SOME PROBLEMS HANDLED BY PARTICLE SWARM OPTIMIZATION IN AUTOMATIC CONTROL

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Abstract: Most of the methods to design automatic control laws rely on the solution to optimization problems. However, straightforward formulations of costs and constraints of these problems are mainly non convex, non smooth or non analytic. That is why the classical approach is to simplify the problem so as to get tractable and exactly solvable optimization problems. The use of direct methods such as metaheuristics is underused in the control community. In this paper, a Particle Swarm Optimization method is used to solve some complex initial problems found in the control field to show the interest in the use of such methods.

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1 INTRODUCTION

Optimization has traditionally brought efficient methods to compute control laws. However, the traditional methodology is concerned with the design of a simplified model of the plant to control. In parallel, costs and constraints are reformulated so as to express all specifications in a well suited framework.

In the automatic control history, numerous examples of this approach can be found: Linear Quadratic methods, optimal control (Kwakernaak and Sivan, 1972), H_2 or H_∞ control design (Zhou et al., 1996), predictive control (Maciejowski, 2002). However, due to the necessity of this specific structure of the optimization model, some of the specifications cannot be directly taken into account in the design process. They have to be a posteriori checked during an analysis phase. This approach may lead to some iteration between the synthesis and the analysis phases.

Nowadays, three points have to be considered: systems to be controlled are more and more complex, specifications are more and more various and precise, industries want to find best performances. Corresponding optimization problems are non convex, non differentiable, with numerous local optima. In such a context, metaheuristic optimization methods appear to be interesting candidate methods to solve these kinds of problems. In this paper, the main focus is on the use of Particle Swarm Optimization method. The goal of this paper is not to present new results (most of them have already been published in the Automatic Control field by the author) but to show to the metaheuristic community that there is a large application field where such algorithms are really underused and have a great potential.

The paper is organised as follows. In section 2, costs and constraints which are commonly encountered in the Automatic Control domain are called up. Two examples of the application of Particle Swarm Optimization are then presented. In section 3, the optimization of the tuning of Proportional-Integral-Derivative (PID) controller is performed. An advanced control methodology is then studied in section 4, namely the H_∞ synthesis problem. Finally, conclusion remarks are drawn in section 5.

2 COST AND CONSTRAINTS IN AUTOMATIC CONTROL

Consider the generic closed loop framework of figure 1. s is the Laplace variable.

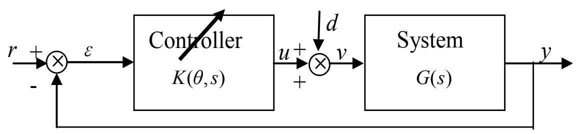

Figure 1: Classical closed loop framework.

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A system $G(s)$ has to be controlled. The control input is v and the output is y. The controller is $K(s,\theta)$ and depends on tuning parameters θ. r is the reference and the disturbance is d. In this closed loop, any transfer function from an input x to an output z is a function of the parameters θ:

$$T_{x\to z}(s) = H(s,\theta) \tag{1}$$

In the same way of thinking, any time response $x(t)$ is a function of θ:

$$x(t) = f(\theta, r(\tau), d(\tau), \tau \in [0,+\infty[\,) \tag{2}$$

Some of the classical criterions in the case of Single Input Single Output (SISO) are:

- Cut-off frequency:

$$\begin{aligned} &\omega_0(\theta) = \arg\min_{\omega_1} \omega_1 \\ &\text{s.t } \left|T_{\varepsilon\to y}(j\omega,\theta)\right| < 1, \forall\omega > \omega_1 \end{aligned} ; \tag{3}$$

- Module margin:

$$\Delta m(\theta) = \min_{\omega}\left|T_{\varepsilon\to y}(j\omega,\theta) - (-1)\right|; \tag{4}$$

- H_∞ norm of the system (computed for Multi Inputs Multi Outputs system):

$$\left\|T_{r\to y}\right\|_\infty(\theta) = \sup_{\omega} \bar{\sigma}(j\omega,\theta) \tag{5}$$

with:

$$\bar{\sigma}(j\omega,\theta) = \max_i \sqrt{\lambda_i(T_{r\to y}(j\omega,\theta)^* T_{r\to y}(j\omega,\theta))} \tag{6}$$

More generally, specifications can be given as temporal templates for transfer function of the system of figure 1. Some classical specifications are given for the Heaviside step response. Once again, all criterion are a function of θ.

