APPLICATION OF MATHEMATICAL TRANSFORM IN DETECTION ALGORITHMS

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Abstract: Recent trends in the design of highly efficient and fully automated systems for processing radar data in terms of a priori uncertainty about the targets and disturbances are causing the researchers to use the latest achievements in the design of real time computing architectures for optimum realization and high performance. The development of new algorithms that can be used to retrieve information about targets, applying a mathematical transformation on the received signals yielding estimates of the parameters of moving targets with extremely high precision in a dynamically changing radar environment is a new and very promising direction in modern information and communication technologies. This article discusses such an approach applying the Hough transform to determine the coordinates of the targets. The approach uses a finite set of preselected patterns of the target movement. The Hough transform, translates the set of measurements received in the space of patterns. Association to one or another specific pattern is done estimating the information about the coordinates extracted from the received signals. Thus the moving target parameters in the surveillance zone are uniquely determined by the parameters of the pattern.

1 INTRODUCTION

In the recent years development of modern highly effective algorithms with optimal statistic characteristics for real time radiolocation data processing is becoming a very actual scientific task. Nowadays algorithms that extract information about target's behavior through mathematical transformation of the signals reflected from a target, find ever-widening practical application. Applying signal transformation allows for higher accuracy of the estimated moving target parameters in dynamic radiolocation environment. That is why development of new robust and reliable algorithms for simultaneous trajectory and target detection applying the Hough transform is a perspective field of research, so the present paper considers this problem. The performance of original Hough detector structures maintaining constant false trajectory detection probability in intensive randomly arriving impulse interference environment is studied. Estimated are the efficiency and quality of the obtained algorithms for data streams with different distribution lows of occurrence of impulse interference. A comparative analysis of the presented Hough detector structures is made. The practical effect of the obtained results lies in the possibility of development of radar signal processing algorithms for automated systems of air traffic control service.

On 18.Dec.1962 the American Patent Service issued a patent 3,069,654 "Method and means for recognizing complex patterns" on the name of Paul V. C. Hough (Hough, 1962). The Hough transform is a mathematical conversion, in which the task for finding specific features of the processed image consisting of points defined in the feature space is transformed to a task for finding groups of points in the parameter space. The Hough transform for straight lines detection is a sub case of the Radon transform which for the Euclidean two dimensional space and arbitrary generalized function F(x, y) is as follows:

$$f(\theta,\rho)=R\{F\}=\iint_D F(x,y)\delta(\rho-x\cos\theta-y\sin\theta)dxdy \qquad (1)$$

where $\delta(.)$ is the Dirac delta function defining the integral on direction of a straight line defined by the

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equation: $\rho = x\cos\theta - y\sin\theta$.

Initially Hough transform proposed in (Hough, 1962) presents the straight lines from a two-dimensional image in the features space (FS) with incidence m and segment c from $y = mx + c$, where x and y are the image coordinates and m and c are the coordinates in parameter space (PS).

The parameter space is sampled to a set of subspaces (accumulators). Each point of the input image is projected onto a straight line with coordinates (*m, c*). Accumulators through which this line passes increase their content by one. Each of these accumulators corresponds to an area in the features space and the presence of a peak in the accumulator corresponds to a straight line or a segment of the image. Lines existing in the features space are detected according to the value accumulated into an accumulator in the parameter space. Hough transform proves to be a major tool in the analysis and algorithms for pattern recognition.

The concept of using the Hough transform for target detection improvement is introduced in (Carlson, Evans and Wilson, 1994). Regardless of the particular application of the Hough transform, different authors point three of its main properties that make it applicable to moving targets detection:

- Applicable for raster images;
- Applicable for fuzzy images processing;
- Effective when there is a lack of necessary information (measurements, observations).

