ADAPTIVE LQG CONTROL WITH LOOP TRANSFER RECOVERY

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Abstract: An adaptive discrete-time LQG control with loop transfer recovery is considered using shift and delta operators. The control problem is analyzed using state-space model and the parameter estimation problem is implemented for corresponding ARMAX model. Analysis of asymptotic performance of delta model approach and continuous-time model case is presented. Computer simulations of third-order system modeled by a second-order model are given to illustrate the robustness properties of the adaptive LQG/LTR controller.

1 INTRODUCTION

Adaptive LQG control is not an area of a great deal of research, in particular for adaptive LQG control with loop transfer recovery (LTR). Adaptive LQG control has been discussed e.g. in (Bitmead et al., 1990; Tay and Moore, 1991; Krolikowski, 1995; Mäkilä et al., 1984), where in (Tay and Moore, 1991) an adaptive LQG/LTR problem was solved augmenting the basic estimator-based controller with a stable proper linear system feeding back the estimation residuals. This idea was also used for non-adaptive continuous-time systems in (Tay and More, 1989) using the H^∞/H^2 optimization technique.
In this paper, an application of LTR technique to adaptive control of discrete-time systems for both z and δ operators is presented. The adaptive continuous-time LQG control algorithm is proposed where the controller/filter parameters are tuned on the basis of δ model identification. Asymptotic performance for $\lim_{T_s \to} 0$ is analyzed. The robustness issue is touched and simulated for third-order ARX system considered as a second-order model.

2 PRELIMINARIES

Consider the following state-space description of the multivariable linear discrete-time system

$$\underline{x}_{t+1} = F\underline{x}_t + G\underline{u}_t + \underline{w}_t \quad (1)$$
$$\underline{y}_t = H\underline{x}_t + \underline{v}_t \quad (2)$$

obtained with ZOH where $\{\underline{w}_t\}$ and $\{\underline{v}_t\}$ are sequences of independent random vector variables with zero mean and covariances $E\underline{w}_t\underline{w}_s^T = \Sigma_w\delta_{t,s}$, $\underline{v}_t\underline{v}_s^T = \Sigma_v\delta_{t,s}$.
The Kalman predictor in steady-state is given by

$$\hat{\underline{x}}_{t+1/t} = F\hat{\underline{x}}_{t/t-1} + G\underline{u}_t + K_p\tilde{\underline{y}}_t^p \quad (3)$$

where $\tilde{\underline{y}}_t^p = \underline{y}_t - H\hat{\underline{x}}_{t/t-1}$ is an innovation of output at time t. The predictor gain is given by

$$K_p = FPH^T[HPH^T + \Sigma_v]^{-1} \quad (4)$$

where P is the solution of Riccati equation

$$P = FPF^T + \Sigma_w - FPH^T[HPH^T + \Sigma_v]^{-1}HPF^T \quad (5)$$

The covariance of the innovation $\tilde{\underline{y}}_t^p$ is $\Sigma_{\tilde{y}} = HPH^T + \Sigma_v$.
Filtered estimate $\hat{\underline{x}}_{t/t}$ in terms of $\hat{\underline{x}}_{t/t-1}$ is

$$\hat{\underline{x}}_{t/t} = \hat{\underline{x}}_{t/t-1} + K_f\tilde{\underline{y}}_t^p \quad (6)$$

and the recursive equation for $\hat{\underline{x}}_{t/t}$ is

$$\hat{\underline{x}}_{t+1/t+1} = F\hat{\underline{x}}_{t/t} + (I - K_fH)G\underline{u}_t + K_f\tilde{\underline{y}}_{t+1}^f \quad (7)$$

where $\tilde{\underline{y}}_{t+1}^f = \underline{y}_{t+1} - HF\hat{\underline{x}}_{t/t}$ and the filter gain

$$K_f = PH^T[HPH^T + \Sigma_v]^{-1}, \quad (8)$$

so $K_p = FK_f$ in view of (4). An alternative version of (7) is

$$\hat{\underline{x}}_{t+1/t+1} = F\hat{\underline{x}}_{t/t} + G\underline{u}_t + K_f\tilde{\underline{y}}_{t+1}^p \quad (9)$$

