EXPERIMENTAL RESULTS OF INTEGRAL SLIDING MODE CONTROLLER FOR A NONHOLONOMIC MOBILE ROBOT

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Abstract: This paper addresses the trajectory tracking problem of a nonholonomic mobile robot. More precisely, we are interested in solving the problem of tracking a reference trajectory in presence of disturbances. A control strategy based on the Integral Sliding Mode is proposed combined with a state feedback linearization. While many studies have considered the kinematic model of the vehicle only, we have used both kinematic and dynamic models. The distinctive property of the proposed controller is its robustness of performance in the presence of uncertainties. To assess the quality of the proposed approach, we performed in addition to simulations the implementation of this controller on the robot Koala, a two-wheel differentially driven mobile robot. Lab work illustrates the real quality and efficiency of this control strategy.

1 INTRODUCTION

The motion control of mechanical systems under nonholonomic constraints has received much attention during past years. Wheeled mobile robots and car-like vehicles are typical examples of such systems. As pointed out in an early paper of Brockett (Brockett, 1983), such control systems cannot be stabilized by continuously differentiable, time invariant, state feedback control laws. Another difficulty in controlling nonholonomic mobile robots is that in the real world there are uncertainties in their modeling. Taking into account intrinsic characteristics of mobile robots such as actual vehicle dynamics, inertia and power limits of actuators and localization errors, their dynamic equations could not be described as a simplified mathematical model. This has attracted interest of researchers to the problem of nonholonomic mobile robot control. Discontinuous state feedback controller is used (Astolfi, 1995), (Astolfi, 1996)), tracking control using direct Lyapunov method (D'Andrea-Novel et al., 1995), time variant state feedback (Samson, 1995), (Walsh et al., 1994). Stabilization and control of nonholonomic systems with dynamic equations are presented in (Bloch et al., 1992), backstepping based methods has been considered in several papers (Fierro and Lewis, 1997), (Jiang and Nijmeijer, 1997), (Tanner and Kyriakopoulos, 2002) and a switched finite-time control algorithm has been proposed in (Banavar and Sankaranarayanan, 2006).

Sliding mode control has been applied to the trajectory control of robot manipulators (Slotine and Sastry, 1983), (Yeung and Chen, 1988), and is receiving increasing attention from researches on control of nonholonomic systems with uncertainties. For example, in (Guldner and Utkin, 1994) a sliding mode control was used to guarantee exact tracking of trajectories made by navigation functions. In (Yang and Kim, 1999) a sliding mode control law is proposed for asymptotically stabilizing the mobile robot to a desired trajectory, where robot posture was represented using polar coordinates. The benefits of the sliding mode command which makes it very important is its robustness with regards to disturbances and structural uncertainties, i. e. the system response depends on the gradient of the sliding surface and remains insensitive to variations of system parameters and external disturbances. However, during the reaching phase (before Sliding Mode occurs), the system has no such insensitivity property; therefore, insensitivity cannot be ensured throughout an entire response. The robustness during the reaching phase is normally improved by high-gain feedback control. Stability problems that arise inevitably limit the application of such high-gain feedback control schemes.

In this paper, we propose to perform a feedback linearization for a class of nonholonomic dynamic systems, combined with an Integral Sliding Mode controller which concentrates on the robustness of the motion in the whole state space. The order of the motion equation in this type of Sliding Mode is equal to the dimension of the state space. Therefore, the robustness of the system can be guaranteed throughout an entire response of the system starting from the

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initial time instance (Utkin and Shi, 1996). To assess the efficiency of this approach, the performance of the proposed controller has been compared to a traditional PID controller.

This paper is organized as follows. In Section 2, the general dynamic model of nonholonomic systems is presented. Feedback linearization is discussed in Section 3. In Section 4 the main result of Integral Sliding Mode is introduced, and its application to solve the tracking problem is discussed in Section 5. Finally in Section 6, experimental results using Koala mobile robot are discussed, before presenting the concluding remarks in Section 7.