2 HOUGH TRANSFORM FOR MOVING TARGET DETECTION

Let us consider the operation of surveillance radar which measures the distance, targets azimuth, elevation and Doppler velocity as a function of time. The sampling time is specified. Trajectory detection by means of Hough transformation can be made either having rotating antenna or phased array (Carlson, Evans and Wilson, 1994). In a single resolution cell in azimuth the traditional radars emit several pulses on a specified carrier frequency. The surveillance area is being consequently scanned with the radar antenna pointing in different directions. This procedure is repeated on successive periods of time equal to the sampling time. In each "azimuth-distance" resolution cell the station processes non-coherent accumulation of the emitted pulses. The target is considered detected if the pulse amplitude in the (azimuth-distance) resolution cell exceeds a preset threshold. This approach has some difficulties to detect fast moving targets, because these objects move quickly from one to another resolution cell during one sampling period.

If received by the radar echo signals are arranged as discrete multidimensional array, i.e. discrete information card (with 5 dimensions - distance, azimuth, elevation, Doppler velocity and time), the target will appear as a curve, which intensity depends on the power level of the echo signals. If this curve can be monitored, it contains all the accumulated information about the target and complete history of its trajectory. Object with constant radial velocity appears as a straight line. The projection of this 5-dimensional information about distance and time is a convenient way to display the curve, while no interest in the other three levels for this target. The result is a so-called "range – time" (r-t) space.

Figure 1 shows (r-t) space of a target with a constant radial velocity, with given direction of the antenna beam to a specified resolution cell of the Doppler velocity. The slope of this line is determined by the radial velocity of the target. The trajectory of a stationary object will appear as a vertical line. All moving objects will have a certain angle, reaching zero for the fastest objects.

Time axis starts from zero to maximum. It is convenient to present the information about the past to a decent level, because too old information is not useful. The current information contains the disturbance, which is an internal white noise of the receiver. It has Rayleigh amplitude distribution and is summed in each cell of (r-t) space. The problem is to find a straight line on the background of the noise.

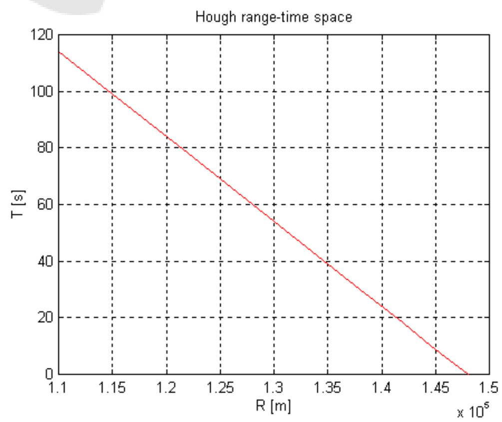


Figure 1: "Range-time" space.

The Hough transform is a method detecting curve elements, often used to detect location lines on the noise background. Other forms of trajectories can be also detected, but so far only straight lines have been investigated.Figure 2 shows several data points that form a straight line in the “range – time” space. In polar coordinates, a straight line can be accurately defined by two parameters:

1. θ - the angle between the perpendicular from the coordinate system origin in (r-t) space to the straight line and the abscissa axis;

2. ρ - length of the perpendicular, i.e. the distance between the coordinates origin in (r-t) space and the line.

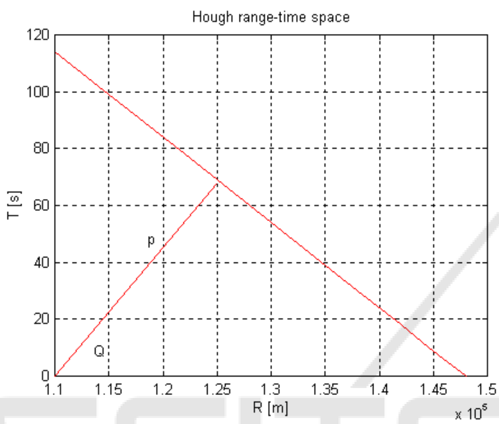


Figure 2: Relation between “range-time” (r-t) space and Hough transform.

The Hough transform translates the points from (r-t) information space to (ρ-θ) or Hough parameter space using the following expression:

$$\rho = r\cos\theta + t\sin\theta \qquad (2)$$

where r and t are coordinates in the (r-t) space.

