DISCRETE-TIME LQG CONTROL WITH ACTUATOR FAILURE

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Keywords: Discrete-time, LQG control, Actuator failure.

Abstract: A discrete-time LQG control with actuator failure is considered. The control problem is analyzed in terms of algebraic Riccati equations. Computer simulations of two-input two-output system are given to illustrate the performance of the reliable LQG controller. An actuator saturation case is also included.

1 INTRODUCTION

Reliable LQG control is an area of some research. This has been discussed e.g. in (Yang et al., 2000b; Yang et al., 2000a; Maciejowski, 2009). In (Yang et al., 2000b) a discrete-time LQ control problem with actuator failure was considered while in (Yang et al., 2000a) a continuous-time LQG control problem with sensor failure was investigated. General ideas on posssible applications of fault-tolerant control are discussed in (Maciejowski, 2009).

In this paper, a discrete-time LQG control problem with actuator failure modelled by a scalling factor is considered. The aim of the paper is to check out how the method presented in (Yang et al., 2000b) for LQ control will work for LQG case. Simultaneous scaling and saturation failure case is also analyzed where the phenomenon similar to the short-term behaviour phenomenon (Chen et al., 1993; Chen et al., 1994) takes place. Numerical comparative simulations for two-input two-output system are given.

2 PROBLEM FORMULATION

Consider the following state-space description of the multivariable linear discrete-time system

$$\underline{x}_{t+1} = F\underline{x}_t + G\underline{u}_t + \underline{w}_t \tag{1}$$

where $\underline{x}_t$ is n-dimennsional state vector, $\underline{u}_t$ is m-dimensional control vector, and $\{\underline{w}_t\}$ is a sequence of independent random n-dimennsional vector variables with zero mean and covariance $E\underline{w}_t\underline{w}_s^T = \Sigma_w\delta_{t,s}$. Consider the stationary loss function

$$J = E(\sum_{t=0}^{\infty} \underline{x}_t^T Q\underline{x}_t + \underline{u}_t^T R\underline{u}_t), \tag{2}$$

where $Q > 0$, $R \geq 0$ are given weighting matrices.

The following actuator failure model is considered (Yang et al., 2000b)

$$u_{t,i}^F = \alpha_i u_{t,i} \quad i = 1,\cdots,m. \tag{3}$$

where $u_{t,i}^F$ denotes the signal from actuator that has failed and

$$0 \leq \underline{\alpha}_i \leq \alpha_i \leq \bar{\alpha}_i \quad i = 1,\cdots,m. \tag{4}$$

with $\underline{\alpha}_i \leq 1$, $\bar{\alpha}_i \geq 1$. In this model $\bar{\alpha}_i = \underline{\alpha}_i$ means the normal case $u_{t,i}^F = u_{t,i}$, $\bar{\alpha}_i = 0$ means the outage case while $\underline{\alpha}_i > 0$ means the partial failure case.

The control law

$$\underline{u}_t = K\underline{x}_t \tag{5}$$

minimizing the loss J is determined by optimal feedback gain

$$K^{opt} = -(G^T P^{opt} G + R)^{-1} G^T P^{opt} F \tag{6}$$

where P^{opt} comes from the Riccati equation

$$P^{opt} = Q + F^T P^{opt} F - \\ -F^T P^{opt} G(G^T P^{opt} G + R)^{-1} G^T P^{opt} F \tag{7}$$

The optimal performance is given then by the loss J^{opt} (Meier et al., 1971)

$$J^{opt} = \bar{\underline{x}}_0^T P^{opt} \bar{\underline{x}}_0 + tr[P^{opt}\Sigma_0] + tr[P^{opt}\Sigma_w] \tag{8}$$

where $\bar{\underline{x}}_0$ is the mean value of the initial state and Σ_0 is its covariance matrix.

The control law is said to be a reliable guaranteed cost associated with a matrix P if P satisfies the equation

$$[F + G\alpha K]^T P[F + G\alpha K] - P + K^T \alpha R\alpha K + \\ +Q \leq 0 \tag{9}$$

Horla D. and Krolikowski A..
DISCRETE-TIME LQG CONTROL WITH ACTUATOR FAILURE.
DOI: 10.5220/0003648105170521
In *Proceedings of the 8th International Conference on Informatics in Control, Automation and Robotics* (ICM-2011), pages 517-521
ISBN: 978-989-8425-74-4

for all α_i satisfying (4). The aim of the algorithm given below is to find a reliable state feedback control law.

