{x}_k^2$ denote the mean values of inventory variables, for each period $k \in [0, N]$. Note that these values are perfectly computed from (3).

5.3 Deterministic Constraints

The serviceable and remanufacturable inventory variables, that is, states of (1.b) and (1.c) systems, are assumed non-negative variables. It is also assumed that there is no upper limit of storage for these systems. These characteristics can be represented as follows:

$$\begin{cases} 0 \le x_k^1 < \mathrm{Inf} \\ 0 \le x_k^2 < \mathrm{Inf} \end{cases} \tag{6}$$

Note that the above characteristic of considering no upper bound to inventory variables is not unusual it seems. In fact, this occurs because many companies produce lots of products that are made to stock, and these lots are rapidly absorbed by the market. This means that these lots remain for a short period of time stored in the serviceable unit. As a consequence, in such a case, it is not necessary to set upper limits into the model. Analogously, products collected from the market remains short periods in the inventory remanufacturing unit. Usually, after a quick inspection, they are sent to remanufacture or to dispose.

Due to the randomness of inventory variables, constraints (6) cannot be used directly into the stochastic problem (1). There are, however, two ways of including (6) into (1): the first one is a classical procedure from mathematical programming theory that consists in penalizing (6) directly into the criterion of the problem (1). The second format consists in taking the constraints in (6) on probability operator. As a result, they become chance-constraints. This last format is usually more realist, particularly because it allows explicitly preserving constraints (6) into the stochastic problem (1); see (1.d) and (1.e). The transformation of these constraints for deterministic equivalent forms is performed in the sequel.

Firstly, using the inventory balance equation (1.b), the chance-constraint (1.d) can be rewritten as follows:

$$\text{Prob.}(x_0^1 + y_k \geq D_k) \geq \alpha \quad (7)$$

or, also, in the form:

$$\text{Prob.}\left(\varepsilon_d \leq \frac{x_0^1 + y_k - \hat{D}_k}{\Theta_k^D}\right) \geq \alpha \quad (8)$$

where $y_k = \sum_{i=1}^{k}\left[(1-\rho)u_i^1 + u_i^2\right]$ and ε_d is a normal random variable related to the variable D_k. Thus, the variable ε_d have the same cumulative distribution function (c.d.f) of D_k, denoted here as Φ_D.

Using basic knowledge of statistical theory, the constraint (8) can be easily handled, and rewritten as shown below:

$$\frac{x_0^1 + y_k - \hat{D}_k}{\Theta_k^D} \geq \Phi_D^{-1}(\alpha) \Rightarrow x_0^1 + y_k - \hat{D}_k \geq \Theta_k^D \cdot \Phi_D^{-1}(\alpha)$$

Taking into account the deterministic system (3) and remembering that $x_0^1 = \hat{x}_0^1$, it is possible to show that $\hat{x}_k^1 = x_0^1 + y_k - \hat{D}_k$. As a result, the determinist equivalent constraint related to the chance constraint (1.d) is given by:

$$x_k^1 \geq \Theta_k^D \cdot \Phi_D^{-1}(\alpha) \;\Rightarrow\; x_k^1 \geq \sqrt{k} \cdot \sigma_d \cdot \Phi_D^{-1}(\alpha) \quad (9)$$

where $\Phi_D^{-1}(.)$ is the inverse distribution of probability of variable D_k.

Proceeding in a similar way for the chance-constraints (1.e), it is possible to find that

$$x_k^2 \geq \Theta_k^R \cdot \Phi_R^{-1}(\beta) \;\Rightarrow\; x_k^2 \geq \sqrt{k} \cdot \sigma_r \cdot \Phi_R^{-1}(\beta) \quad (10)$$

where $\Phi_R^{-1}(.)$ is the inverse distribution of probability of variable R_k.

As a last observation, note that the inventory levels x_k^1 and x_k^2 are random variables that, due to the linearity of the system, follow similar distribution of probability of D_k and R_k.