The Q grid in Hough space is formed by consequent change of θ angle form 0° to 180° and calculating the corresponding ρ. Sometimes another form of the Hough transform is used:

$$\rho = \sqrt{r^2 + t^2}\sin\left(\theta + arctg\frac{r}{t}\right) \qquad (3)$$

The transform results in a sinusoid with phase and amplitude defined by the (r-t) value of the information point. The maximal ρ value is equal to the length of the diagonal in the (r-t) space. The transformation according (2) is shown on Figure 3. Each point in (ρ-θ) space corresponds to a separate straight line in (r-t) space defined by the values of ρ and θ. Each sinusoid presents a set of possible straight lines through the point. If there are points forming a straight line in the (r-t) space this corresponds to an intersection point of set of sinusoids in the Hough space. The (r-t) space is sampled to cells which number is equal to the number of the distance resolution elements and the sample numbers. The primary threshold is used for signal detection in each (r-t) cell. When the signal value in a specified (r-t) cell exceeds the primary threshold, its power gets added to the (ρ-θ) cell being intersected by the corresponding sinusoid in the Hough space. Thus the value of an accumulator cell in the intersection of several sinusoids will become higher.

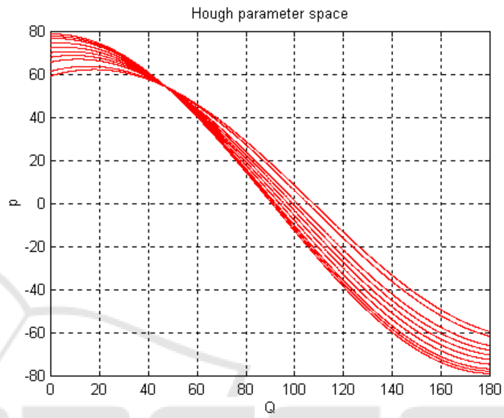


Figure 3: Hough parameter space.

The secondary threshold applied to each cell in the parameter space may declare straight line trajectory detection. This is the accumulated for several scans moving target echo signal. The ρ and θ parameters of the tracked straight line trajectory in Hough space might be transformed back to the (r-t) space indicating the current object position. Transition from (r-t) to parameter space is being made by means of a simple matrix manipulation. Matrix D contains I number of elements where the signal value exceeding the primary threshold.

$$D = \begin{bmatrix} r_1 & r_2 & \dots & r_I \\ t_1 & t_2 & \dots & t_I \end{bmatrix} \qquad (4)$$

Transformation matrix H consists of sinusoids and cosinusoids from (2) defined as:

$$H = \begin{bmatrix} \sin\theta_1 & \cos\theta_1 \\ \sin\theta_2 & \cos\theta_2 \\ \dots & \dots \\ \sin\theta_{N_S} & \cos\theta_{N_S} \end{bmatrix} \qquad (5)$$

where θ are discrete values of Q from 0° to 180°, obtained during the sizing of the parameter space.

The product of H and D is a matrix R of size ($N_S \times I$), which contains the corresponding ρ values.

The indices of the ρ elements in matrix R are the indices of the points in (r-t) space where the primary threshold has been exceeded.

$$R = HD = \begin{bmatrix} \rho_1, \theta_1 & \dots & \rho_I, \theta_1 \\ . & . & . \\ . & . & . \\ . & . & . \\ \rho_1, \theta_{N_S} & \dots & \rho_I, \theta_{N_S} \end{bmatrix} \qquad (6)$$

Each column or the R matrix contains the ρ values for one sinusoid on the parameter space. It is plain to see the more points in (r-t) space exceed the primary threshold the bigger D and R will be. The result is increasing number of calculations. The size of matrix H depends only on the parameter space sampling.

The advantage of the Hough transform application is the simultaneous target and its trajectory detection. The target is considered detected when its straight line trajectory is localized in the Hough space, i.e. ρ and θ parameters. When applying the Hough transform additional non-coherent integration of the signal obtained in several consequent scans is done. This signal integration for fast moving targets increases the detection probability compared to the conventional radars. In the recent years the Hough transform finds wide application in moving targets detection (Carlson, Evans and Wilson, 1994). This is a new and perspective direction of the Hough transform application and the results presented in (Kabakchiev, Doukovska, Garvanov, 2005, Doukovska, 2007, Doukovska 2007, Doukovska, 2008, Doukovska 2008, Doukovska, 2010) are dedicated to this problem.