The following notations are adopted

$$\bar{\alpha} = diag(\bar{\alpha}_1, \bar{\alpha}_2, \cdots \bar{\alpha}_m) \quad (10)$$
$$\underline{\alpha} = diag(\underline{\alpha}_1, \underline{\alpha}_2, \cdots \underline{\alpha}_m) \quad (11)$$
$$\alpha = diag(\alpha_1, \alpha_2, \cdots \alpha_m) \quad (12)$$

3 CONTROL ALGORITHM

The following algorithm (Yang et al., 2000b) is taken for consideration

- Step 1 Solve (7) for P^{opt}, then choose diagonal R_0 satisfying
$$R_0 \leq (G^T P^{opt} G + R)^{-1}. \quad (13)$$
- Step 2 Solve
$$P = Q + F^T PF - F^T PJ_0 PF \quad (14)$$
for stabilising P and then check
$$R_0 \leq (G^T PG + R)^{-1}, \quad (15)$$
where
$$J_0 = G(I - \beta_0^2)[(G^T PG + R)(I - \beta_0^2) + R_0^{-1}\beta_0^2]^{-1}G^T. \quad (16)$$
- Step 3 (i) If eqn.(15) holds for R_0 and P then increase R_0 and go to Step 2. (ii) If eqn.(15) does not hold for R_0 and P then decrease R_0 and go to Step 2.
- Step 4 When eqn.(15) holds for R_0 and stabilising P fulfils (14), but eqns. (14) and (15) have no positive solution for any R_{01} with $R_0 < R_{01} \leq (G^T P^{opt} G + R)^{-1}$, stop. In this case the feedback gain is given by
$$K = -\beta^{-1}\{I - (X^{-1} - R_0)[(I - \beta_0^2) + \beta_0^2 R_0^{-1} X^{-1}]^{-1}$$
$$\times \beta_0^2 R_0^{-1}\} X^{-1} G^T PF, \quad (17)$$
where $X = G^T PG + R$.

The following notations have been adopted

$$\beta = diag(\beta_1, \beta_2, \cdots \beta_m), \quad (18)$$
$$\beta_0 = diag(\beta_{10}, \beta_{20}, \cdots \beta_{m0}), \quad (19)$$

where

$$\beta_i = \frac{\bar{\alpha}_i + \underline{\alpha}_i}{2}, \quad (20)$$
$$\beta_{i0} = \frac{\bar{\alpha}_i - \underline{\alpha}_i}{\bar{\alpha}_i + \underline{\alpha}_i}. \quad (21)$$

Moreover, matrix P is said to be a stabilising solution to the Riccati equation

$$P = Q + F^T PF - F^T PG(G^T PG + R)^{-1} G^T PF \quad (22)$$

if it satisfies this equation and the matrix $F - G(G^T PG + R)^{-1} G^T PF$ is stable.

4 AMPLITUDE-CONSTRAINED CONTROL

The case of amplitude-constrained control input which can be treated as an actuator saturation can also be considered as a kind of actuator failure (Zuo et al., 2010). In this case the control input can be expressed as

$$u_{t,i}^F = sat(\underline{k}_i^T \underline{x}_t; \beta_i) \qquad i = 1, \cdots, m \quad (23)$$

where β_i is the value of constraint for $u_{t,i}$ and $\underline{k}_i^T$ is the $i-th$ row of feedback gain matrix K. The method for calculating optimal feedback gain for stochastic systems under the saturation constraint was proposed for example in (Toivonen, 1983; Krolikowski, 2004).

Illustration of actuator failure given by (4) and the actuator saturation given by (23) is shown in Fig.1 for single input system.
In this figure the model failure

$$u_{t,i}^F = sat(\alpha_i u_{t,i}; \beta_i) \qquad i = 1, \cdots, m \quad (24)$$

being the superposition of models (3) and (23) is also illustrated (shadowed area).