3 LOOP TRANSFER RECOVERY: z OPERATOR FORMULATION

Consider the stationary loss function

$$J = E\sum_{t=0}^{\infty}\underline{y}_t^T\underline{y}_t \quad (10)$$

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and assume that the system is square and $\det(HG) \neq 0$.
The control law

$$\underline{u}_t = K_c \underline{x}_{t/t} \quad (11)$$

minimizing the loss J is then determined by

$$K_c = -(HG)^{-1} HF \quad (12)$$

and the matrix $H^T H$ is the solution of the corresponding Riccati equation.
The transfer function in z operator $G_f(z)$ of compensator defined by (7) and (11) can be manipulated into the form

$$G_f(z) = zK_c[zI - (I - K_f H)(F - GK_c)]^{-1} K_f \quad (13)$$

The filter's open-loop return ratio is

$$\Phi(z) = H(zI - F)^{-1} K_p \quad (14)$$

In (Maciejowski, 1985) it was shown that if $G(z) = H(zI - F)^{-1}G$ is minimum-phase and K_c takes a form of (12) then the perfect recovery takes place, that is

$$\Delta(z) = G(z)G_f(z) - \Phi(z) = 0. \quad (15)$$

When $G(z)$ is nonminimum-phase then the perfect recovery is in general not possible, however the possibility of recovery is frequently realized in closed-loop bandwidth (Maciejowski, 1985).
In the case of the Kalman predictor feedback, the controller is

$$\underline{u}_t = K_c \underline{x}_{t/t-1} \quad (16)$$

and its transfer function is

$$G_p(z) = K_c[zI - F + GK_c + K_p H]^{-1} K_p \quad (17)$$

Again the perfect recovery cannot be achieved in this case even for minimum-phase system.

4 LOOP TRANSFER RECOVERY: δ OPERATOR FORMULATION

State equation (1) in δ operator formulation takes a form

$$\delta \underline{x}_t = F_\delta \underline{x}_t + G_\delta \underline{u}_t + \underline{w}'_t \quad (18)$$

where in view of (1) $F_\delta = \frac{1}{T_s}(F - I)$, $G_\delta = \frac{1}{T_s}G$ and $\underline{w}'_t = \frac{1}{T_s}\underline{w}_t$, $\underline{v}'_t = \frac{1}{T_s}\underline{v}_t$ are sequences with spectral densities W and V, respectively. Usually, the δ operator discretization is used for small T_s when ZOH discretization makes numerical problems. The filter's open-loop return ratio at the output node of the plant is

$$\Phi(\gamma) = H(\gamma I - F_\delta)^{-1} K_p \quad (19)$$

where δ transform operator with the sampling period T_s is $\gamma = \frac{z-1}{T_s}$.

The LQG controller is defined by the control law

$$\underline{u}_t = K_c \underline{x}_{t/t} \quad (20)$$

with the Kalman filter given by

$$\hat{\underline{x}}_{t/t} = \hat{\underline{x}}_{t/t-1} + +T_s K_f \tilde{\underline{y}}^p_t \quad (21)$$

where

$$\delta \hat{\underline{x}}_{t/t-1} = F_\delta \hat{\underline{x}}_{t/t-1} + G_\delta \underline{u}_t + K_p \tilde{\underline{y}}^p_t \quad (22)$$

Moreover, it holds $K_p = (I + T_s F_\delta) K_f$, and an explicit recursive equation for Kalman filter is

$$\delta \hat{\underline{x}}_{t/t} = F_\delta \hat{\underline{x}}_{t/t} + G_\delta \underline{u}_t + K_f \tilde{\underline{y}}^p_{t+1}. \quad (23)$$