3 TARGET DETECTION SIGNAL MODEL

The radar receives signal, noise and randomly arriving impulse interference. In the present paper a Swerling II target signal model is used. This model is a package of echo signals with Rayleigh distribution, reflected from a fast moving target. The noise is a stationary and internal for the receiver. The noise has normal distribution law which corresponds to a Rayleigh distribution of the envelope. Distribution function of the envelope of the signal and the noise and the corresponding density are $F_{s1}(x)$ and $f_{s1}(x)$. If there is there is a possibility for randomly arriving impulse interference (RAII) - e_0, (Poisson stream) the radar resolution cells are filled with signal, noise and RAII (Akimov, P., F. Evstratov, S. Zaharov, 1989). The function and density distribution of the envelope in this case are $F_{s2}(x)$ and $f_{s2}(x)$.

The overall distribution function of the above described case is obtained taking into account the probability of absence of RAII is $(1-e_0)$ and for presence of RAII - (e_0) respectively. Now the distribution function of the envelope is (Akimov, P., F. Evstratov, S. Zaharov, 1989):

$$F_{sP}(x) = (1-e_0)F_{s1}(x) + e_0 F_{s2}(x) \qquad (7)$$

The distribution density function is:

$$f_{sP}(x) = (1-e_0)f_{s1}(x) + e_0 f_{s2}(x) \qquad (8)$$

Here it is assumed that the probability of RAII appearing in a resolution cell is an infinitely small quantity compared to the probability of single impulse occurrence. This is a typical feature for a Poisson stream.

When the duration of the impulse disturbance is not negligible compared to the average period of recurrence (high probability for RAII), a binomial model of the stream distribution is used. In this case the distribution function of the envelope is (Akimov, P., F. Evstratov, S. Zaharov, 1989):

$$F_{sB}(x) = (1-e)^2 F_{s1}(x) + 2e(1-e)F_{s2}(x) + e^2 F_{s3}(x) \qquad (9)$$

For the distribution density of the envelope we have:

$$f_{sB}(x) = (1-e)^2 f_{s1}(x) + 2e(1-e)f_{s2}(x) + e^2 f_{s3}(x) \qquad (10)$$

where $F_{s3}(x)$ and $f_{s3}(x)$ - are function and distribution density of the signal, the noise and two pulse interferences.

In the presented paper it is assumed that the distribution of the signal plus noise and the mixture of signal, noise and RAII after the quadratic detector have an exponential density (Akimov, P., F. Evstratov, S. Zaharov, 1989):

$$f_{s1}(x) = \frac{1}{\lambda_0(1+s)} \exp\left(\frac{-x}{\lambda_0(1+s)}\right) \qquad (11)$$

$$f_{s2}(x) = \frac{1}{\lambda_0(1+s+r_j)} \exp\left(\frac{-x}{\lambda_0(1+s+r_j)}\right) \qquad (12)$$

where s is the average value of the signal to noise ratio. In this case the probability density function for

Poisson distribution model of the RAII has the following expression (Bird J., 1982) – see (8):

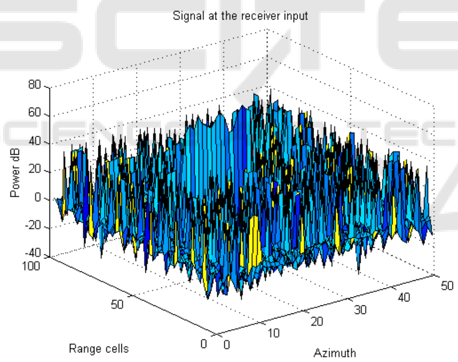
$$f_{sP}(x) = \frac{(1-e_0)}{\lambda_0(1+s)}\exp\left(\frac{-x}{\lambda_0(1+s)}\right) + \\ + \frac{e_0}{\lambda_0(1+s+r_j)}\exp\left(\frac{-x}{\lambda_0(1+s+r_j)}\right) \quad (13)$$