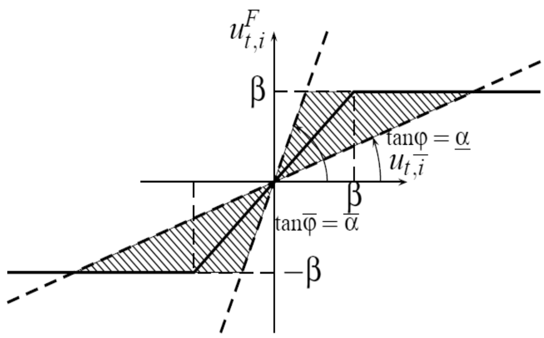

Figure 1: Illustration of actuator failure.

5 SIMULATIONS

The following two-input two-output system is given by matrices

$$F = \begin{bmatrix} 1 & 0.1 \\ -0.5 & 0.9 \end{bmatrix}, \quad G = \begin{bmatrix} 1 & -0.4 \\ -0.8 & 0.9 \end{bmatrix},$$

Table 1: Feedback gains and loss function values; failure (3).

$\underline{\alpha}_i$ / $\overline{\alpha}_i$	$\underline{k}_1^T$ / $\underline{k}_2^T$	$\overline{J}$
0.9	–0.9004, –0.3319	–
1.1	–0.1572, –1.0898	2.2794
0.7	–0.7492, –0.0924	–
1.3	0.0342, –0.7964	2.4626
0.5	–0.6357, 0.0989	–
1.5	0.1967, –0.5417	2.7634
0.3	–0.6141, 0.1750	–
1.7	0.3034, –0.3381	3.1764
0.1	–0.9880, 0.1943	–
1.9	0.5037, –0.2898	4.6344

$$\Sigma_w = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$Q = I$, $R = 0.1I$ and $\underline{x}_0 = \underline{0}$.

The feedback gains $K^T = (\underline{k}_1, \underline{k}_2)$ and the corresponding values of the loss function for few configurations of system failure $\underline{\alpha}_i, \overline{\alpha}_i$ are shown in Table 1. The values of the loss function J were averaged over 10000 runs.

The feedback gains and the corresponding values of the loss function for few constraints β_1, β_2 for failure (23) are shown in Table 2. In this case, the suboptimal feedback gains $\underline{k}_1, \underline{k}_2$ were calculated with iterative procedure given in (Toivonen, 1983; Krolikowski, 2004) for given constraints β_1, β_2.

The optimal value of the loss function (also seen from Table 2 for $\beta_1 = \beta_2 = \infty$) is $J^{opt} = 2.256$. Finally, the feedback gains and the corresponding values of the loss function for failure (24) and different constraints β_1, β_2 are shown in Tables 3, 4, 5, 6. It should be noticed that in this case the amplitude constraint β_i in (24) is realized as a simple cut-off that is obviously not an optimization approach like in the previous case.

The exemplary run of inputs and outputs with actuator failure (3) with $\underline{\alpha}_i = 0.75, \overline{\alpha}_i = 1.25, \quad i = 1,2$ is shown in Fig.2 where the corresponding loss is $\overline{J} = 2.4018$, and the corresponding run under actuator failure (24) with $\underline{\alpha}_i = 0.75, \overline{\alpha}_i = 1.25, \beta_i = 1.5, \quad i = 1,2$ is shown in Fig.3 where the corresponding loss is $\overline{J} = 2.5440$.

Analyzing the values of the loss function given in Tables 3, 4, 5, 6 one can observe a phenomenon like *the short-term behaviour phenomenon* described in (Chen et al., 1993; Chen et al., 1994) which takes place when the minimum variance control is considered and the cutoff method is used to constrain the control signal. This means that even though more control effort is applied to the system, the closed-loop system performance does not improve. The effect of

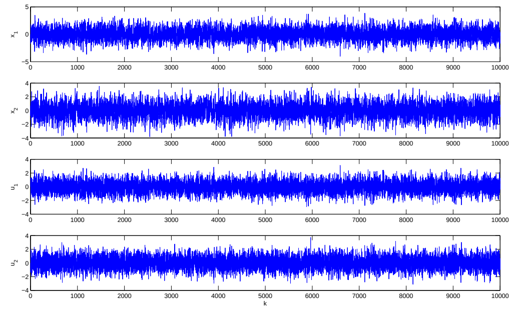

Figure 2: Input and output signals for failure (3).