- $\alpha\%$ time response:

$$T_e(\theta) = \inf_{T>0}\left\{\begin{array}{l} T \setminus \forall t > T : \\ \left|\varepsilon(t,\theta)\right| \le \alpha/100 \cdot r(t) \end{array}\right\}; \tag{7}$$

- Maximum of the control input:

$$u_{\max}(\theta) = \max_t\left|u(t,\theta)\right|. \tag{8}$$

It appears that the mathematical expressions of these constraints are often non smooth (computations of min/max, absolute value, no analytical expression...). Some classical approaches do exist to compute a controller which satisfies a given set of specifications. However, the problem is now not only to satisfy a set of constraints, but to optimize the performances of the system and to take into account all constraints in the design procedure. Finally, corresponding optimization problems are hard to solve. That is why the use of metaheuristics optimization methods appears as a very interesting approach to explore.

3 PID TUNING AND OPTIMIZATION

In this section, we want to optimize a PID controller for a magnetic levitation, represented in figure 2.

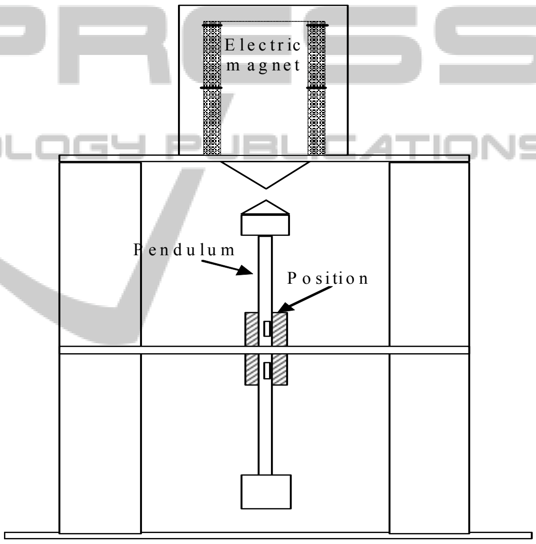

Figure 2: Magnetic levitation system.

To control the system, PID controller with high frequencies filtering is used:

$$C(s) = K\left(1 + \frac{1}{T_i s} + \frac{T_d s}{1+\tau_d s}\right)\cdot\frac{1}{1+T_f s}, \quad \text{with } \tau_d = T_d/10. \tag{9}$$

The parameters of the optimization problem are:

$$\theta = (K, T_i, T_d, T_f)^T. \tag{10}$$

The specifications to be achieved are: control input limitations: $\max|u(t)| \le 10\text{V}$, module margin: $\Delta m \ge 0.5$, 5% time response as low as possible.

The problem can be expressed as the following minimization problem:

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$$\min_{\theta} \inf_{T>0} \begin{Bmatrix} T \setminus \forall t > T : \\ |\varepsilon(t)| \le 5/100 \cdot r(t) \end{Bmatrix} + J_1(\theta) + J_2(\theta)$$

$$\begin{aligned} &\text{if } \max_t(|u(t)|) > 10 : \\ &\quad J_1(\theta) = \exp(\lambda (\max_t(|u(t)|) - 10)) \\ &\text{if } \min_\omega |G(j\omega) - (-1)| < 0.5 : \\ &\quad J_2(\theta) = \alpha \cdot (\min_\omega |G(j\omega) - (-1)| - 0.5)^2 \end{aligned} \tag{11}$$

A Particle Swarm Optimization method is used to solve the initial problem (Eberhart and Kennedy, 1995), with standard values of parameters (Kennedy and Clerc, 2006). Statistical optimization results are given in table 1. Computation times are 10 seconds with Matlab 2007b on a Pentium IV, 2.0GHz.