2 MODELLING

The dynamical model of a nonholonomic system is expressed as (see (Campion et al., 1991b) for details)

$$\begin{aligned} \mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{f}(\mathbf{q},\dot{\mathbf{q}}) &= \mathbf{B}(\mathbf{q})\tau + \mathbf{A}(\mathbf{q})\lambda \\ \mathbf{A}^T(\mathbf{q})\dot{\mathbf{q}} &= 0 \end{aligned} \tag{1}$$

where $\mathbf{q} \in \mathbb{R}^n$ is an n-vector of generalized configuration variables, $\mathbf{M}(\mathbf{q}) \in \mathbb{R}^{n\times n}$ is a positive definite symetric inertia matrix, $\mathbf{f}(\mathbf{q},\dot{\mathbf{q}}) \in \mathbb{R}^n$ denotes the friction vector, $\mathbf{A}^T(\mathbf{q}) \in \mathbb{R}^{m\times n}$ is the matrix associated with nonholonmic constraints, $\lambda \in \mathbb{R}^{m\times 1}$ is a vector of Lagrange multipliers and $\mathbf{B}(\mathbf{q})\tau \in \mathbb{R}^n$ is the set of generalized forces applied to the system. As shown in (Campion et al., 1991b) it can be written in state space form as

$$\begin{aligned} \dot{\mathbf{q}} &= \mathbf{G}(\mathbf{q})\mathbf{v} \\ \mathbf{J}(\mathbf{q})\dot{\mathbf{v}} + \mathbf{m}(\mathbf{q},\mathbf{v}) &= \mathbf{G}^T(\mathbf{q})\mathbf{B}(\mathbf{q})\tau \end{aligned} \tag{2}$$

where $\mathbf{v} \in \mathbb{R}^m$ is the vector of pseudo-velocities and we have $\dot{\mathbf{q}} = \mathbf{G}(\mathbf{q})\mathbf{v}$, where $\mathbf{G}(\mathbf{q})$ is a matrix whose columns are a basis for the null space of $\mathbf{A}^T(\mathbf{q})$, so that $\mathbf{A}^T(\mathbf{q})\mathbf{G}(\mathbf{q}) = \mathbf{0}$ and we have

$$\begin{aligned} \mathbf{J}(\mathbf{q}) &= \mathbf{G}^T(\mathbf{q})\mathbf{M}(\mathbf{q})\mathbf{G}(\mathbf{q}) \\ \mathbf{m}(\mathbf{q},\mathbf{v}) &= \mathbf{G}^T(\mathbf{q})\mathbf{M}(\mathbf{q})\dot{\mathbf{G}}(\mathbf{q}) + \mathbf{G}^T(\mathbf{q})\mathbf{f}(\mathbf{q},\dot{\mathbf{q}}) \end{aligned} \tag{3}$$

Under the assumption that $\det(\mathbf{G}^T(\mathbf{q})\mathbf{B}(\mathbf{q})) \neq 0$, it is possible to perform a partial linearization via feedback on (2) by letting

$$\tau = \left(\mathbf{G}^T(\mathbf{q})\mathbf{B}(\mathbf{q})\right)^{-1}(\mathbf{J}(\mathbf{q})\mathbf{u} + \mathbf{m}(\mathbf{q},\mathbf{v})) \tag{4}$$

where $\mathbf{u} \in \mathbb{R}^m$ is the *pesudo-acceleration* vector. The resulting system is then

$$\begin{aligned} \dot{\mathbf{q}} &= \mathbf{G}(\mathbf{q})\mathbf{v} \\ \dot{\mathbf{v}} &= \mathbf{u} \end{aligned} \tag{5}$$

By defining the state $\mathbf{q}_g = (\mathbf{q},\mathbf{v})$, system (5) can be expressed as

$$\dot{\mathbf{q}}_g = \begin{bmatrix} \mathbf{G}(\mathbf{q}_g)\mathbf{v} \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix}\mathbf{u} \tag{6}$$

which is known as the second-order kinematic model of the constrained mechanical system.

The following two properties of the system (6) have been established in (Campion et al., 1991a)

- The nonholonomic system (6) is controllable.
- The equilibrium point $x_* = 0$ of the nonholonomic system (6) can be made Lagrange stable, but can not be made asymptotically stable by a smooth state feedback.

3 REVIEW OF FEEDBACK LINEARIZATION

For the reader convenience, let's review feedback linearization as shown in (Campion et al., 1991b). For a nonholonomic system with n degrees of freedom and $n-m$ actuators, there exists an output vector function $\mathbf{y} = h(\mathbf{q})$ and a static state feedback control $\mathbf{u}(\mathbf{q},\mathbf{v})$ such that the closed loop is stable, and the output $\mathbf{y} = h(\mathbf{q})$ asymptotically converges to zero. This can be achieved by feedback linearization.

