

SYNTHESIS OF THE SYSTEM FOR AUTOMATIC FORMATION OF UNDERWATER VEHICLE'S PROGRAM VELOCITY

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Abstract: In this paper a method of automatic formation of program signal of underwater vehicle's (UV) movement is proposed. This method allows providing its movement on desires spatial trajectory with maximal velocity and desired accuracy. For this purpose an additional control loop is included in UV's control system. This control loop provides a tuning of UV's desirable velocity of movement along desirable trajectory. If current UV's deviation from this trajectory more than allowable value then this control loop decreases a value of UV's desirable velocity and vice versa. Proposed approach provides to increase efficiency of using of existing UV's control systems.

1 INTRODUCTION

It has already created a lot of methods for synthesis of high accuracy control systems (robust, self adjustment est.) of underwater vehicle's (UV) movement on spatial trajectories (Yuh, 1995; Fossen, 1994; Antonelli, 2006 and est.). These methods provide the high dynamic accuracy of control. It is possible if UV's actuators will be able simultaneously to realize the UV's moving program signals and compensate the interactions between different degrees of freedom. When UV is being moved on trajectory's parts with large curvature some actuators will be able to reach of saturation. As a result the UV will be able to deviate from desirable trajectory.

It can eliminate this situation if it uses more powerful actuators. But in this case the mass and size of UV grows. Another decision it is moving with small velocity which provides high accuracy of UV's movement along whole trajectory. But in this case on the rectilinear parts of trajectory the UV will be moves with velocity smaller then it possible.

So for more fully using of UV's potential it is necessary change of its velocity by dependence from curvature of current trajectory's part. On the part with large curvature UV can significantly deviate from desired trajectory and we must decrease velocity. In this case the interaction between control channels will be decreased and, consequently, the level of control signals will be decreased too. On the

rectilinear parts of trajectory we can increase the UV's velocity, because in this case the interaction between control channels are small.

The basic problem which arises during solving of this task it is variability and uncertainty of UV's parameters during it moving in viscous environment (Yuh, 1995); (Fossen, 1994); (Antonelli, 2006); (Filaretov, 2006). Therefore it is possible to select a UV's motion mode only approximately. Also it is ordinary situation when single actuator makes control force for movement on several degrees of freedom (Ageev, 2005). It is significantly complicate selection of movement mode in cases when movement takes place on several degrees of freedom simultaneously.

In work (Lebedev, 2004) on the base of kinematics equations the approach to automatic formation of UV's velocity was suggested. But this approach not allows take into account the saturations of actuators and dynamical properties of UV. In the work (Repoulias, 2005) the desirable mode of UV's movement is formed on the base equations of dynamic and kinematics which describe work of UV and it's actuators complex. But in this work the open-loop system is developed. This system not provide accuracy UV's movement.

In this paper the approach based on the automatic formation of desirable UV's velocity depending on value its deviation from desirable trajectory is offered. This approach not requires of identification of UV's parameters and has simple practical realization.

2 PROBLEM DEFINITION

Let UV already have control system (CS)

$$u(t) = F_u(\varepsilon(t), X^*(t)), \tag{1}$$

where $u(t) \in R^n$ is a vector of control signals of UV's actuators; n is a number of UV's actuators; $\varepsilon(t) = X^*(t) - X(t) \in R^3$ is a vector of UV's dynamical error; $X^*(t) = (x^*(t), y^*(t), z^*(t))^T \in R^3$ and $X(t) = (x(t), y(t), z(t))^T \in R^3$ are vectors of desirable and current UV's position, respectively. CS (1) provide for UV desirable control quality.