Table 1: Results for the time response minimization with penalization on the control input and module margin.

Best	Worst	Mean	Standard deviation
30.8 10^{-3}s	39.7 10^{-3}s	31.4 10^{-3}s	1.1 10^{-3}s

4 REDUCED ORDER H∞ SYNTHESIS

4.1 Problem Statement

H∞ synthesis is an efficient tool in automatic control to compute controllers in a closed-loop framework, achieving high and various performances (Gahinet and Apkarian, 1994); (Zhou et al., 1996).

H∞ synthesis relies on the reformulation of the closed loop problem of figure 1 into a standard form corresponding to the block diagram of figure 3.

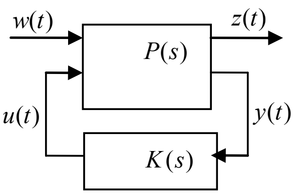

Figure 3: Standard form of a closed loop.

The idea of H∞ synthesis is to solve the following optimization problem:

$$\min_{K(s)} \|T_{w\to z}(s)\|_\infty . \tag{12}$$

This optimization problem can easily be solved as it can be expressed either by a Linear Matrix Inequality (LMI) problem (Zhou et al., 1996). The main drawback is the controller order: the controller computed by the H∞ synthesis procedure has the same order of the synthesis model.

To get low order controllers, matrices rank constraints can be added, leading to Bilinear Matrix Inequality (BMI) problems and so to non-convex ones. More recently, some new techniques have began to emerge, adding some random process in the deterministic search algorithm (Arzelier, et al., 2010), and achieving results which are almost similar to those obtained with the HIFOO standard (Burke, et al., 2006).

Consider the state space representation of the plant $P(s)$ of figure 3:

$$\Sigma : \begin{cases} \dot{x}(t) = \mathbf{A}x(t) + \mathbf{B}_1 w(t) + \mathbf{B}u(t) \\ z(t) = \mathbf{C}_1 x(t) + \mathbf{D}_{11} w(t) + \mathbf{D}_{12} u(t) \\ y(t) = \mathbf{C}x(t) + \mathbf{D}_{21} w(t) \end{cases} , \tag{13}$$

In this paper, we look for a static feedback:

$$u(t) = \mathbf{K}y(t) , \tag{14}$$

where $\mathbf{K}$ is a constant matrix of gains. This closed loop is stable if and only if:

$$\Lambda(\mathbf{A} + \mathbf{BKC}) \in \Re^- , \tag{15}$$

where $\Lambda(\mathbf{M}) \in \Re^-$ denotes the spectrum of $\mathbf{M}$. Considering the direct solution to the optimal H∞ static output feedback, the problem finally refers to the following optimization problem:

$$\begin{aligned} &\min_{\mathbf{K} \in \Re^{m\times r}} \|T_{w\to z}(s)\|_\infty \\ &s.t. \begin{cases} u(t) = \mathbf{K}y(t) \\ \Lambda(\mathbf{A} + \mathbf{BKC}) \in \Re^- \end{cases} , \end{aligned} \tag{16}$$

This kind of criterion can be optimized by PSO which does not require any particular formulation of the cost function. Finally, the design of an H∞ static output feedback relies on the tuning of a matrix $\mathbf{K} \in \Re^{m\times r}$ and so to the tuning of $m \times r$ variables. For this possibly relatively large scale problem, we use the algorithm given in (van den Bergh and Engelbrecht, 2002).

4.2 Numerical Results

For comparison, the algorithm is tested on the benchmark examples given in the COMPleib library (Leibfritz, 2004).

Results obtained with the PSO algorithm have been compared with those obtained with the HIFOO

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package (Burke, et al. 2006), which is a deterministic solver and considered as one of the best effective tool for the synthesis of static output feedback, and those obtained in (Arzelier, et al., 2010). Corresponding results are given in table 2.