We start by choosing the output function

$$\mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_{n-m} \end{bmatrix} \tag{7}$$

which depends on the configuration state variable $\mathbf{q}$ only, but not on the state $\mathbf{v}$, such that the largest linearizable subsystem is obtained by differentiating this output function as follows

$$\begin{aligned} \dot{\mathbf{y}} &= \nabla_{\mathbf{q}} h(\mathbf{q})\dot{\mathbf{q}} \\ &= \nabla_{\mathbf{q}} h(\mathbf{q})\mathbf{G}(\mathbf{q})\mathbf{v} \end{aligned} \tag{8}$$

By differentiating again, one may write

$$\ddot{\mathbf{y}} = \mathbf{F}(\mathbf{q},\mathbf{v}) + \mathbf{D}(\mathbf{q})\mathbf{u} \tag{9}$$

where

$$\mathbf{F}(\mathbf{q},\mathbf{v}) = \frac{\partial}{\partial \mathbf{q}}\left[\nabla_{\mathbf{q}} h(\mathbf{q})\mathbf{G}(\mathbf{q})\mathbf{v}\right]\mathbf{G}(\mathbf{q})\mathbf{v} \tag{10}$$

$$\mathbf{D}(\mathbf{q}) = \nabla_{\mathbf{q}} h(\mathbf{q})\mathbf{G}(\mathbf{q}) \tag{11}$$

By choosing $h(\mathbf{q})$ in such a way that the matrix $\mathbf{D}(\mathbf{q})$ is nonsingular for all $\mathbf{q}$, then linearization is achieved by the following feedback control

$$\mathbf{u} = \mathbf{D}^{-1}(\mathbf{q})(\mathbf{z} - \mathbf{F}(\mathbf{q},\mathbf{v})) \tag{12}$$

where $\mathbf{z} \in \mathbb{R}^{n-m}$ is the new external control input. And the resulting system is

$$\ddot{\mathbf{y}} = \mathbf{z} \tag{13}$$

Let's consider a wheeled mobile robot (WMR) that is moving on a horizontal plane as shown in Fig. 1. The robot has two independently driven wheels on a single common axle. The centre of mass of the robot is located in $P(x, y)$, which is the origin of the local coordinate frame that is attached to the robot body and is located on the wheels' axis. The point $B(x_L, y_L)$ is a virtual reference point on x axis of the local frame at a distance L (lookahead distance) of P. If the generalized coordinates vector is selected to be $\mathbf{q} = [x \;\; y \;\; \theta]^T$, one velocity constraint is obtained as $x\sin\theta - y\cos\theta = 0$. Thus, we define the vector $\mathbf{v}$ of the nonholonomic robot as $\mathbf{v} = [v \;\; \omega]^T$, with v and ω denote the linear and angular velocities of the robot, respectively.

The dynamical equations of the mobile can be expressed in the matrix form (1) where

$$\mathbf{M}(\mathbf{q}) = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & I \end{bmatrix} \quad \mathbf{f}(\mathbf{q}, \dot{\mathbf{q}}) = \mathbf{0}$$

$$\mathbf{B}(\mathbf{q}) = \begin{bmatrix} \frac{1}{R}\cos\theta & \frac{1}{R}\cos\theta \\ \frac{1}{R}\sin\theta & \frac{1}{R}\sin\theta \\ \frac{D}{2R} & -\frac{D}{2R} \end{bmatrix} \quad \mathbf{A}(\mathbf{q}) = \begin{bmatrix} \sin\theta \\ -\cos\theta \\ 0 \end{bmatrix}$$

$$\lambda = -mv\omega$$

input-output linearizability is guaranteed through this choice of output function

$$\mathbf{y} = h(\mathbf{q}) = [x + L\cos\theta \;\; y + L\sin\theta]^T \tag{14}$$

with $L \neq 0$.

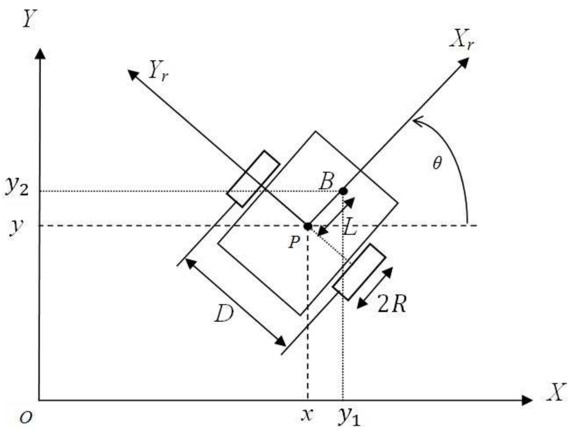

Figure 1: Unicycle mobile robot.