Let the desirable trajectory in Cartesian space is described by following expressions:

$$\begin{cases} y^* = g_y(x^*) \\ z^* = g_z(x^*) \end{cases} \tag{2}$$

In order to CS (1) provides the movement of UV along trajectory (2), it is necessary to form vector $X^*(t) = (x^*(t), y^*(t), z^*(t))^T$. It gets more comfortable to do if we set the desirable velocity along trajectory (2). Vector $X^*(t)$ will be calculated by formulas [6]:

$$\begin{aligned} \dot{x}^*(t) &= v^*(t) / \Phi(x^*), \quad y^*(t) = g_y(x^*(t)), \\ z^*(t) &= g_z(x^*(t)), \end{aligned} \tag{3}$$

where $v^*(t)$ is a law of change UV's desirable velocity of movement on trajectory (2),

$$\Phi(x^*) = \sqrt{1 + \left(\frac{dg_y(x^*)}{dx^*}\right)^2 + \left(\frac{dg_z(x^*)}{dx^*}\right)^2}.$$

The dynamic error of tracing $\varepsilon(t)$ arises at the movement of UV with CS (1). Presence of $\varepsilon(t)$ leads to deviation of UV from desirable trajectory (2) on value $e(t)$ (see fig. 1). This deviation often is most important characteristic of UV's control quality. The more trajectory's part curvature is the more value $e(t)$ is. It is obvious that for $e(t)$ follow condition is performed:

$$0 \leq e(t) \leq \|\varepsilon(t)\|. \tag{4}$$

From inequality (4) it is clearly that for movement of UV along desirable trajectory (2) with deviation

$$e(t) \leq e_{\max}, \tag{5}$$

(where e_{\max} is a maximum allowed deviation of UV from trajectory) is necessary restricts value $\|\varepsilon\|$. Moreover (by dependence on current trajectory

curvature) the value $\|\varepsilon\|$ can have different restriction. The value $\|\varepsilon\|$ on rectilinear parts of trajectory for correctness of inequality (5) can be more then the one on parts with large curvature. We can restrict $\|\varepsilon\|$ if we restrict speed of changing of program signals or, according to expression (5), UV's desirable velocity. Regarding what is set above in this work the following problem is set and solved. Let UV include the CS (1). The vector of program signals $X^*(t)$ forms by expressions (3). It is necessary create such law of change of UV's desirable velocity $v^*(t)$ along trajectory (2) which provides the correctness inequality (5).

3 DESCRIPTION OF SYSTEM FOR FORMATION OF PROGRAM SIGNALS OF MOVEMENT

In this section we will considered the proposed synthesis method of system for formation of UV's desirable velocity. This system must automatically form a maximum possible value $v^*(t)$ by dependence on curvature of current trajectory's part and provide correctness of expression (5).

We could solve this task if we will get the analytical expressions for formation of $v^*(t)$ by dependence on properties of UV and its trajectory. But it is getting of these expressions practically impossible because the mathematical model of UV and its actuators complex is very complicated. So in this work other approach proposes. It lies in creation of addition control loop which automatically form $v^*(t)$ by dependence on current deviation of UV from desirable trajectory (2).

The general structure of proposed system is shown on the figure 2.

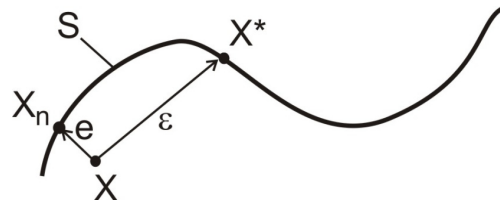


Figure 1: The errors occurring at UV's movement on spatial trajectory.

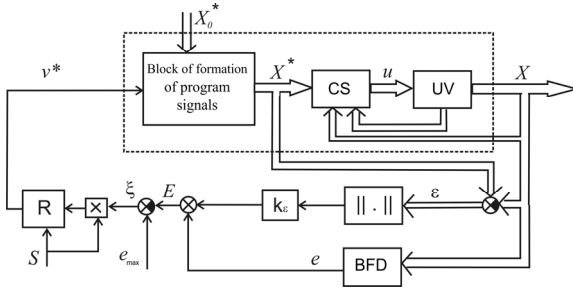


Figure 2: The block diagram of system for automatic formation of UV' desirable velocity.