In (Tadjine et al., 1994) it was shown that if the system (18), (2) is stabilizable, detectable, left invertible and inversely stable, and weighting matrices in the performance index $Q = H^T H$, $R = \rho I$ then asymptotically as $\rho \to 0$, K_c takes the forms

$$K_c = -\frac{1}{T_s}(HG_\delta)^{-1} H(I + T_s F_\delta) \quad (24)$$

and the perfect recovery takes place, that is

$$\Delta(\gamma) = G(\gamma)G_f(\gamma) - \Phi(\gamma) = 0, \quad (25)$$

where $G(\gamma) = H(\gamma I - F_\delta)^{-1} G_\delta$, and

$$G_f(\gamma) = (1 + T_s \gamma) K_c [\gamma I - F_\delta + G_\delta K_c)]^{-1} K_f \quad (26)$$

is the transfer function of the controller

$$\underline{u}_t = K_c \underline{x}_{t/t} \quad (27)$$

where now

$$\hat{\underline{x}}_{t/t} = \hat{\underline{x}}_{t/t-1} + +T_s K_f \underline{y}_t \quad (28)$$

and

$$\delta \hat{\underline{x}}_{t/t-1} = F_\delta \hat{\underline{x}}_{t/t-1} + K_p \underline{y}_t. \quad (29)$$

The above results from the fact that as soon as recovery is obtained the coupling between the observation error and the observer output should vanish.

5 LOOP TRANSFER RECOVERY: CONTINUOUS-TIME FORMULATION

The dynamics of the system is given by the transfer function matrix from control input to the output

$$G(s) = C(sI - A)^{-1} B, \quad (30)$$

where A, B, C are matrices in the standard state-space equation, and $C = H$. It is worthy to note that asymptotically i.e. for $T_s \to 0$, $G(\gamma) \to G(s)$, however $G(z) \to 0$.

Transfer matrix of the controller is

$$G_f(s) = K_c^c[sI - A + BK_c^c + K_f^cC]^{-1}K_f^c \quad (31)$$

To compute K_c^c for the LQG/LTR controller the following Riccati equation is to be solved

$$P_\rho A + A^T P_\rho + C^T C - \frac{1}{\rho} P_\rho BB^T P_\rho = 0 \quad (32)$$

for $\rho \to 0$ and then the controller gain K_c^c is calculated as

$$K_{c,\rho}^c = -\frac{1}{\rho} B^T P_\rho. \quad (33)$$

The following LTR result holds (Athans, 1986): if the plant $G(s)$ is minimum-phase then $\lim_{\rho \to 0} G(s)G_{f,\rho}(s) = \Phi(s)$, where $\Phi(s) = C(sI - A)^{-1}K_f^c$ and $G_{f,\rho}(s)$ is calculated from (31) for $K_{c,\rho}^c$. The dual LTR result, i.e. when the weighting matrix $Q = Q_0 + \rho M$ for $\rho \to \infty$ can be found in (Kulcsar, 2000). It is easy to see from (26) that asymptotically

$$\lim_{T_s \to 0} G_f(\gamma) = G_f(s) = K_c[sI - A + BK_c)]^{-1}K_f \quad (34)$$

and full recovery holds that is $G(s)G_f(s) = \Phi(s)$, so the δ model approach and continuous-time case are asymptotically equivalent. Obviously, it holds $K_p = K_f$ for $T_s \to 0$.
To compute K_f^c in (31) the following Riccati equation is to be solved

$$AP_\mu + P_\mu A^T + L^T L - \frac{1}{\mu} P_\mu C^T CP_\mu = 0 \quad (35)$$

and then the filter gain K_f^c is calculated as

$$K_{f,\mu}^c = \frac{1}{\mu} P_\mu C^T. \quad (36)$$

where μI and $L^T L$ are intensity matrices for measurement and system noise, respectively.

6 ADAPTIVE CONTROL

The SISO ARMAX model is given by

$$A(q^{-1})y_t = B(q^{-1})u_t + C(q^{-1})e_t \quad (37)$$

where $A(q^{-1}), B(q^{-1})$ and $C(q^{-1})$ are polynomials in the backward shift operator q^{-1}, i.e. $A(q^{-1}) = 1 + a_1q^{-1} + ... + a_nq^{-n}, B(q^{-1}) = b_1q^{-1} + ... + b_nq^{-n}, C(q^{-1}) = 1 + c_1q^{-1} + ... + c_nq^{-n}$ and y_t is the output , u_t is the control input, and $\{e_t\}$ is assumed to be a sequence of independent variables with zero mean and variance σ_e^2. Unknown system parameters $\underline{\theta} = (a_1, ..., a_n, b_1, ..., b_n, c_1, ..., c_n)^T$ (or corresponding parameters of δ model) are estimated on-line to obtain an updated model at time t, i.e. $\underline{\hat{\theta}}_t$ (or corresponding δ model) which is in turn used for updating the lqg adaptive control of the system. The parameter estimates of δ model can be used for tuning the continuous-time LQG/LTR control assuming the sampling period is small enough. In this way a continuous-time system identification problem can be omitted.
ARMAX model (31) has an equivalent innovation state space representation

