

# VARYING DIMENSIONAL PARTICLE SWARM OPTIMIZERS FOR DESIGN OF SWITCHING SIGNALS

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**Abstract:** This paper presents varying dimensional particle swarm optimizers for design of switching signals in circuits and systems. The particle position and dimension correspond to the switching phase and the number of switches, respectively. The dimension can vary depending on an objective function in the search process, i.e., the number of switches is adjustable automatically. The algorithm is defined for a multi-objective problem described by the hybrid fitness consisting of analog functions and digital logic. The algorithm is defined in a general form and the performance is investigated in an example: design of a switching signal for single phase inverters with a two-objective problem corresponding to total harmonic distortion and power sufficiency.

## 1 INTRODUCTION

The particle swarm optimizer (PSO) is a population-based optimization algorithm (Engelbrecht, 2005). The PSO is simple in concept, is easy to implement and is applicable to a variety of systems: image/signal processing (Wachowiak et al., 2004), artificial neural networks (Garro et al., 2009), power electronics (Ono and Saito, 2009), etc. The PSO has been improved in order to challenge various problems: multi-objective problems, multiple solutions, escape from a trap of local optima, variable swarm topology, etc. (Engelbrecht, 2005) (Parsopoulos and Vrahatis, 2004) (Miyagawa and Saito, 2009).

This paper presents a varying dimensional particle swarm optimizer (VDPSO) for application to design of switching signals in circuits and systems. The switching signals, which determine the on/off timing, are characterized by the switching phases. Design of such switching signals is a key in a variety of circuits and systems: digital communications (Maggio et al., 2001), switching power converters (Giral et al., 1999) (Sundareswaran et al., 2007), etc. In the VDPSO, the particle positions and their dimension correspond to the switching phases and the number of switches, respectively. The goal of the VDPSO is optimizing the objective function of the switching phases to realize a desired circuit operation. The particle dimension is adjustable automatically in the search process following the objective function: two switching phases

can equalize (dimension reduction) and can separate (dimension recovery) if some conditions are fulfilled. Such dimension control is important because of several reasons including 1) The number of switchings affects performance of circuits and systems, however, the optimal number is unknown in many cases; 2) In optimization problems, the objective solution is often restricted in a lower dimensional subspace in the whole search space, however, automatic identification of the target subspace is hard. The VDPSO is defined in multi-objective problems (MOP) described by the hybrid fitness consisting of analog objective functions with criterion and digital logic (Ono and Saito, 2009). Some fitness component can increase below the criterion and this increase can help escape from a trapping solution.

After definition of the general form, the VDPSO performance is investigated in a two-objective problem of basic dc/ac inverters whose solution is known. The two-objective hybrid fitness function represents the total harmonic distortion and power sufficiency of the output waveform. Performing numerical experiments, we can confirm that the dimension varies suitably, the flexible search is realized and the particles approach to the solution. Note that we have used the basic problem whose solution is known because such a problem is convenient to investigate/evaluate the algorithm performance: this paper aims at proposal of a prototype of the VDPSO and investigation of the algorithm performance. The results provide ba-

sic information to improve/establish the VDPSO and to realize effective applications in various circuits and systems. Preliminary results along these lines can be found in (Kawamura and Saito, 2010).

## 2 VARYING DIMENSIONAL PSO

Here we define the VDPSO for an optimization problem of switching signals  $y(t)$  with period  $T$ :

$$y(t) = \begin{cases} 0 & \text{for } a_{l-1} \leq t < a_l \\ 1 & \text{for } a_l \leq t < a_{l+1} \end{cases} \quad y(t+T) = y(t) \quad (1)$$

where  $l \in \{1, 3, \dots, N\}$ ,  $a_0 \equiv 0 < a_1 \leq \dots \leq a_N < a_{N+1} \equiv T$  and  $a_i$  denotes the switching phase as shown in Fig. 1 (a).  $y = 1$  and  $y = 0$  correspond to switch-on and -off, respectively. Let an objective problem be described by a set of functions of the switching phases  $a_i$ :