5.4 The Equivalent Problem

Gathering together all the transformed parts as exhibited above, an equivalent, but deterministic, problem can be stated to represent the stochastic problem (1). It is formulated as follows:

$$\min_{u_1,u_2,u_3} \hat{Z} = h_1\hat{x}_N^1 + h_2\hat{x}_N^2 + \sum_{k=0}^{N-1}\left\{ h_1\hat{x}_k^1 + h_2\hat{x}_k^2 + c_1u_1 + c_2u_2 + c_3u_3 \right\}$$

s.t.

$$\hat{x}_k^1 = x_0^1 + \sum_{i=1}^{k}\left[(1-\rho)u_i^1 + u_i^2\right] - \hat{D}_k \quad (11)$$

$$\hat{x}_k^2 = x_0^2 + \sum_{i=1}^{k}\left(\rho u_i^1 - u_i^2 - u_i^3\right) + \hat{R}_k$$

$$\hat{x}_k^1 \geq \sqrt{k} \cdot \sigma_d \cdot \Phi_D^{-1}(\alpha)$$

$$\hat{x}_k^2 \geq \sqrt{k} \cdot \sigma_r \cdot \Phi_R^{-1}(\beta)$$

$$0 \leq u_k^1 \leq \bar{u}_1;\; 0 \leq u_k^2 \leq \bar{u}_2;\; u_k^3 \geq 0$$

Some advantages of problem (11) are:

- Linearity of criterion (1.a) and inventory balance equations, given by (1.b) and (1.c), are totally preserved.
- In the same way, chance-constraints (1.d) and (1.e) are converted to equivalent determinist constraints, in which are maintained original statistical characteristics of the problem (1); see that first and second statistical moments, given in (2), are explicitly into the constraints (9) and (10).
- The equivalent problem (11) is very simple to be solved. Classical techniques of linear programming, available in the literature, can be applied.

6 AN EXAMPLE

A company sells a kind of product that can be remanufactured. Structurally speaking, the production/remanufacturing process of this company can be seen as a closed-loop system, as that one shown by Figure 1. In the forward direction of this system, new and remanufactured products are stored into a serviceable inventory unit. These products are previously stored in serviceable unit in order to meet the demand that fluctuates in an uncertain way over weekly periods. On the other hand, in the reverse (backward) direction, used-products are collected weekly from the market, with base on an uncertain return rate. They are stored into the remanufacturable inventory unit and, after being inspected, they are sent to remanufacture or to discard. Besides, a very small part of products stored in the remanufacturable inventory are defective products. They are new products that fail during the quality inspection process.

The demand fluctuation for serviceable products and the rate of return products are considered uncorrelated random variables that follow stationary normal processes. Such processes are well-defined by their first and second statistics moments. They follow the same equations given in (2) to describe mean and standard-deviation.

The objective of the company is to analyze if, under certain conditions related to inventory and production costs, it is advantageous to increase the return rate of used products. More specifically, the question to be answered is: can the increase of the weekly return rate improve the performance of processes (1.b) and (1.c)?; and what is the implication on the total production cost?

To carry out such an analysis, the company decided to develop production plans for the next eight weeks (N=8), which are based on the solution of deterministic problem (11).

Before proceeding with the deployment of this study, it is necessary to introduce general data of the problem, which are mainly available in the Tables 1 and 2.

Table 1 presents the first and second statistic moments related to the normal distribution function of demand variable.

Table 1: Weekly statistics of demand d_k.

k	1a	2a	3a	4a	5a	6a	7a	8a
$\hat{d}_k$	600	595	610	615	605	585	600	610
Standard-deviations: $\sigma_d \approx 20, \forall k$								

From the Table 1, it is important to calculate the absolute mean value (*amv*) of demand variable d_k. This absolute value is based on the arithmetic mean of first statistical moments of demand, known a prior for each period k of the planning horizon N (i.e, $\hat{d}_1$, $\hat{d}_2$, $\hat{d}_3$, ..., $\hat{d}_N$). Thus, considering the mean values provided in Table 1, for 8 weekly periods (N=8), the absolute mean value for this example is computed as follows:

$$amv = \sum_{i=1}^{8} \hat{d}_i \cong 602. \tag{12}$$

The *amv* value (12) will be used ahead to determine the mean value of the return rate of used-products.