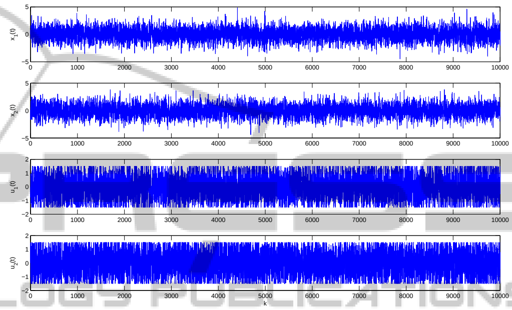

Figure 3: Input and output signals for failure (24).

Table 2: Feedback gains and loss function values; failure (23).

β_1 / β_2	$\underline{k}_1^T$ / $\underline{k}_2^T$	$\overline{J}$
∞	–0.9322, –0.3821	–
∞	–0.1975, –1.1526	2.2567
1.5	–0.8594, –0.2113	–
1.5	–0.0544, –1.0186	2.4879
1.0	–0.7465, –0.0614	–
1.0	0.1423, –0.8302	2.7932
0.5	–0.5987, 0.1166	–
0.5	0.4141, –0.4282	4.3363
0.3	0.6192, 0.0785	–
0.3	0.5146, –0.2542	9.1401

that kind happens for $\underline{\alpha}_i = 0.0, 0.1$, $\overline{\alpha}_i = 2.0, 1.9$ as illustrated in Fig.4, where the notation $\underline{\alpha}_i = 1 - \delta$, $\overline{\alpha}_i = 1 + \delta$ is used. In Fig.5 where $\delta = 0.2, 0.3$ the effect is not seen. It can be concluded that the bigger δ the stronger is the effect.

A similar phenomenon can be observed for given β_i and variable δ, for example for $\beta_i = 1.0$, $\beta_i = 1.5$ that is illustrated by Fig.6. Fig.7 shows the case $\beta_i = 0.3$, $\beta_i = 0.5$ where the effect is not seen.

Table 3: Feedback gains and loss function values; failure (24), $\beta_1 = \beta_2 = 1.5$.

$\underline{\alpha}_i$ / $\overline{\alpha}_i$	$\underline{k}_1^T$ / $\underline{k}_2^T$	– / $\overline{J}$
0.9	–0.9004, –0.3319	–
1.1	–0.1572, –1.0898	2.5658
0.7	–0.7492, –0.0924	–
1.3	0.0342, –0.7964	2.5467
0.5	–0.6357, 0.0989	–
1.5	0.1967, –0.5417	2.7846
0.3	–0.6141, 0.1750	–
1.7	0.3034, –0.3381	3.1765
0.1	–0.9880, 0.1943	–
1.9	0.5037, –0.2898	4.2560

Table 4: Feedback gains and loss function values; failure (24), $\beta_1 = \beta_2 = 1.0$.

$\underline{\alpha}_i$ / $\overline{\alpha}_i$	$\underline{k}_1^T$ / $\underline{k}_2^T$	– / $\overline{J}$
0.9	–0.9004, –0.3319	–
1.1	–0.1572, –1.0898	3.3789
0.7	–0.7492, –0.0924	–
1.3	0.0342, –0.7964	2.9068
0.5	–0.6357, 0.0989	–
1.5	0.1967, –0.5417	2.9351
0.3	–0.6141, 0.1750	–
1.7	0.3034, –0.3381	3.2345
0.1	–0.9880, 0.1943	–
1.9	0.5037, –0.2898	4.0399

6 CONCLUSIONS

The problem of reliable LQG control for discrete-time stochastic system is presented. An example of a two-input system described by state-space equation is taken for simulation. Simulation results show an

Table 5: Feedback gains and loss function values; failure (24), $\beta_1 = \beta_2 = 0.5$.