$$F_j(\vec{a}) \geq 0, \quad \vec{a} \equiv (a_1, \dots, a_N), \quad j = 1 \sim N_f \quad (2)$$

where the minimum values are normalized as zero. Let the desired circuit operation corresponds to the minimum value of  $F_i$  for all  $i$ . The cases  $N_f = 1$  and  $N_f \geq 2$  correspond to the uni-objective problems and MOP, respectively. Since it is hard to find the exact minimum value of  $F_i$  for all  $i$ , we try to find an approximate solution  $\vec{a}_s$  satisfying

$$0 \leq F_i(\vec{a}_s) < C_i, \quad i = 1 \sim N_f \quad (3)$$

where  $C_i$  is the criterion of the  $i$ -th component. Note that the problem has a margin  $[0, C_1]$ .

Let a swarm contain  $N_p$  pieces of  $N$ -dimensional particles. The  $i$ -th particle at a discrete time  $n$  is characterized by its position  $\vec{a}_i(n) \equiv (a_{i1}, \dots, a_{iN})$  and velocity  $\vec{v}_i(n) \equiv (v_{i1}, \dots, v_{iN})$  where  $i = 1 \sim N_p$  is the index of the particles. The positions correspond to the switching phases  $\vec{a} \equiv (a_1, \dots, a_N)$  and the velocity controls their movement. Each particle tries to approach a solution using two key pieces of information: the personal best particle  $\vec{a}_{p_i}$  ( $pbest_i$ ) that has the best evaluation in the past history, and the global best particle  $\vec{a}_g$  ( $gbest$ ) that is the best of the  $pbest_i$  for all  $i$ . The  $\vec{a}_g$  is the occasional solution at time  $n$ . The search space is the one period of time axis and is divided into  $N$  subintervals as illustrated in Fig. 1 (b):

$$I_k \equiv [(k-1)d, (k+1)d), \quad d = T/N, \quad k = 1 \sim N \quad (4)$$

Note that two successive subintervals overlap  $I_k \cup I_{k+1} = [kd, (k+1)d]$ . This overlapping plays an important role in the dimension control in the algorithm. The algorithm is defined in the following 6 steps.

**Step 1.** Let  $n = 0$ . As illustrated in Fig. 1: the  $k$ -th element of the  $i$ th particle position is assigned randomly

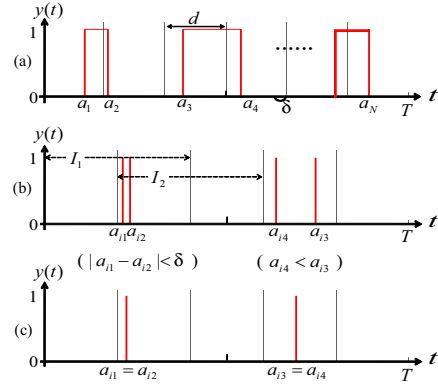


Figure 1: Particles assignment and dimension control.

in  $I_k$ :  $a_{ik} \in I_k$ ,  $k = 1 \sim N$ . Note that if  $I_k$  consists of  $N_l$  lattice points then the possible number of  $\vec{a}_i$  is  $N_l^N$ . The brute force search becomes hard as  $N$  increases. Other variables are also initialized: velocity  $\vec{v}_i(n) = \vec{0}$ , personal best  $\vec{a}_{p_i} = \vec{a}_i(n)$  and global best  $\vec{a}_g = \vec{a}_1(n)$ . **Step 2.** If Eq. (5) is satisfied for all  $i$  then the algorithm is terminated, otherwise go to Step 3.

