A COMPARATIVE STUDY OF SOLVING QUADRATIC ASSIGNMENT PROBLEMS USING SOME STANDARD MINLP SOLVERS

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Abstract: The Quadratic Assignment Problem (QAP) has important application areas, for example, facility layout (Dickey and Hopkins, 1972) and electronic component placement (Rabak and Sichman, 2003). The NP-hard problem already becomes difficult and time consuming to solve satisfactorily for small applications. It is therefore of interest to investigate how well standard MINLP methods can provide good solutions within a reasonable time, even though global optimality can not be guaranteed. In this study we focus on solving a subset of 50 problems in the QAP library (Burkard et al., 1997). We use a standard Mixed-Integer Non-Linear Programming (MINLP) formulation modelled in the General Algebraic Modeling System (GAMS) (Rosenthal, 2010). The solution quality and solution time is evaluated for the solvers AlphaECP, Bonmin, DICOPT and SBB. We compare the solvers when a 1 hour time limit per problem is used, where the solvers are started from 3 random start points, i.e. initial variable levels. Furthermore, we investigate how well the most promising solver DICOPT performs when started from 50 random start points for 22 problems for which the global optimal solution is known.

1 INTRODUCTION

The Quadratic Assignment Problem is a general model formulation and has been applied to many different application areas, of which the following could be mentioned: assignments of buildings in a University Campus (Dickey and Hopkins, 1972); locating hospital departments (Elshafei, 1977); zoning forest for different uses (Bos, 1993); electronic component placement on a printed circuit card (Rabak and Sichman, 2003) and computer motherboard design (Miranda et. al., 2005).

Originally the QAP formulation was introduced for the facility layout problem where n facilities are placed in n locations. The objective is to minimize the sum of the products of flows and distances between the facilities. Let a_{ij} define the distances between locations i and j and let b_{kl} define the material flows between the facilities k and l, i.e. two $n \times n$ constant matrices. Let x_{ij} be the decision variable that facility i is placed at location j. The QAP problem was first presented by Koopmans and Beckmann (1957) in the following form:

$$min \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{n} \sum_{l=1}^{n} a_{ij} b_{kl} \cdot x_{ik} x_{jl} \quad (1)$$

Subject to

$$\sum_{i=1}^{n} x_{i,j} = 1, \qquad j = 1, \dots, n \quad (2)$$

$$\sum_{j=1}^{n} x_{i,j} = 1, \qquad i = 1, \dots, n \quad (3)$$

$$x_{ij} \in \{0,1\}, \qquad i, j = 1, \dots, n \quad (4)$$

The bilinear objective function (1) results in a non-convex formulation, hence optimality can not be guaranteed with convex optimization techniques. Therefore the QAP problem is often reformulated in convex or linear form, having the disadvantage that the number of variables and constraints will increase substantially. Exact algorithms based on reformulation and convexification guarantee global optimality but problems with the size $n \geq 30$ are already very difficult to solve to proven optimality (Çela, 1998). Heuristic algorithms can, however,

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often provide good solutions within a reasonable time but not verify the quality of them. Among heuristic methods one finds: Construction Methods (CM), Limited Enumeration Methods (LEM), Improvement Methods (IM), Tabu Search (TS), Simulated Annealing (SA), Genetic Algorithms (GA), Greedy Randomized Adaptive Search Procedures (GRASP) and Ant Systems (AS). The goal of this study is to compare how a set of standard MINLP solvers perform, when the bilinear QAP formulation is used.

The MINLP solvers are standard GAMS solvers and uses the following solution techniques: AlphaECP solves the problem by cutting plane techniques; Bonmin (Basic Open-source Nonlinear Mixed Integer programming) uses a simple branch and bound algorithm and solves a Non-Linear Programming (NLP) problem in each node; DICOPT (Discrete and Continuous Optimizer) is an outer-approximation method; SBB combines a standard branch and bound method with some of the NLP solvers in GAMS.