Table 2 introduces the current information about the current state of the production/remanufacturing system, and about parameters and costs of the problem (9).

Based on these data, two situations will be analyzed by managers of the company: in the first one, managers consider that the mean value of the return rate variable is set equal to 50% of absolute mean value (*amv*) of demand, that is, $\hat{r}_k = 301, \forall k$.

Table 2: Other data of the problem.

Initial inventories	$x_0^1 = 300$ and $x_0^2 = 225$
Inventory costs	$h_1 = \$2$ and $h_2 = \$1$
Production costs	$c_1 = \$1$ and $c_2 = \$1{,}20$
Disposal cost	$c_3 = \$0{,}14$
Probabilistic indexes	$\alpha = 95\%$ and $\beta = 80\%$
Rate of defective	$\rho = 5\%$

This means that on an average flow of returned products is 50% smaller than the flow of orders placed (i.e. demand for serviceable products).

The second situation considers that on average 100% of used-products are weekly collected. This means that the return rate is close to $\hat{r}_k = 602, \forall k$.

Additionally, it pertinent to mention that, for both situations, the second statistical moment will be set exactly equal to 10 units, that is, $\sigma_r = 10, \forall k$.

In the sequel, the two situations are implemented in the format given by (11), and their solutions are compared.

6.1 1st Situation: 50% Returnable

In this situation, used-products that are weekly collected from the market have the same weekly average return rate of $\hat{r}_k = 301$, with standard-deviation of 15%.

Figures 2 and 3 illustrate optimal inventory-production trajectories for forward and reverse channels of the system illustrated generically in the Figure 1. It is interesting to note that serviceable and remanufacturable inventory levels show a similar sharp in their optimal trajectories. In fact, except for the first week, when inventory-production process (1.b) and (1.c) are still being adjusted to the initial conditions of the problem (i.e., x_0^1 and x_0^2), the remaining weeks show the continuous growing of serviceable and remanufacturable inventory levels over the weekly periods. As a result, safety-stocks are providing. In the case of serviceable inventory, the idea of a safety-stock is to guarantee ready delivery of products to meet demand; and, in the case of remanufacturable inventory, the idea is to provide a buffer of used products that can be used in remanufacturing process, during future periods, for avoiding backlogging occurrences.

It is worth mentioning that these safety-stocks are due to the original stochastic nature of the problem (1). During the transformation process, from (1) to the equivalent problem (11), safety-stocks are created to preserve the feasibility of (11); to realize this, see constraints (9) and (10).

The figure 2 shows optimal rates of manufacturing and remanufacturing, which evolve over weekly periods of the planning horizon. It is possible to observe that the manufacture of new products has met quite completely the weekly fluctuation of demand. The remanufactured products only complement a small part of serviceable products. Practically, the weekly amount of disposal products is very close to the amount of products that are weekly remanufactured. It is important to remember that 5% of products, which are processed in the remanufacturing unit, are defective products.

Production and inventory costs associated to the operation of this system are presented in Table 3, given ahead. The idea is to compare total production costs for the two situations analyzed.

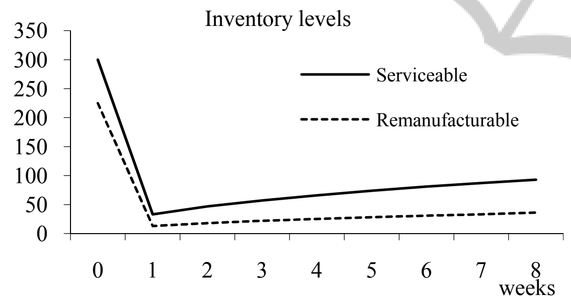

Figure 2: Serviceable and remanufacturable levels.

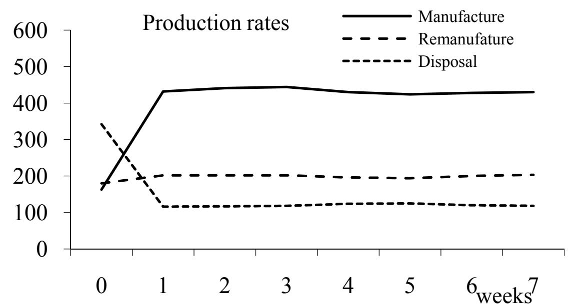

Figure 3: Production rates.