$$0 \leq F_i(\vec{a}_g) < C_i, \quad i = 1 \sim N_f \quad (5)$$

**Step 3** (Renewal of velocity and position).

$$\begin{aligned} \vec{v}_i(n+1) &= w_i \vec{v}_i(n) + \rho_1 (\vec{a}_{p_i} - \vec{a}_i(n)) + \rho_2 (\vec{a}_g - \vec{a}_i(n)) \\ \vec{a}_i(n+1) &= \vec{a}_i(n) + \vec{v}_i(n+1), \quad i = 1 \sim N \end{aligned} \quad (6)$$

where  $w$ ,  $\rho_1$  and  $\rho_2$  are deterministic parameters, not random parameters as standard PSOs. In order to avoid overflow, speeding and stagnation; we apply

$$\begin{aligned} \text{If } a_{ik}(n) \notin I_k \text{ then } a_{ik}(n) &= \text{RND}(I_k) \\ \text{If } v_i(n) \notin [-V_L, V_L] \text{ then } v_{ik}(n) &= \text{RND}([-V_L, V_L]) \\ \text{If } |v_i(n)| < \varepsilon \text{ then } v_{ik}(n) &= qv_{ik}(n) \end{aligned}$$

where  $q$ ,  $\varepsilon$  and  $V_L$  are control parameters.  $\text{RND}(I_k)$  means a random number on the  $k$ -th subinterval  $I_k$ .

**Step 4** (Dimension Control). As illustrated in Fig. 1 (b) and (c), the particle position is equalized if two successive particles are sufficiently close or the order of particles are inverted:

$$\begin{aligned} a_{ik} = a_{i(k+1)} &= \frac{a_{ik} + a_{i(k+1)}}{2} \quad \text{if} \\ |a_{ik} - a_{i(k+1)}| \leq \delta \text{ or } a_{i(k+1)} &< a_{ik} \end{aligned} \quad (7)$$

where  $k = 1 \sim M-1$ . This equalization  $a_{ik} = a_{i(k+1)}$  means dimension reduction of  $\vec{a}$  in principle. If Eq. (7) is not satisfied in the future, then  $a_{ik}$  can be separated from  $a_{i(k+1)}$  and the dimension can be recovered in Steps 3 and 4. If "the maximum dimension"  $N$  is large then the particle can express very wide-band switching signals.

**Step 5** (Hybrid Fitness). The  $i$ th personal best,  $i = 1 \sim N_p$ , is renewed if some fitness component(s)

is improved and other component(s) satisfies the criterion:

$$\begin{aligned} \vec{a}_{p_i} &= \vec{a}_i(n+1) \text{ if either (A) or (B) is satisfied} \\ \text{(A) } C_j &< F_j(\vec{a}_i(n+1)) < F_j(\vec{a}_{p_i}) \text{ for all } j = 1 \sim N_f \\ \text{(B) } F_j(\vec{a}_i(n+1)) &< C_j \text{ for some } j \text{ and} \\ C_k &< F_k(\vec{a}_i(n+1)) < F_k(\vec{a}_{p_i}) \text{ for } k \neq j \end{aligned} \quad (8)$$

Since Eq. (8) is constructed by analog function  $F_j$  and digital logic, we refer it to as hybrid fitness. The  $g_{best}$  is renewed as the best of all the personal bests.

**Step 6.**  $n = n + 1$ , go to Step 2 and repeat until the maximum time limit  $n_{max}$ .

Note that the dimension control in Step 4 can find the suitable number (dimension) of switchings automatically. Note also that some fitness  $F_j$  can increase below the criterion  $C_j$  before Eq. (5) is fulfilled. This flexibility can help to search for a suitable solution. Existing algorithms often use a weighted sum of the objectives for the MOPs, however, it is difficult to get the best weight values (Engelbrecht, 2005).