For high probability of RAII, when the model is binomial the noise density distribution function is used as well as two pulse interferences:

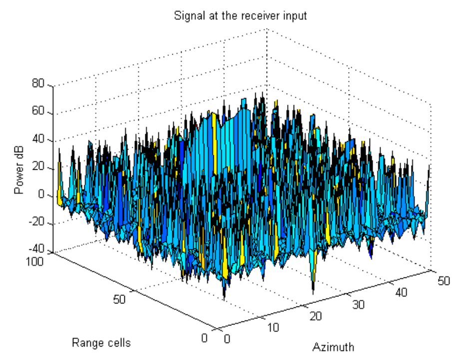
$$f_{s3}(x) = \frac{1}{\lambda_0(1+s+2r_j)}\exp\left(\frac{-x}{\lambda_0(1+s+2r_j)}\right) \quad (14)$$

In this case the probability density function for binomial distribution of the RAII – see (10) is:

$$f_{sB}(x) = \frac{(1-e)^2}{\lambda_0(1+s)}\exp\left(\frac{-x}{\lambda_0(1+s)}\right) + \\ + \frac{2e(1-e)}{\lambda_0(1+s+r_j)}\exp\left(\frac{-x}{\lambda_0(1+s+r_j)}\right) + \\ + \frac{e^2}{\lambda_0(1+s+2r_j)}\exp\left(\frac{-x}{\lambda_0(1+s+2r_j)}\right) \quad (15)$$



a) Poisson RAII model



b) Binomial RAII model

Figure 4: Radar resolution cell filled with signal (s=70dB), noise (λ_0=1) and impulse interference (INR=30dB, e_0=0.1).

Figure 4 shows the two streams – a) Poisson and b) Binomial. The cells that do not contain useful signal are filled with receiver internal noise and impulse interference. The cells containing signal are filled according (13) and (15). The results are obtained for: average receiver noise level λ_0=1, signal to noise ratio - s=70dB, impulse interference to noise ratio - INR=30dB and probability for RAII 0.1 for both distributions.

4 EXPERIMENTAL RESULTS

Recently a lot of robust moving target detection algorithms for processing signals from noisy environments are developed. As a result a bank of Hough detectors making use of one and two dimensional signal processors was created (Kabakchiev, Doukovska, Garvanov, 2005, Doukovska, 2007, Doukovska 2007, Doukovska, 2008, Doukovska 2008, Doukovska, 2010). All these structures have been analytically studied and by means statistical analysis has been compared to each other as well as to results obtained by other authors (Carlson B., E. Evans, S. Wilson, 1994). On Figure 5 is presented the overall structure scheme of an adaptive to the environmental conditions detector. It consists of two main modules – signal processor and Hough detector. Maintaining constant false alarm rates at the detector's output depends on the chosen scalar factor (T_α) of the CFAR signal processor. The system input signal reflected from the target is filtered with a simple sinusoidal signal (complex signal compression), then it enters the quadratic detector where the signal matrix of the receiver is generated. This signal matrix is fed to the CFAR processor. As a result at the output a binary signal matrix is generated containing zeros and ones presenting absence or presence of a signal in a given radar resolution cell. The binary matrix is visualized on the plot extractor. Results are stored in the so called target coordinates record determined by the i-th radar observation. For several consecutive scans an interscan gathering of the plots of the target is done. Then the (r-t) space is formed and using the already processed data the trajectory is being determined. The Hough transform is applied over the points from the (r-t) space in order to transfer them to the Hough space. As a result there is a bunch of sinusoids which intersection point accumulates the energy reflected from the target. Comparing the value accumulated in this point (sum of zeros and ones) to a preliminary chosen threshold is the way to detect a target if the radar range resolu-