2 SETUP

The basic QAP formulation in (1-3) is modelled in GAMS 23.6.2 and all solvers are used with default parameter settings. We use here the following abbreviations: ECP for AlphaECP, BON for Bonmin and DOP for DICOPT. 50 problems from the QAP library are solved: retrieved March 22nd, 2011, from http://www.seas.upenn.edu/qaplib/. The problems are selected from the QAP library (Hahn and Anjos, 2002) with the following criteria: $30 \leq n \leq 90$, where n is the size of the square matrices ($n \times n$). Thus, the selected set contains problems that are very difficult to solve to proven optimality with exact algorithms. Two problems, lipa90a and lipa90b, with $n = 90$ are, however, not included in the comparison because all solvers stopped within 4 minutes because of a system or memory limitation. In the first test the solvers are set to solve 50 problems with a 1 hour time limit per problem per starting point, hence the total solution time may raise to 3 hours when the solvers are started from 3 random points. The second test batch consists of the 22 problems from the first set, which are the problems where the global optimal solution is known. In this test DICOPT is called with a 1 hour time limit per problem per start point from 50 random points. The test computer is an Intel core i7 with 4 cores of 2,8GHz and 6GB of memory.

3 RESULTS

Table 1 shows how well, in general, the four solvers solved the problems. Table item "Avg. % from best solution" describes the average (avg.) deviation in percentage from the best solutions known reported in the QAPLIB (Hahn and Anjos, 2002), for the 13 problems where all the solvers found a solution. Table item "Number of best solutions" denotes for how many problems a solver found a better solution than the other three solvers. Note that SBB found a better solution than the three other solvers for 26 problems, but could not find any solution for 8 problems. Furthermore, it is worth noting the exceptionally short solution time for DICOPT to find good solutions.

Table 1: Overall performance of the 4 solvers.

	ECP	DOP	SBB	BON
Problems solved	50	50	42	13
Avg. solution time (min)	36	4	35	43
Avg. % from best solution	4.7	5.6	3.7	4.3
Number of best solutions	8	5	26	4

In Table 2 the problem size is included in the name. The table reveals, for each solver, the best solution when the solver is started from the 3 starting points. The star in Table 2 indicates that the best known solution is a global optimal one. None of the solvers are able to find exceptionally good solutions compared to the other solvers. Table 3 reveals the improvement for DICOPT when the solver is started from 50 random starting points instead of 3. The standard deviation denotes the standard deviation in the obtained value of the objective function.

4 CONCLUSIONS

In this study 50 challenging problems from the QAP-library were solved with some standard MINLP solvers from GAMS. The compared solvers were: AlphaECP, Bonmin, DICOPT and SBB. AlphaECP found good solutions for all the problems, but typically used the total solution time available before termination. Bonmin found a solution only for 13 problems, but 4 of them were better than any of the other solvers. DICOPT solved the problems significantly faster than the three other solvers, but was unable to significantly improve the solution quality when the solver was started from 50 random start points instead of 3.

Table 2: Solution and average time usage when starting the solvers from 3 starting points for 50 problems.