In the beginning of work of this system the signal $v^*(t)$ is fed on the input of block of program signals formation. On the output of this block the vector of signals $X^*(t) = (x^*(t), y^*(t), z^*(t))^T$ is formed by expressions (3). This vector is fed on input of UV's CS which forms a control signals $u(t)$ for UV's actuators (see expression (1)).

By using of current values of elements of vectors $X^*(t)$ and $X(t)$ the signal $E(t)$ is formed:

$$E(t) = k_e \varepsilon_m(t) + e(t), \quad (6)$$

where $\varepsilon_m(t) = \|\varepsilon(t)\|$; $k_e = \text{const}$ is a positive coefficient. Expression (6) defines accuracy of UV's movement on trajectory.

After the system (see fig. 2) is turned on the regulator R form the signal $v^*(t)$ by dependence on current value $E(t)$. Using this signal and expressions (2) and (3) the vector $X^*(t) = (x^*(t), y^*(t), z^*(t))^T$ is calculated. The work of system finish when UV arrived to final point of trajectory. In this case the signal $v^*(t)$ is been set to zero by help of zero signal S (see fig. 2).

Value $e(t)$ in (6) is calculated as distance from current UV's position (vector X) to nearest point on desirable trajectory $X_n(x_n, y_n, z_n)$ (see fig. 1) by means of block of formation of deviation (BFD). Coordinates of point X_n we can find as follows.

The vector X_τ of trajectory's tangent line in point X_n has coordinates $X_\tau = f_\tau(x_n) = (1, dg_y(x^*)/dx^*|_{x^*=x_n}, dg_z(x^*)/dx^*|_{x^*=x_n})^T = (1, g'_y(x_n), g'_z(x_n))^T$ [8]. Vector which connect points X and X_n perpendicular to vector X_τ . Therefore the following equality is correctness:

$$X_\tau \cdot (X_n - X) = (x_n - x) + g'_y(x_n)(y_n - y) + g'_z(x_n)(z_n - z) = 0 \quad (7)$$

Having solved the equation system (7) and (2), we will be able to define coordinates of point X_n .

Regulator $R(\xi)$ is working as follows. If $\xi(t) < 0$, then inequality $e(t) > e_{\max}$ is correctness. In this case regulator $R(\xi)$ will be decreases $v^*(t)$. It is going to result in decreasing of value $\|\varepsilon(t)\|$ and in accordance with (4) – to decreases of $e(t)$. If $\xi(t) > 0$, then $e(t) < e_{\max}$ and regulator $R(\xi)$ will be increases $v^*(t)$.

Notice that value $v^*(t)$ must be nonnegative. It is essentially important in the beginning of UV's movement when error between initial UV's position and initial point of trajectory may be large. In this case nonnegativeness of $v^*(t)$ allow to fix a position of desirable point X^* while UV come to it on distance less then e_{\max} . Only after that the value of $v^*(t)$ begin to increase.

Also we have to restrict the value $v^*(t)$ when $e(t)$ is small (for example when UV movement along straight line). For this purpose we take into account the value of $\|\varepsilon(t)\|$ at the formation of signal $E(t)$ (see expression (6)). First term in expression (6) will be the main at the UV's movement on the rectilinear part of trajectory and second term will be the main at one's movement on the curvilinear part of trajectory.

In the next section we will considered a problem of selection of regulator R (ξ).

4 THE SYNTHESIS OF UV'S DESIRABLE VELOCITY CONTROLLER

In the beginning we will get a mathematical model of control object (CO) for regulator R. In this case the signal v^* is fed on the CO's input and value E is formed on the CO's output by expression (6). This CO include the block of formation of desirable signals (3), UV's CS and own UV (see fig. 2). It is obvious that model of this CO is nonlinear. Moreover a kind of trajectory is previously unknown and it use of this model for synthesis of regulator very difficult.

Therefore we will use the estimations of values E and e instead their real values for simplification of synthesis procedure. In this case the regulator R will be synthesized independently on a kind of UV's trajectory and regulator's structure simplify significantly.