6.2 2nd Situation: 100% Returnable

In this situation, used-products that are weekly collected from the market have the same weekly average return rate of $\hat{r}_k = 602$, with standard-deviation of 15%.

The optimal inventory and production trajectories for forward and reverse channels are respectively exhibited in Figures 4 and 5. Note that the trajectories, depicted in the Figure 4, are exactly equal to trajectories exhibited in Figure 2. Note also that the characteristic of inventory levels increase over the week periods of the planning horizon is typical behavior imposed by chance constraints (9) and (10). Once again it is important to emphasize that such a behavior creates safety-stock to protect inventory units against stockout occurrences.

Contrasting with the behavior of inventories trajectories that do not change with the increase of the return rate (as seen in Figures 2 and 4), the behavior of manufacturing, remanufacturing, and disposal variables, over weekly periods, change completely (when compared with last situation). In fact, in the Figure 3, it is observed that the level of remanufactured products only complements the level of new manufactured products. This means that the amount of "new" products predominates in the weekly production of serviceable products. On the other hands, the Figure 5 shows that the weekly remanufacturing rate is greater than the weekly rate of manufacturing. Therefore, in such a situation, the amount of remanufactured products is that effectively predominate. This characteristic reveals the importance of increasing return rate percentage.

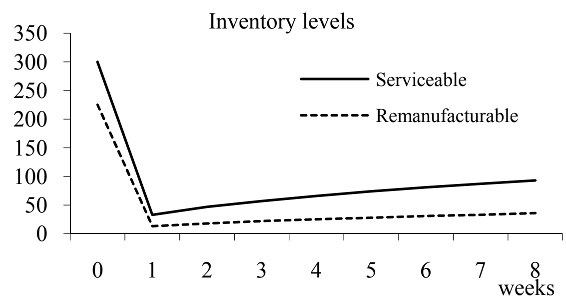

Figure 4: Serviceable and remanufacturable levels.

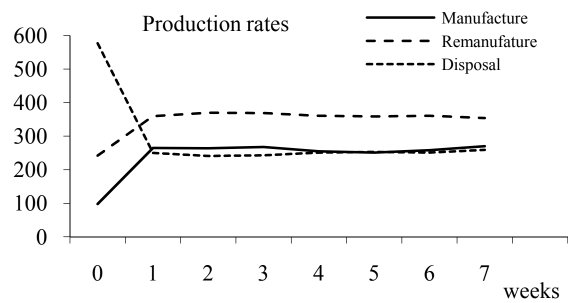

Figure 5: Production and disposal rates.

6.3 Assumptions, Costs and Final Comments, and Future Perspectives

With the objective of developing the situations 1 and 2, the problem (11) took into account some particular features of the company's production environment. These features had strong implications on optimal costs provided by this problem, and allowed some reflections about other perspectives for using the current model. In the sequel this features are discusses as problem's assumptions.

6.3.1 Some Assumptions

a) Serviceable holding cost (h_1) is the double of the remanufacturing holding cost (h_2). It is considered here that serviceable products need special care with storage, packing, etc.
b) Cost of the remanufacturing process (c_2) is 20% more expensive than the cost for manufacturing new products (c_1). In practice, remanufacturing process is usually an expensive activity because it involves a series of sub-activities like dismantling, replacing parts, reprocessing of parts, testing, etc.
c) It is assumed that the cost of disposing is quite insignificant when compared with the cost of remanufacturing. This means that can be advantageous to discard used products.
d) It was not taken into account raw material purchase costs for manufacturing new products.
e) Transport costs are indirectly included in holding inventory and production costs.

6.3.2 Costs Evaluations

Features, listed previously, explain the optimal plans provide by situation 1 and 2, whose costs generated are given in Table 3.

Table 3: Total and individual costs.