$$\underline{x}_{t+1} = F\underline{x}_t + \underline{g}u_t + \underline{k}_p e_t \quad (38)$$

$$y_t = \underline{h}^T \underline{x}_t + e_t \quad (39)$$

where $\underline{g} = (b_1, ..., b_n)^T$, $\underline{k}_p = (c_1 - a_1, ..., c_n - a_n)^T$, $\underline{h}^T = (1, 0, ..., 0)$

$$F = \begin{bmatrix} -a_1 & 1 & \dots & 0 \\ . & . & \dots & 0 \\ -a_{n-1} & . & \dots & 1 \\ -a_n & . & \dots & 0 \end{bmatrix},$$

$\underline{k}_p$ is the stationary gain vector for the associated Kalman predictor corresponding to (3)

$$\underline{\hat{x}}_{t+1/t} = F\underline{\hat{x}}_{t/t-1} + \underline{g}u_t + \underline{k}_p \tilde{y}_t^p \quad (40)$$

where $\tilde{y}_t^p = y_t - \underline{h}^T \underline{\hat{x}}_{t/t-1}$ and $\sigma_{\tilde{y},p}^2$ is the variance of $\tilde{y}_t^p$ for which it holds $\sigma_{\tilde{y},p}^2 = \sigma_e^2$.
The actual model used for LQG/LTR control signal u_t calculation is obtained for current parameter estimates $\underline{\hat{\theta}}_t$.
The investigated problem is to check out how the approximated δ model used in adaptive LQG/LTR control can be used in tuning the continuous-time LQG/LTR control.
The issue of stability of the proposed adaptive LQG/LTR control system is of course crucial. This depends on the asymptotic convergence of parameter estimates, particularly taking into account that in general the parameter estimation in LQG adaptive control even in the lack of modelling error, does not assure the convergence to the true parameters. Closed loop stability and good performance cannot be guaranteed especially during the transient stage.

7 SIMULATIONS

Consider as an example a third-order minimum-phase actual system obtained by discretizing the continuous-time system

$$G(s) = \frac{s+5}{(s+1)(s+2)(s+3)} = \frac{-s+1}{(s+1)(s+2)} + \frac{1}{s+3}$$

with ZOH and sampling period $T_s = 0.5s$ whose nominal part has a standard state space representation

$$A = \begin{bmatrix} -3 & 1 \\ -2 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

and which yields the following transfer function in q^{-1} operator

$$G(q^{-1}) = \frac{0.09771q^{-1} + 0.06925q^{-2} - 0.005945q^{-3}}{1 - 1.198q^{-1} + 0.4406q^{-2} - 0.04979q^{-3}} = \\ = \frac{-0.1612q^{-1} + 0.2856^{-2}}{1 - 0.9744q^{-1} + 0.223^{-2}} + \frac{0.259^{-1}}{1 - 0.223^{-1}}. \quad (41)$$

The first part of $G(q^{-1})$ is taken for undermodelling. Substituting $q^{-1} = (1 + \delta T_s)^{-1}$ the corresponding discrete-time δ model is obtained as

$$y_t^\delta + \alpha_1 y_{t-1}^\delta + \alpha_2 y_{t-2}^\delta = \beta_1 u_{t-1}^\delta + \beta_2 u_{t-2}^\delta. \quad (42)$$

where $\alpha_1 = \frac{2+a_1}{T_p}$, $\alpha_2 = \frac{1+a_1+a_2}{T_p^2}$, $\beta_1 = \frac{b_1}{T_p}$, $\beta_2 = \frac{b_1+b_2}{T_p^2}$ and $y_t^\delta = \frac{y_t - 2y_{t-1} + y_{t-2}}{T_p^2}$, $y_{t-1}^\delta = \frac{y_{t-1} - y_{t-2}}{T_p}$, $y_{t-2}^\delta = y_{t-2}$, $u_{t-1}^\delta = \frac{u_{t-1} - u_{t-2}}{T_p}$, $u_{t-2}^\delta = u_{t-2}$.