First we estimate the value of E . Using the inequality (4), we can write

$$e = k_e \varepsilon_m, \quad (8)$$

where $0 \leq k_e \leq 1$. Putting (9) in (6) we will get:

$$E = (k_e + k_e) \varepsilon_m = \tilde{k}_e \varepsilon_m, \quad (9)$$

where $k_e \leq \tilde{k}_e \leq k_e + 1$.

The using of value ε_m is going to result in big difficulties at synthesis of regulator R . Therefore it gets more comfortable to replace this value for value $\tilde{\varepsilon}_m = |\varepsilon_x| + |\varepsilon_y| + |\varepsilon_z|$, where $\varepsilon_x, \varepsilon_y, \varepsilon_z$ are corresponding elements of vector $\varepsilon(t)$.

For this replacement we use fact that norms of vector $\|a\|_1 = \sum_{i=1}^n |a_i|$ and $\|a\|_2 = (\sum_{i=1}^n |a_i|^2)^{1/2}$ satisfy following inequality (Korn, 1968):

$$\|a\|_2 \leq \|a\|_1 \leq n^{1/2} \|a\|_2. \quad (10)$$

From expression (10) the justice of equality follows:

$$\varepsilon_m = k_m \tilde{\varepsilon}_m, \quad (11)$$

where k_m is coefficient which satisfy the inequality $1/\sqrt{3} \leq k_m \leq 1$.

We suppose that using of CS (1) provides to describe the UV's dynamic to each degree of freedom by transfer functions $W_x(s), W_y(s), W_z(s)$. These transfer functions, written relatively errors, have form:

$$\begin{aligned} W_{\varepsilon_x}(s) &= 1 - W_x(s), W_{\varepsilon_y}(s) = 1 - W_y(s), \\ W_{\varepsilon_z}(s) &= 1 - W_z(s). \end{aligned} \quad (12)$$

Therefore take into account of equality $|a| = \text{sign}(a)a$ and expressions (9)-(12) we can write:

$$\begin{aligned} E(s) &= (\text{sign}(\varepsilon_x)W_{\varepsilon_x}(s)x^*(s) + \text{sign}(\varepsilon_y)) \times \\ &W_{\varepsilon_y}(s)y^*(s) + \text{sign}(\varepsilon_z)W_{\varepsilon_z}(s)z^*(s))k_m \tilde{k}_e. \end{aligned} \quad (13)$$

If it supposes that $W(s) = W_x(s) = W_y(s) = W_z(s)$ then expression (13) take form:

$$\begin{aligned} E(s) &= k_m \tilde{k}_e W_\varepsilon(s) (\text{sign}(\varepsilon_x)x^*(s) + \\ &+ \text{sign}(\varepsilon_y)y^*(s) + \text{sign}(\varepsilon_z)z^*(s)), \end{aligned} \quad (14)$$

where $W_\varepsilon(s) = 1 - W(s)$.

From expressions (3) it is simply to get:

$$\begin{aligned} \dot{y}^*(t) &= \frac{dg_y(x^*)}{dx^*} \frac{dx^*}{dt} = \frac{g'_y(x^*)}{\Phi(x^*)} v^*(t), \\ \dot{z}^*(t) &= \frac{dg_z(x^*)}{dx^*} \frac{dx^*}{dt} = \frac{g'_z(x^*)}{\Phi(x^*)} v^*(t). \end{aligned} \quad (15)$$

If we apply the Laplace transformation to expression (15) then we get:

$$\begin{aligned} x^*(s) &= k_{vx} v^*(s) / s, y^*(s) = k_{vy} v^*(s) / s, \\ z^*(s) &= k_{vz} v^*(s) / s, \end{aligned} \quad (16)$$

where $k_{vx} = 1/\Phi(x^*)$, $k_{vy} = (dg_y(x^*)/dx^*)/\Phi(x^*)$, $k_{vz} = (dg_z(x^*)/dx^*)/\Phi(x^*)$ are current value of corresponding functions.