For the sake of completeness, in the next section we briefly present the major result of Integral Sliding mode technique presented in (Utkin and Shi, 1996).

4 INTEGRAL SLIDING MODE

For a given dynamic system represented by the following state space equation

$$\dot{x} = f(x) + B(x)u \tag{15}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, we suppose that there exists a feedback control law $u = u_0(x)$, such that system (15) can be stabilized in a desired way (e.g. its state trajectory follows a reference trajectory with a given accuracy). We denote this ideal closed loop system as

$$\dot{x}_* = f(x) + B(x)u_0 \tag{16}$$

where x_* denotes the state trajectory of the ideal system under control u_0. However, systems like (15) are normally operating under some uncertainty conditions that may be generated by parameter variations, unmodeled dynamics and external disturbances etc. Under this consideration a real control system may be summarized with

$$\dot{x} = f(x) + B(x)u + h_d(x,t) \tag{17}$$

in which function $h_d(x,t)$ represents the whole perturbation described above and we assume that it is bounded and fulfills the *uncertainty matching condition*, in other words

$$h_d(x,t) = B(x)u_h \quad u_h \in \mathbb{R}^m \tag{18}$$

For system (16), firstly, we design a control like

$$u = u_0 + u_1 \tag{19}$$

where u_0 is the ideal control defined in (16) and u_1 is designed to be discontinuous for rejecting the perturbation term $h_d(x,t)$. Secondly, we design our *switching function s* as

$$s = s_0(x) + \mu \tag{20}$$

with $s, s_0(x), \mu \in \mathbb{R}^m$.

This *switching function* consists of two parts; the first part $s_0(x)$ may be designed as the linear combination of the system states (similar to the conventional Sliding Mode design); and, the second part μ induces the integral term and will be determined below.

To derive the Sliding Mode equation, the time derivative of s on the system trajectories should be made equal to zero; the differential equation $\dot{s} = 0$ should be solved with respect to control input and the solution u_{eq} referred to as the *Equivalent Control* should be substituted into the motion equation for u (Utkin, 1992).

The control philosophy is to design an integral feedback such that the *Equivalent Control* is

$$u_{1eq} = -u_h \tag{21}$$

Condition (21) holds if

$$\dot{\mu} = \frac{\partial s_0}{\partial x}\left(f(x) + B(x)u_0\right) \\ \mu(0) = -s_0(x(0)) \tag{22}$$

where $\mu(0)$ is determined based on the requirement $s(0) = 0$ (Sliding Mode occurs starting from the initial time) . The motion equation of the system in Sliding Mode will be ideal system (16).

5 TRAJECTORY TRACKING

Given a smooth bounded reference trajectory

$$\mathbf{y}_d(t) = h(\mathbf{q}_d(t)) \tag{23}$$

which is generated by a trajectory generator which satisfies nonholonomic constraints, then the tracking control problem is to design a feedback control law for linearized system (13) with output equation $\mathbf{y}(t) = h(\mathbf{q}(t))$ such that the tracking error

$$\mathbf{e}(t) = \mathbf{y}(t) - \mathbf{y}_d(t) \tag{24}$$

is bounded and asymptotically tends to zero. By differentiating (24) twice, one may write using (13)

$$\ddot{\mathbf{e}} = \ddot{\mathbf{y}} - \ddot{\mathbf{y}}_d \\ = \mathbf{z} - \ddot{\mathbf{y}}_d + h_d(\mathbf{e}, t) \tag{25}$$

By applying the algorithm of previous section, we need to design control $\mathbf{z}$ as stated in equation (19): $\mathbf{z} = \mathbf{z}_0 + \mathbf{z}_1$, where $\mathbf{z}_0$ is predetermined such that system $\mathbf{x} = \mathbf{z}_0$, follows a given trajectory with satisfactory accuracy. For example, $\mathbf{z}_0$, may be obtained through linear feedback control, like $\mathbf{z}_0 = -\mathbf{k}^T\mathbf{x} + \ddot{\mathbf{y}}_d, \mathbf{k} \in \mathbb{R}^{(n-m)\times 1}$ in which gain vector $\mathbf{k}$ can be determined by Pole Placement or Linear Quadratic Regulator (LQR) methods.