As already mentioned, in both cases, a second-order ARX model was taken for identification and *certainty equivalence principle* was used to implement the adaptive control system to demonstrate the robustness of adaptive LQG/LTR controller with respect to undermodelling. The simulation of continuous -time adaptive LQG control with LTR and estimation based on the δ model are shown in Figs.1,2 for $\rho = 0.001$ and $T_s = 0.5, 0.2$, respectively. An output variance in steady state was calculated: for the case of Fig.1 it equals to 0.2090, and for the case of Fig.2 it is 0.6051.

The case with δ model is shown in Figs.3,4 for $T_s = 0.5, 0.2$, and corresponding variances equal to $0.2956, 0.5584$, respectively. In both cases the adaptive control system performs well, however the continuous-time LQG/LTR adaptive control system with δ model tuning is superior with respect to output variance.

System parameters were identified using the standard recursive least squares (RLS) algorithm for $t = 1, \ldots, 300$ and $\sigma_e^2 = 0.1$. Obviously, in the general case of ARMAX model the recursive pseudolinear regression (RPLR) or recursive prediction error (RPEM) algorithm must be used. It was shown in (Nilsson and Egardt, 2010), that RPEM is more suitable in the considered undermodelled situations taking into account the asymptotic properties of the algorithm.

8 CONCLUSIONS

The problem of using loop transfer recovery for adaptive LQG control is presented in both z and γ domains. In the latter case an asymptotic equivalence ($T_s \to 0$) with the continuous-time system is investigated. Example of third-order actual system described by a second-order ARX model is taken for simulation. Simulation results show an effectivness of the LTR technique as a method for robustifying the

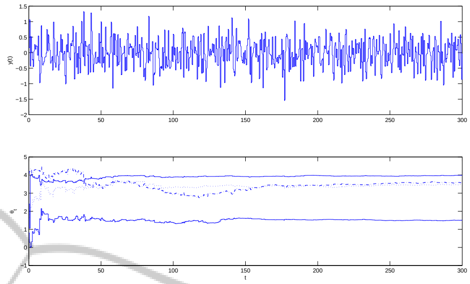

Figure 1: Output signal and estimates for $T_s = 0.5$, $\rho = 0.001$.

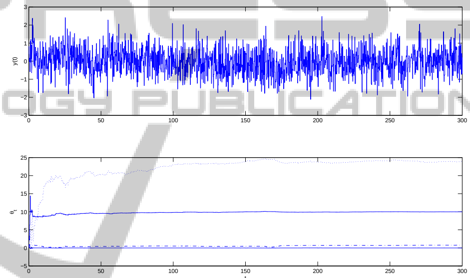

Figure 2: Output signal and estimates for $T_s = 0.2$, $\rho = 0.001$.

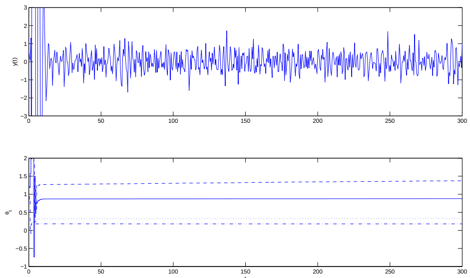

Figure 3: Output signals and estimates for $T_s = 0.5$, δ model.

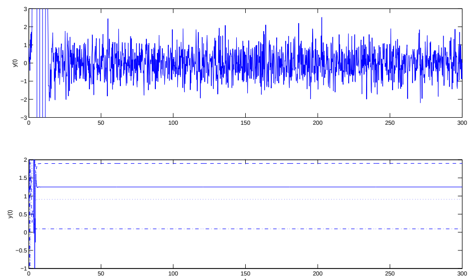

Figure 4: Output signals and estimates for $T_s = 0.2$, δ model.

adaptive LQG control.

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