Problem	Best solution known	Solution value ECP	DOP	SBB	BON	% from best ECP	DOP	SBB	BON	Avg. time (min) ECP	DOP	SBB	BON
esc32a	130	148	154	142	146	13.8	18.5	9.2	12.3	0.2	0.0	0.4	46.2
esc32b	168	192	192	184	184	14.3	14.3	9.5	9.5	0.1	0.0	1.8	28.0
esc32c	642	642	642	642	NA	0.0	0.0	0.0	NA	60.0	0.0	60.0	54.9
esc32d	200	210	200	200	NA	5.0	0.0	0.0	NA	0.9	0.0	1.4	60.3
esc32e	2*	2	2	2	2	0.0	0.0	0.0	0.0	0.0	0.0	1.9	39.1
esc32g	6*	6	6	6	6	0.0	0.0	0.0	0.0	0.0	0.0	2.0	21.6
esc32h	438	440	448	440	NA	0.5	2.3	0.5	NA	60.0	0.0	60.0	61.2
esc64a	116	116	116	116	NA	0.0	0.0	0.0	NA	60.0	0.1	60.0	0.0
kra30a	88900*	92610	90760	92810	91400	4.2	2.1	4.4	2.8	26.3	0.1	2.8	53.4
kra30b	91420*	94490	94490	92290	94400	3.4	3.4	1.0	3.3	60.0	0.0	6.0	45.6
kra32	88700*	91420	90470	89820	NA	3.1	2.0	1.3	NA	60.0	0.1	17.9	29.6
lipa30a	13178*	13413	13469	13438	13485	1.8	2.2	2.0	2.3	60.0	0.1	60.0	37.8
lipa30b	151426*	174540	174202	172718	174318	15.3	15.0	14.1	15.1	44.9	0.2	26.3	65.0
lipa40a	31538*	32104	32187	NA	NA	1.8	2.1	NA	NA	60.0	0.5	60.0	0.0
lipa40b	476581*	559251	559417	554548	NA	17.3	17.4	16.4	NA	60.0	0.8	38.2	0.0
lipa50a	62093*	63046	63275	NA	NA	1.5	1.9	NA	NA	60.0	1.4	60.0	0.0
lipa50b	1210244*	1422035	1423875	1421683	NA	17.5	17.7	17.5	NA	60.0	2.9	60.0	0.0
lipa60a	107218*	108560	108835	NA	NA	1.3	1.5	NA	NA	60.0	4.0	60.1	0.0
lipa60b	2520135*	2999484	2994479	2986313	NA	19.0	18.8	18.5	NA	60.0	5.9	60.1	0.0
lipa70a	169755*	171800	171954	NA	NA	1.2	1.3	NA	NA	60.0	7.9	60.2	0.0
lipa70b	4603200*	5527251	5506114	NA	NA	20.1	19.6	NA	NA	60.0	11.2	60.1	0.0
lipa80a	253195*	255986	256237	NA	NA	1.1	1.2	NA	NA	60.1	16.9	60.2	0.0
lipa80b	7763962*	9350196	9329120	NA	NA	20.4	20.2	NA	NA	60.0	24.4	60.1	0.0
nug30	6124*	6218	6280	6156	6154	1.5	2.5	0.5	0.5	5.7	0.1	4.5	61.4
sko42	15812	15960	15904	15838	15818	0.9	0.6	0.2	0.0	21.2	0.7	29.0	42.5
sko49	23386	23628	23738	23556	NA	1.0	1.5	0.7	NA	53.6	1.2	35.8	0.0
sko56	34458	35088	34688	34598	NA	1.8	0.7	0.4	NA	35.1	2.5	60.1	0.0
sko64	48498	49466	48798	48724	NA	2.0	0.6	0.5	NA	44.9	4.7	60.1	0.0
sko72	66256	67202	66852	66716	NA	1.4	0.9	0.7	NA	60.0	8.7	60.2	0.0
sko81	90998	92558	91396	91250	NA	1.7	0.4	0.3	NA	60.0	13.3	58.5	0.0
sko90	115534	117116	116260	116560	NA	1.4	0.6	0.9	NA	60.0	26.7	60.3	0.0
ste36a	9526*	9838	10228	9754	10402	3.3	7.4	2.4	9.2	1.7	0.1	11.4	41.2
ste36b	15852*	16212	16620	16316	15852	2.3	4.8	2.9	0.0	0.2	0.0	0.3	7.9
ste36c	8239110*	8287134	8720778	8306974	NA	0.6	5.8	0.8	NA	44.2	0.1	60.0	27.3
tai30a	1818146	1868648	1879624	1857106	NA	2.8	3.4	2.1	NA	1.9	0.1	3.5	0.0
tai30b	637117113	705935352	711585738	698531894	NA	10.8	11.7	9.6	NA	17.9	0.1	4.1	36.9
tai35a	2422002	2495070	2500438	2470970	NA	3.0	3.2	2.0	NA	5.2	0.3	11.0	0.0
tai35b	283315445	287808844	285343286	283334598	NA	1.6	0.7	0.0	NA	43.4	0.2	20.1	9.6
tai40a	3139370	3218910	3216932	3209658	NA	2.5	2.5	2.2	NA	1.4	0.7	13.6	0.0
tai40b	637250948	698738895	682857820	674921066	NA	9.6	7.2	5.9	NA	60.0	0.3	40.1	39.3
tai50a	4938796	5073328	5029626	5023310	NA	2.7	1.8	1.7	NA	3.8	1.5	13.8	0.0
tai50b	458821517	495201830	478598493	477360768	NA	7.9	4.3	4.0	NA	7.6	0.7	60.0	0.0
tai60a	7205962	7422590	7362494	7356088	NA	3.0	2.2	2.1	NA	12.8	5.5	30.1	0.0
tai60b	608215054	633529622	627096107	626306798	NA	4.2	3.1	3.0	NA	60.0	1.7	60.0	0.0
tai64c	1855928	1890604	1988186	1988186	NA	1.9	7.1	7.1	NA	0.2	0.0	0.0	6.0
tai80a	13499184	13874028	13745826	13738850	NA	2.8	1.8	1.8	NA	32.6	17.6	56.0	0.0
tai80b	818415043	880698860	850580839	NA	NA	7.6	3.9	NA	NA	60.0	11.1	60.1	0.0
tho30	149936*	151328	153210	152118	150810	0.9	2.2	1.5	0.6	1.1	0.1	0.9	39.6
tho40	240516	244676	244120	241234	NA	1.7	1.5	0.3	NA	2.4	0.2	1.7	0.0
wil50	48816	49138	49016	48920	NA	0.7	0.4	0.2	NA	60.0	2.7	60.1	0.0