We continue by designing the sliding surface

$$s = \mathbf{c}^T\mathbf{x} + \mu \tag{26}$$

$$\dot{\mu} = -\mathbf{c}^T(\mathbf{z}_0) \tag{27}$$

$$\mu(0) = -\mathbf{c}^T\mathbf{x}(0) \tag{28}$$

in that case the motion equation of the Sliding Mode coincides with that of the ideal system $\mathbf{x}_i = \mathbf{z}_0$, without perturbation. Further more, since $s(0) = \mathbf{c}^T\mathbf{x} + \mu(0) = 0$, Sliding Mode will occur from the initial time $t = 0$. The second part of the control i.e. μ_1 can be designed as following where $m_0(\mathbf{x}) \geq |h_0|$.

The Sliding Mode can be then enforced using the control

$$\mathbf{z}_1 = -M(x)sign(s) \tag{29}$$

where $M(x)$ is positive definite diagonal matrix, under the condition that matrix $\frac{\partial s_0}{\partial x}B(x)$ is definite and the elements of matrix $M(x)$ are large enough.

6 EXPERIMENTAL RESULTS

In our experiments, we used the wheeled robot Koala illustrated in Fig. 2. Koala is a mid-size robot designed for real-world applications. Koala has the functionality necessary for use in practical applications, rides on 6 wheels for indoor operations. It has two-motorized wheels (the middle wheel of each side), with 0.4 m/s maximum speed, the wheels have a radius of 4.5 cm and are mounted on an axle 30 cm long. The chassis of the robot measures 30x30x20 cm (l/w/h) and its total weight is 3.6 kg. Each motor is equipped with an incremental encoder counting 5850 pulses/turn. The robot is equipped with 16 Infrared proximity and ambient light sensors in addition to a camera mounted on a turret. Data acquisition and control implementation are performed at sampling period $T_s = 0.05\ s$.

Figure 2: Mobile robot Koala.

In this section, we will report the experimental results of Koala in tracking an eight shape reference trajectory defined by

$$y_{d1}(t) = y_{d1max}\sin(2\pi\frac{t}{T}) + y_{d1i} \\ y_{d2}(t) = y_{d2max}\sin(2\pi\frac{t}{2T}) + y_{d2i} \tag{30}$$

for $t \in [0, 2T]$. We choose $y_{d1max} = 2m$, $y_{d2max} = 2m$, $(y_{d1i}, y_{d2i}) = (1.0, 0.0)$ and $T = 40s$.

We apply feedback linearization discussed above, we let $q(0) = (0m, 0m, 0rad)$, i.e., starting with an initial state error with respect to the assigned trajectory $q_d(0) = (1.0m, 0.0m, 0.4636rad)$. In the first set of experiments, PID controller is applied with $k_p = 9.17, k_i = 0.72$ and $k_d = 10.59$. As we can see from Figs. 3, a relatively high tracking errors (up to 10.0 cm) are observed on x, y. These tracking errors are resulting from unmodeled dynamics (motors dynamics and unmodeled friction forces) and measurements errors. In addition, there is a large transient error resulting from the initial posture being different from the desired trajectory starting point.

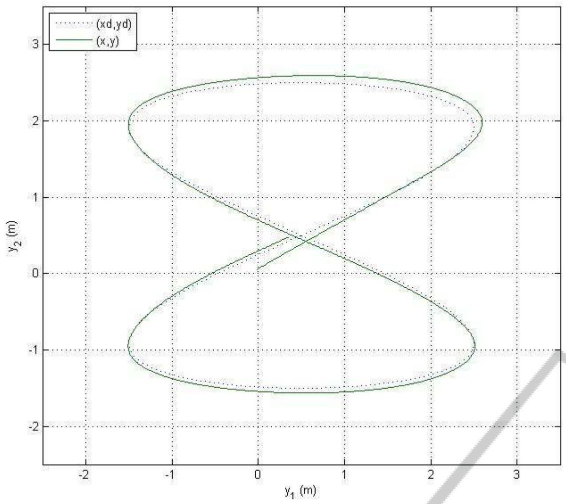

Figure 3: Asymptotic trajectory tracking using linear PID controller to track an eight shape trajectory on the (x, y) plane.