Costs	1st situation	2nd situation
Serviceable holding	1.672,80	1.672,80
Remanufacturing holding	430,85	430,85
Manufactured rate	3.191,90	1.928,70
Remanufactured rate	1.829,10	3.332,10
Disposal rate	165,21	325,49
Total cost	7.289,86	7.689,94

The costs of holding inventory for serviceable and remanufacturable units are exactly the same for both situations. This means that the optimal inventory trajectories, as shown in Figures 2 and 4 did not vary with increasing rate of return used. The justification for such a result is that the weekly demand is sufficient to absorb all products available in the serviceable inventory unit (i.e. "new" and remanufactured products that are processed concurrently; see Figures 3 and 5). Other interesting point is that the cost for disposing is very cheap, so all used products that return or defective products are immediately remanufactured or discarded.

6.3.3 Final Comments

It is important to note that although the total cost of the second situation has been 5.5% higher than the total cost verified by the first situation, the resulting optimal production plan shows a balanced distribution on the amount of "new" products (i.e. manufactured products) and remanufactured products. This characteristic can be observed through Figures 3 and 5. In fact, serviceable products are now composed of approximately 35% of manufactured (i.e. new products), 5% of recovered products (they were identified as defectives) and around 60% of remanufactured products; Figure 5 illustrates partially this characteristic. Particularly, the main advantage of this type of operation is on the cost reduction for raw-material purchasing. Thus, it is sure that a large amount of remanufactured products implies in lower raw-material acquisition. As a result, total production costs can be minimized. Unfortunately, Table 2 does not reflect such a reduction in production costs.

6.3.4 Further Extensions of the Model

As prospects for future study, it might be considered an extended version of the model here proposed. Such a version should be idealized to allow a greater realism in the formulation of the original stochastic planning problem (1). Some improvements into the model are, for instance:

a) to adopt more realistic functions to describe the production and inventory costs. Thus, one can consider nonlinear functions, but, in particular, it is desirable that they have convexity and concavity properties in order that deterministic transformations can be easily performed;

b) to include upper storage limits for serviceable and remanufacturable stores and consider them into chance-constraints formulation;

c) to consider multi-products and include new constraints on the remanufacturing and manufacturing processes that allow sharing production operations among these products in their respective processing units; and

d) to consider sequential solutions based on rolling horizon techniques, which allow optimal adjustment of the generated plans in order to follow actual demand fluctuations during each period of the planning horizon, see Pereira and De Sousa (1997).

With these improvements, the model will be closer to reality, and, as a result, it will be possible to develop more accurate studies, and to provide more efficient plans for management purposes.

7 CONCLUSIONS

From a linear structure that represents an inventory-production dynamic system with forward and reverse flows, a production planning problem based on a stochastic linear programming model with chance constraints was proposed. The solution of this optimization problem can be very useful for those companies that deal with operations involving both used-products return, and defective products, as well.

The main difficulty here is that a global solution to this stochastic problem cannot be trivial. Thus, it was assumed without lost of generality that the statistical behavior of the demand could be approximated by a normal process. As an immediate consequence, an easy-to-solve equivalent deterministic problem was proposed. This provides an open-loop sub-optimal solution that would be optimal global if current demand observed at each period of the planning horizon was exactly equal to mean demand. However, this sub-optimal solution is a good estimate to the original stochastic problem, once demand levels and return rates are both stationary processes.

A simple example was proposed to illustrate the applicability of the equivalent deterministic problem, and, at the same time, to compare the effect of increasing the return rate of used-products on the optimal production policies provided by solving the problem (11). In order to make the analysis interesting, it was considered that the cost of remanufacture would be 20% higher than the cost to manufacture new products. In this case, it was expected that the manufacture of new products would be preferred instead of the remanufacture of used-products, but this was not effectively observed. Indeed, the results showed that as the return rate of used-products becomes close to the average absolute value (*amv*), the remanufactured products becomes more attractive to the company. This kind of operating system (1) is illustrated by Figure (5).

The model considered here is, therefore, an interesting management tool that allows not only develop a production plan for implementation within the hierarchy of business decisions, but also allows helping in the process of decision making on new strategies, as discussed in the example of section 5.

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