Take into account (16) we can transform the expression (14) to from:

$$\begin{aligned} E(s) &= k_m \tilde{k}_e \frac{W_\varepsilon(s)}{s} (\text{sign}(\varepsilon_x)k_{vx} + \\ &+ \text{sign}(\varepsilon_y)k_{vy} + \text{sign}(\varepsilon_z)k_{vz}) v^*(s). \end{aligned} \quad (17)$$

As CO (17) is inertial objects then point X^* always outstrip current UV's position. So we can confirm that signature of UV's desirable velocity on corresponding coordinate coincide with the signature of dynamic error on this coordinate. It is aside from small trajectory's parts where the change of signature of UV's velocity on separate coordinates takes place. The signature of dynamic error is unequal to signature of desirable velocity on corresponding coordinates on small time interval. So in further reasoning we can ignore it phenomenon.

Regarding what is set above and $\Phi(x^*) > 0$ we can present the penult multiplier in expression (17) in form:

$$k_v^* = |k_{vx}| + |k_{vy}| + |k_{vz}|. \quad (18)$$

We can see from expressions (3) and (16) that equality $k_v = (k_{vx}^2 + k_{vy}^2 + k_{vz}^2)^{1/2} = 1$ always will be correct. Therefore take into account inequality (8) it can write:

$$1 \leq k_v^* \leq \sqrt{3}. \quad (19)$$

As result the mathematical model of CO for regulator R take into account expressions (17)-(19) and entered assumptions can be presented in form:

$$E(s) = k_v^* k_m \tilde{k}_e W_\varepsilon(s) v^*(s) / s, \quad (20)$$

where $k_e \sqrt{3} / 3 \leq \tilde{k}_e k_m k_v^* \leq (1 + k_e) \sqrt{3}$.

Further we synthesize the regulator R for UV which includes the CS discussed in work (Filaretov, 2006). This CS provides to describe a UV's spatial

movement on all translational degrees of freedom by matrix differential equation:

$$\ddot{X} + C\dot{X} + KX = C\dot{X}^* + KX^* \quad (21)$$

where $C \in R^{3 \times 3}$, $K \in R^{3 \times 3}$ are diagonal matrices with positive elements.

It is simply to get from equation (21) a transfer function relatively error for each control channel of UV's translational movement:

$$W_e(s) = s^2 / (s^2 + cs + k), \quad (22)$$

where $c = c_i > 0$, $k = k_i > 0$, $i = \overline{1,3}$ are elements of matrices C and K respectively.

Take into account expressions (20) and (22) the CO's transfer function can be present as

$$W_p(s) = E(s) / v^*(s) = \tilde{k}_e k_m k_v s / (s^2 + cs + k). \quad (23)$$

It is obvious that transfer function of regulator $R(s)$ will has a form:

$$R(s) = k_r (T_r s + 1) / s, \quad (24)$$

then take into account expression (23) the transfer function of open loop system will be in following:

$$W_r(s) = R(s)W_p(s) = \frac{k_r \tilde{k}_e k_m k_v (T_r s + 1)}{s^2 + cs + k}. \quad (25)$$

As we can see from expression (25) the stability of close loop system not depend from value of coefficients $k_r, \tilde{k}_e, k_m, k_v$. So parameters of regulator R we can choose of any value by depend on required characteristics of work quality.

5 THE SIMULATION OF WORK OF SYNTHESIZED SYSTEM

The mathematical simulation for checking of workability of synthesized system was carried out. It supposes that UV already include the CS which provides to describe of UV's dynamic properties by equation analogous the equation (20) (Filaretov, 2006). Therefore it supposes that UV has identical actuators which are described as aperiodic links.

The UV's mathematical model described in work (Lebedev, 2004) was used during the simulation. The CS's parameters was selected so that the elements of matrices C and K in equation (20) have following values: $k_i = 1$, $c_i = 0.3$, $i = \overline{1,3}$. The parameters of regulator $R(s)$ had values: $k_r = 10$,

$T_r = 0.5 s$, $k_e = 0.2$, $e_{max} = 0.2 m$. During simulation the UV's horizontal movement along trajectory $z^*(t) = 10 \sin(\pi x^*(t) / 20)$. In this case it supposes that $x_0^* = 0$, $z_0^* = 0$ and UV's long axis always directed to point X^* during UV's movement.