5 CONCLUSIONS

Optimization has always played an important role in the field of Automatic Control. Indeed, most of the existing control design methodologies are concerned with the solution to optimization problems.

Table 2: Computation of H_∞ static output feedbacks.

Ex.	*n*	*m*	*r*	HIFOO	Ar. et al	PSO
A1	5	3	3	$4.14\ 10^{-7}$	$1.76\ 10^{-6}$	$4.7\ 10^{-22}$
A2	5	3	3	0.1115	0.1115	0.1115
A5	4	2	2	669.56	661.7	665.09
A9	10	4	5	1.0029	1.0061	1.098
A10	55	2	2	Inf	Inf	Inf
A11	5	2	4	2.8335	2.8375	2.8609
A12	4	3	4	0.3120	0.6165	0.3134
A13	28	3	4	163.33	395.0404	167.36
A14	40	3	4	101.7203	319.31	101.96
A18	10	2	2	12.6282	10.6214	27.18
H1	4	2	1	0.1539	0.1538	0.1529
H3	8	4	6	0.8061	0.8291	0.8399
H4	8	4	6	22.8282	22.8282	23.43
H5	4	2	2	8.8952	17.6061	10.0031
H6	20	4	6	192.3445	401.7698	195.86
H7	20	4	6	192.3885	353.9425	194.24
D2	3	2	2	1.0412	1.0244	1.0255
D4	6	4	6	0.7394	0.7404	0.7863
D5	4	2	2	1035.5	1030.82	1028
J2	21	3	3	183.3512	365.09	192.17
J3	24	3	6	5.0963	9.194	5.138
R1	4	2	3	0.8694	0.8661	0.8738
R2	4	2	2	1.1492	1.1482	1.1451
R3	12	1	3	74.2513	74.2513	74.2513
W1	10	3	4	4.0502	4.1055	6.4843
B2	82	4	4	0.6471	2.90	1.0345
S	60	2	30	0.0201	0.02	0.0200
P	5	1	3	32.2258	0.0087	0.0571
T1	7	2	4	0.3736	0.3799	0.4038
T2	7	2	3	5200	5200	5200
T3	7	2	3	0.4567	0.3264	0.5829
N1	3	1	2	13.9089	13.458	13.8189
N2	2	1	1	2.2216	2.2050	2.2049
N5	7	1	2	266.54	266.5445	266.4023
N6	9	1	4	5602	5602	5593
N7	9	1	4	74.0757	74.0372	74.0326
N9	5	3	2	28.6633	31.03	30.1549
N12	6	2	2	16.3925	16.3116	17.7568
N13	6	2	2	14.0589	14.0579	14.4829
N14	6	2	2	17.4778	17.4757	17.5063
N15	3	2	2	0.0982	0.0980	0.0980
N16	8	4	4	0.9556	0.9556	0.9560
N17	3	2	1	11.2182	11.2182	11.4864

Table 2: Computation of H_∞ static output feedbacks. (cont.)

F10	5	2	3	79853	82314	80658
F11	5	2	3	7719	78248	77213
F14	5	2	4	53156	557008	535040
F15	5	2	4	17521	202610	178900
F16	5	2	4	44432	465790	447500
F17	5	2	4	30024	303380	300240
F18	5	2	2	124.7259	154.9970	126.6402
TM	6	2	4	2.5267	2.1622	2.8015
FS	5	1	3	96925	87160	84727

However, in the classical approach, particular expressions and reformulation of initial costs and constraints functions are used to get an optimization problem which can be exactly solved. To capture the difficulties of the initial optimization problems an underused approach relies on the use of stochastic algorithms which are able to deal with whatever costs and constraints. In this paper, the main focus is on the use of Particle Swarm Optimization algorithm to solve some generic Automatic Control problems: PID optimization, and reduced order H_∞ synthesis. All these results are much than satisfactory, showing the interest of using such algorithms, as results are quite similar to standard deterministic algorithms.

Finally, Automatic Control appears as a large, mostly unexplored, field of applications for the metaheuristic community.

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