### 3 NUMERICAL EXPERIMENTS

In order to investigate the basic performance, we consider the two-objective problem for switching signal of the dc/ac inverter. Figure 2 shows an output waveform  $y(t)$ . Since this is odd-symmetric, it is sufficient to consider in the first quarter:

$$y(t) = \begin{cases} 0 & \text{for } a_{l-1} \leq t < a_l \\ 1 & \text{for } a_l \leq t < a_{l+1} \end{cases} \quad (9)$$

where the period is normalized as  $2\pi$ ,  $a_0 \equiv 0 < a_1 \leq \dots \leq a_{N_s} < a_{N_s+1} \equiv \pi/2$ ,  $l \in \{1, 3, \dots, N_s\}$  and  $N_s$  is the number of the switches. Let  $\vec{a} \equiv (a_1, \dots, a_{N_s})$ . Let us define two positive definite objective functions.

$$F_1(\vec{a}) = 1 - \frac{b_1^2}{2P(\vec{a})}, \quad F_2(\vec{a}) = \left| 1 - \frac{P(\vec{a})}{P_d} \right| \quad (10)$$

$$y(t) = \sum_m b_m \sin mt, \quad P(\vec{a}) = \frac{1}{2\pi} \int_0^{2\pi} y(t)^2 dt.$$

where  $b_m$  is the Fourier sine coefficient,  $0 < P(\vec{a}) < 1$  is the normalized average power of  $y(t)$  and  $0 < P_d < 1$  is a desired average power.  $F_1$  relates to the total harmonic distortion (THD) and  $F_1 = 0$  means pure sinusoidal waveform.  $F_2$  describes the power sufficiency and  $F_2(\vec{a}) = 0$  gives the desired power. If both  $F_1$  and  $F_2$  are minimized, we can obtain the output having desired power with low distortion. It is suitable for continuous control of the ac power. Substituting  $N = N_s$  and  $N_f = 2$  into the algorithm, we can implement the

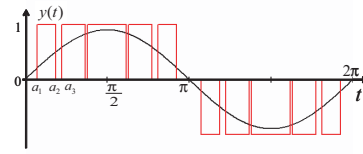


Figure 2: PWM control signal of dc/ac inverters.

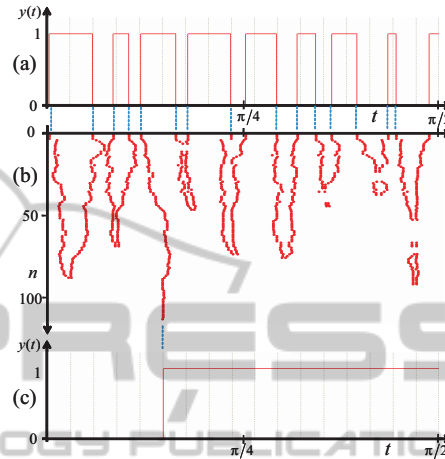


Figure 3: Search process for  $P_d = 0.7$ ,  $C_1 = 8 \times 10^{-2}$  and  $C_2 = 8 \times 10^{-3}$ . (a) initial waveform at  $n = 0$ , (b) position transition of the  $g_{best}$  particle, (c) waveform of the criterion attainment at  $n = 112$ :  $(F_1, F_2) = (7.99 \times 10^{-2}, 7.40 \times 10^{-3})$ .

VDPSO for this problem. The goal is described by  $F_1(\vec{a}_g) \leq C_1$  and  $F_2(\vec{a}_g) \leq C_2$ . If  $P(\vec{a})$  in  $F_2$  is given, the optimal solution is  $a_1 = \dots = a_{N_s} = \pi(1 - P(\vec{a}))/2$ . That is, the optimal dimension is one. We investigate how the algorithm reduces the dimension automatically. We note again that our purpose is investigation of basic performance of the algorithm and the search process. For the numerical experiments, we select  $C_1$ ,  $C_2$  and  $P_d$  as control parameters and the other 10 parameters are fixed after trial-and-errors:  $N_p = 20$ ,  $N_s = 17$ ,  $w = 0.8$ ,  $\rho_1 = \rho_2 = 2$ ,  $V_L = 0.2$ ,  $\varepsilon = 10^{-15}$ ,  $q = 0.1$ ,  $\delta = 0.01$  and  $n_{max} = 400$ . Fig. 3 shows a typical result: as  $n$  increases, the number of the switching phases decreases by the dimension control in Step 4 and the criterion is attained. Fig. 3 (b) shows position transition of the  $g_{best}$  particle where the number of the switches changes from 20 to 1 by repeating dimension control. Fig. 4 shows the search process. As  $n$  increases,  $F_2$  decreases and reaches the criterion  $C_2$  rapidly. Below the criterion  $C_2$ , the  $F_2$  can increase and this increase helps decrease of  $F_1$  and attainment of the criterion  $C_1$ . That is, the VDPSO can adjust the number of switches automatically and can give the desired solution. Table 1 summarizes results of 50 trials for various parameter values and initial conditions. We have used the three measures: SR (The successful