$\underline{\alpha}_i$ / $\overline{\alpha}_i$	$\underline{k}_1^T$ / $\underline{k}_2^T$	– / $\overline{J}$
0.9	–0.9004, –0.3319	–
1.1	–0.1572, –1.0898	8.7100
0.7	–0.7492, –0.0924	–
1.3	0.0342, –0.7964	5.8636
0.5	–0.6357, 0.0989	–
1.5	0.1967, –0.5417	5.0188
0.3	–0.6141, 0.1750	–
1.7	0.3034, –0.3381	4.8076
0.1	–0.9880, 0.1943	–
1.9	0.5037, –0.2898	4.9832

Table 6: Feedback gains and loss function values; failure (24), $\beta_1 = \beta_2 = 0.3$.

$\underline{\alpha}_i$ / $\overline{\alpha}_i$	$\underline{k}_1^T$ / $\underline{k}_2^T$	– / $\overline{J}$
0.9	–0.9004, –0.3319	–
1.1	–0.1572, –1.0898	18.3355
0.7	–0.7492, –0.0924	–
1.3	0.0342, –0.7964	12.9966
0.5	–0.6357, 0.0989	–
1.5	0.1967, –0.5417	11.3516
0.3	–0.6141, 0.1750	–
1.7	0.3034, –0.3381	10.7430
0.1	–0.9880, 0.1943	–
1.9	0.5037, –0.2898	10.2722

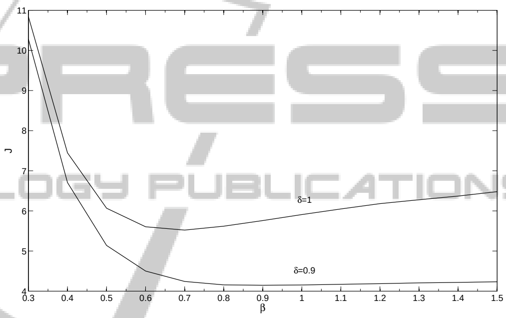

Figure 4: Loss function vs β for $\delta = 1.0, 0.9$.

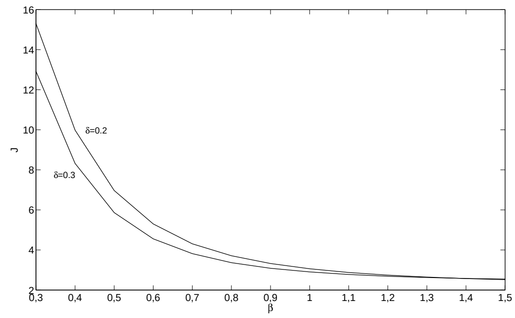

Figure 5: Loss function vs β for $\delta = 0.2, 0.3$.

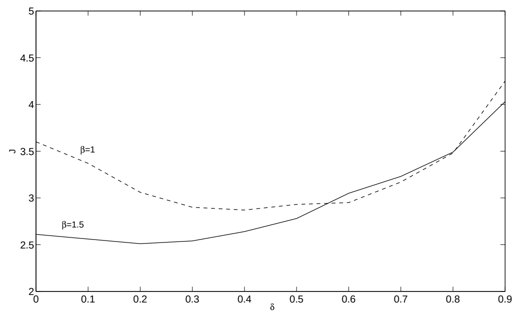

Figure 6: Loss function vs δ for $\beta = 1.0, 1.5$.

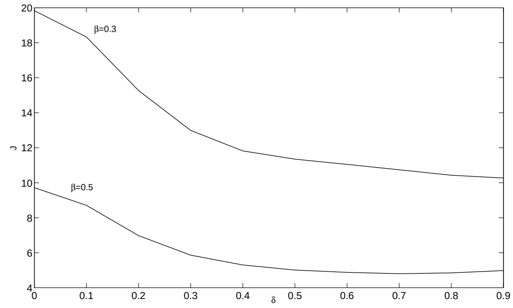

Figure 7: Loss function vs δ for $\beta = 0.3, 0.5$.

effectivness of the proposed control algorithms as a method for coping with actuator failure in the case of state feedback LQG control. A failure in form of actuator saturation (23) has stronger impact on the loss function than actuator failure given by (3), especially for tight constraints. In that case the failure (24) has even more stronger impact.

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