Table 3: The performance of DICOPT when starting the solver from 50 random start points.

Problem	Solution					Time (sec)
	Optimal	Best solution found	% from optimal	Improvement	Standard deviation	Avg. time/ problem
esc32e	2*	2	0.0	0	4	0.1
esc32g	6*	6	0.0	0	1	0.1
kra30a	88900*	90760	2.1	0	1842	3.0
kra30b	91420*	91910	0.5	2580	1462	2.8
kra32	88700*	88700	0.0	1770	2424	3.5
lipa30a	13178*	13469	2.2	0	36	7.9
lipa30b	151426*	174202	15.0	0	1277	12.2
lipa40a	31538*	32135	1.9	52	46	28.5
lipa40b	476581*	555183	16.5	4234	3148	50.2
lipa50a	62093*	63122	1.7	153	85	90.1
lipa50b	1210244*	1412355	16.7	11520	6067	175.1
lipa60a	107218*	108750	1.4	85	109	225.3
lipa60b	2520135*	2979395	18.2	15084	8800	449.0
lipa70a	169755*	171821	1.2	133	108	496.2
lipa70b	4603200*	5469943	18.8	36171	15075	929.9
lipa80a	253195*	255993	1.1	244	173	1015.8
lipa80b	7763962*	9283379	19.6	45741	21762	1817.1
nug30	6124*	6158	0.6	122	66	6.6
ste36a	9526*	9994	4.9	234	326	4.9
ste36b	15852*	16478	3.9	142	307	2.5
ste36c	8239110*	8472932	2.8	247846	121457	5.5
tho30	149936*	150724	0.5	2486	402	3.5

SBB found a better solution for 26 problems than the other three solvers, but was unable to find a solution for 8 problems. When any of the 4 solvers terminated with a solution, then the best solution of 3 was always less than 21 % from the best solution known.

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REFERENCES

Bos, J., 1993. Zoning in Forest Management: a Quadratic Assignment Problem Solved by Simulated Annealing. *Journal of Environmental Management*, 37, pp. 127-145

Burkard, R. E., Karisch, S. E., Rendl, F., 1997. QAPLIB – A Quadratic Assignment Problem Library. *Journal of Global Optimization* 10,pp. 391-403.

Çela, E., 1998.The Quadratic Assignment Problem: Theory and Algorithms. *Kluwer Academic Publishers.*

Dickey, J. W., Hopkins J. W., 1972. Campus building arrangement using TOPAZ. *Transportation Research*, 6, pp.59-68.

Elshafei, A. N., 1977. Hospital Layout as a Quadratic Assignment Problem. Operational Research Quarterly, *Pergamon Press* Vol. 28 (1ii), pp.167-179.

Hahn, P., Anjos, M., 2002. QAPLIB - A Quadratic Assignment Problem Library Online. *University of Pennsylvania, Engineering and Applied Science*.

Koopmans, T. C., Beckmann, M., 1957. Assignment Problems and the Location of Economic Activities. *Econometrica,* Vol. 25 (1), pp. 53-76.

Miranda G., Luna, H. P. L., Mateus, G. R., Ferreira, R. P. M., 2005.A performance guarantee heuristic for electronic components placement problems including thermal effects. *Computers & Operations Research*, 32, pp. 2937-2957

Rabak, C. S., Sichman, J. S., 2003. Using A-Teams to optimize automatic insertion of electronic components *Advanced Engineering Informatics*, 17, pp. 95–106.

Rosenthal R. E., 2010, GAMS -A User's Guide, GAMS Development Corporation, <http://www.gams.com/dd/docs/bigdocs/GAMSUsersGuide.pdf> accessed 25.3.2011