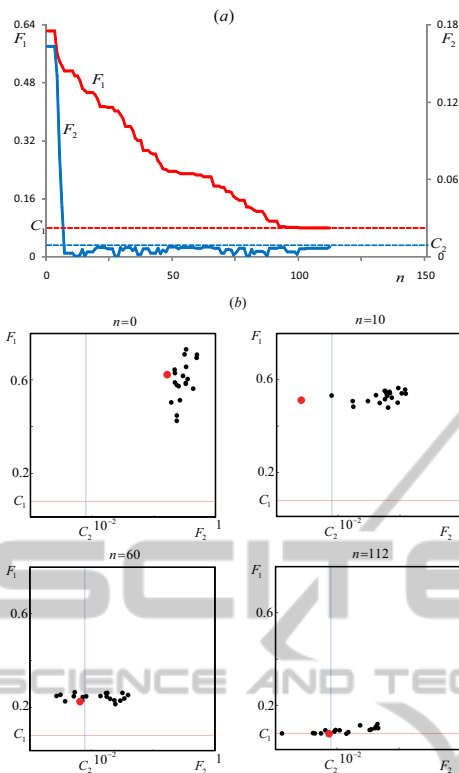


Figure 4: Search process for  $C_1 = 8 \times 10^{-2}$  and  $C_2 = 8 \times 10^{-3}$ . (a) Time evolution of the two fitness functions. (b) Behavior of particles. The red point is the global best.

rate of the criterion attainment within the time limit), #ITL (The average number of iterations to attain the criterion for successful run) and #SW (The average number of switches at the criterion attainment in successful runs).

The SR is good around  $P_d = 0.7$  and decreases as  $P_d$  is apart from it. Note that #SW=2.2 for  $p_d = 0.7$  and  $C_1 = 0.08$  means that the criterion can be attained even in the case of 3 switchings. As  $C_1$  decreases for  $P_d = 0.7$ , #SW approaches 1. As  $C_1$  increases, the #ITE and #SW tend to increase: the criteria are attained before the #SW is optimized. These results suggest that the VDPSO is efficient if the desired power  $P_d$  is in some range, however, several improvements are required for finding solution in a wider range of  $P_d$ . We have also confirmed that the desired operation is hard to be given if the dimension control or/and hybrid fitness is not used.

## 4 CONCLUSIONS

The VDPSO has been studied and its performances have been investigated in a simple example. It is confirmed that the dimension control can work effec-

Table 1: Basic performances ( $N_s = 17$ ,  $C_2 = 8 \times 10^{-3}$ ).

$P_d$	$C_1$	SR	#ITE	#SW
0.9	0.08	n/a	n/a	n/a
	0.12	74	157	1.0
	0.15	100	46	4.6
	0.19	100	37	5.9
0.7	0.08	100	97	2.2
	0.12	100	65	5.4
	0.15	100	52	7.4
	0.19	100	43	9.6
0.5	0.15	n/a	n/a	n/a
	0.19	92	175	1.0

tively together with the hybrid fitness function. However, this paper provides a first step to develop an efficient optimization algorithm with dimension control. Future problems are many, including analysis of search process, role of key algorithm parameters, comparison with other kinds of PSOs and application to switching signals in various circuits and systems.

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