On the fig. 3 the processes of change of values $v^*(t)$, $v(t)$, $e(t)$, $E(t)$ and $z(t)$ during UV's movement on desired trajectory are shown.

On this figure we can see that minimal and maximum value $v^*(t)$ is formed when UV move on trajectory with minimal and maximum velocity respectively (see curve $z(t)$). On the trajectories parts which close to rectilinear $e \rightarrow 0$, value $v^*(t)$ increase, the UV's actuators reaches to saturations and value $v(t)$ reaches to maximum. It is going to result in fast increasing of ε_m and $E(t)$. As result $E(t)$ become more then e_{max} and value $v^*(t)$ begin to decrease. During the UV's movement its desirable velocity $v^*(t)$ changed from 0,85 m/s to 2,2 m/s and real velocity $v(t)$ from 0,9 m/s to 2 m/s.

For comparison on fig. 4 the simulation results is shown for case of UV's movement without using of synthesized system of automatic formation of desirable velocity.

In this case such value $v^*(t) = 0,9 m/s = const$ is selected which provide correctness inequality $e(t) < e_{max}$ on whole trajectory. On this figure we can see that during UV's movement with constant value of v^* the value of e less then 0,2 m too. But in this case the UV spends for running of one period of trajectory on 66 seconds and when the synthesized system is used it spends only 45 seconds.

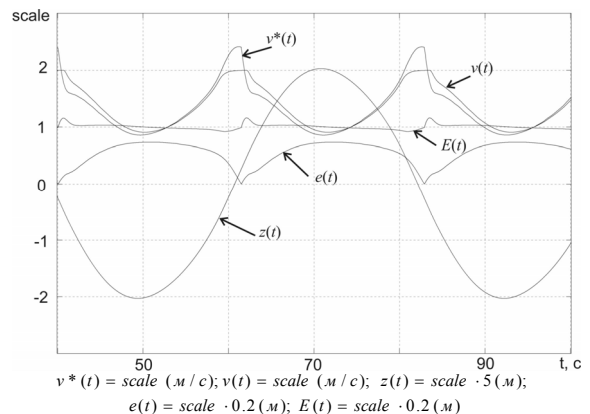


Figure 3: Processes of change $v^*(t)$, $z(t)$, $e(t)$, $E(t)$ when the automatic system for formation of program signal of UV's movement is used.

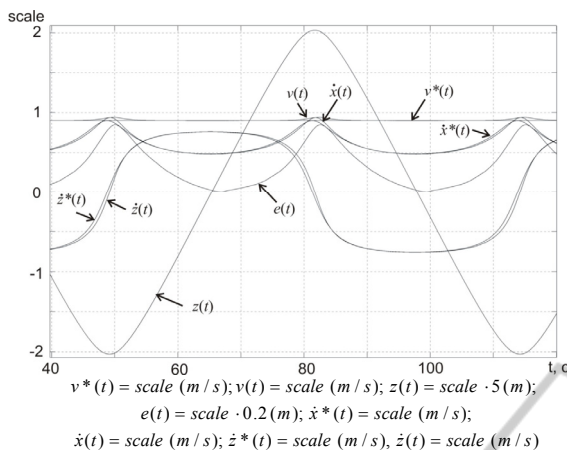


Figure 4: UV's movement without using of automatic system for formation of program signals.

Thus the simulation results confirm the workability of synthesized system for automatic formation of UV's desirable velocity.

6 CONCLUSIONS

Thus in this paper the new method for formation of UV's program signals is suggested. This method allow automatically set such UV's desirable velocity which provide UV's movement on desirable trajectory with deviation less then allowable value. The formation of this desirable velocity takes place by using of current deviation of UV from desirable trajectory and norm of dynamic error vector. In this work the block diagram of this system was suggested and the selection of desirable velocity regulator was proved. It has carried out the mathematical simulation which shows the workability and efficiently suggested approach.

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