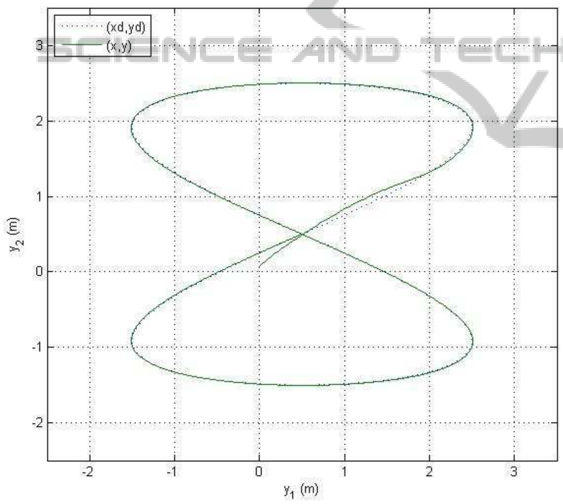

Figure 4: Asymptotic trajectory tracking using Integral Sliding Mode controller of an eight shape trajectory on the (x, y) plane.

In the second set of experiments, we add Integral Sliding Mode controller as described in Section 5. As we can see from Figs. 4 and 5, the tracking of the reference trajectory is quite accurate. Residual errors (1 cm Maximum) are mainly due to quantization and discretization of velocity commands. Figs. 6 and 7 show resulting linear and angular velocities and the applied torques respectiveley.

7 CONCLUSIONS

In this paper, a control strategy was proposed to

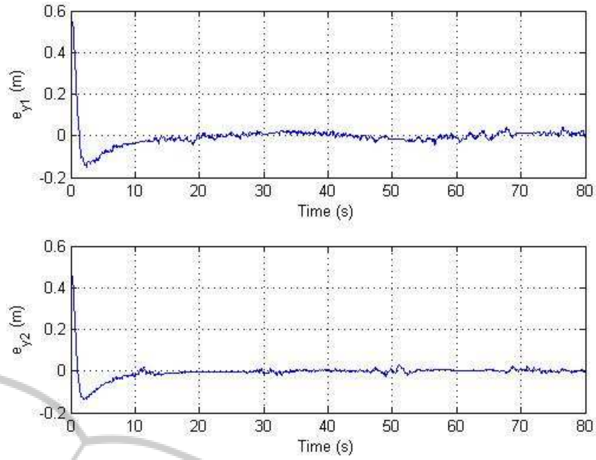

Figure 5: Output tracking errors using Integral Sliding Mode controller of an eight shape trajectory on the (x, y) plane.

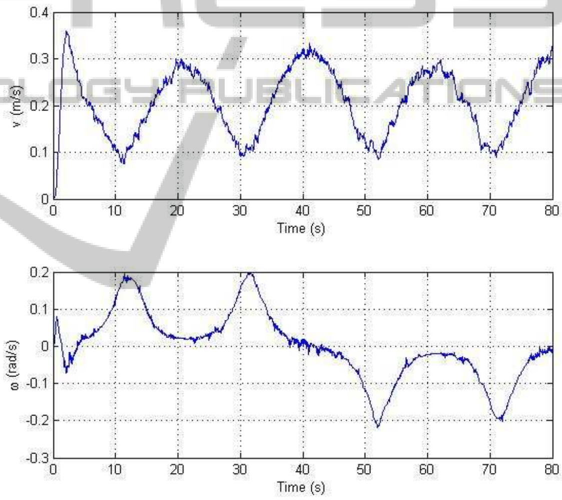

Figure 6: Linear and angular velocities.

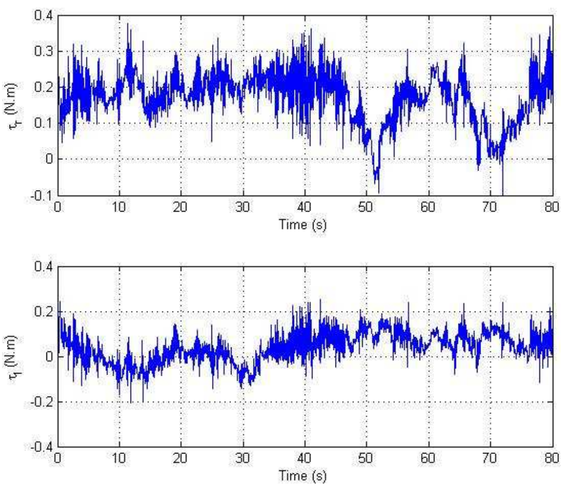

Figure 7: Applied torques.

solve the trajectory tracking problem of nonholonomic robotic systems in presence of external disturbances and parametric uncertainties. This strategy is based on Integral Sliding Mode combined with state feedback linearization technique that is extended to include both kinematic and dynamic models. Experimental results on a nonoholonomic mobile robot as a case study have shown that this combination can effectively stabilize the robot about a reference trajectory, and the results are much better in terms of robustness when compared to a traditional linear PID controller.

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