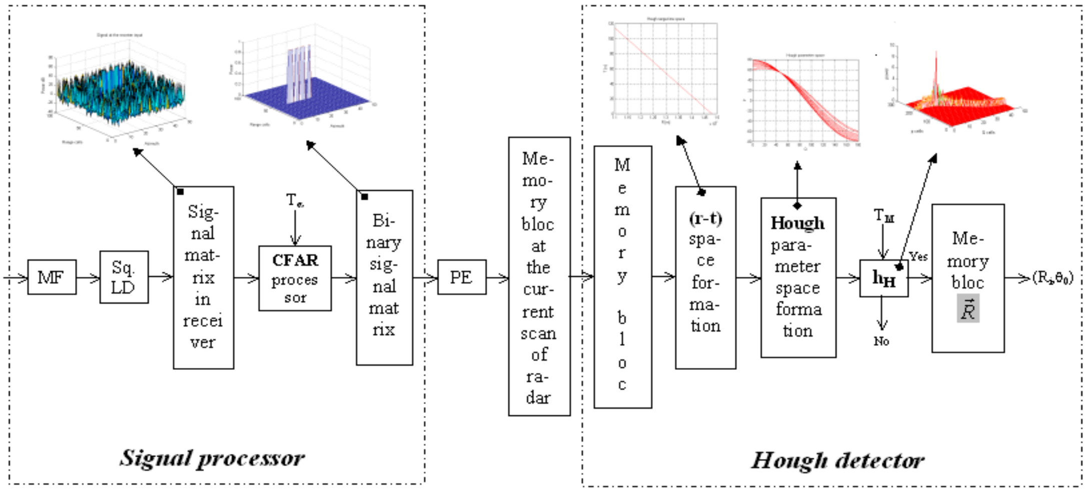


Figure 5: Generalized Hough detector structure.

tion cell. It the result is positive a reverse Hough transform is applied in order to determine the distance to the target for a given azimuth.

Presented paper considers the results obtained from the analysis of different Hough detectors with one and two dimensional signal processors maintaining constant false alarm rates. To make the results applicable they were compared in equal conditions using equal criteria. The efficiency of the Hough transform application was estimated by the profit, gained during the detection process, expressed by the signal to noise ratio as per the criterion presented in (Rohling H., 1983).

Choosing the appropriate threshold constants assures good detection results even for low values of the SNR (Doukovska, 2010). Table 1 presents the obtained threshold constants in equal experimental conditions for the different detection structures and different values of the binary rule in the Hough parameter space.

Table 1: Threshold constants for different Hough detectors.

Hough detectors	T_M=2/20	T_M=T_{Mopt}/20
CA Hough CFAR	672	1.186
EXC Hough CFAR	21880	3.225
Hough CFAR BI	0.000494	0.0000858
EXC Hough CFAR BI	1.1285	0.3161
API Hough CFAR	7.5	1.535

For comparison are shown the achieved results for the detection probability of different Hough detector structures, calculated for non optimal and optimal values of binary rule in Hough parameter space - T_M= T_{Mopt}/20, for following environment parameter values - average power of the receiver noise λ_0=1, average interference-to-noise ratio (INR) r_j=30dB, probability for the appearance of impulse interference with average length e_0=0.1, N=16, L=16 and for probability of false alarm P_{FA}=10^{-4}.

The results presented in this paper are obtained after statistical analysis of the Hough detectors detection probability in intensive noise environment with very high probability for randomly arriving impulse interference. Different Hough detector structures with one and two dimensional CFAR signal processors are studied.

All analytical conclusions necessary to convey the experiments are considered in details in (Kabakchiev, Doukovska, Garvanov, 2005, Doukovska, 2007, Doukovska 2007, Doukovska, 2008, Doukovska 2008, Doukovska, 2010).

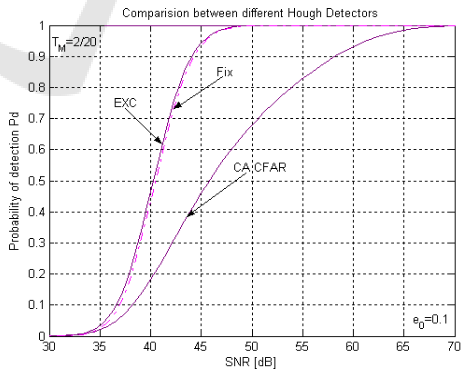


Figure 6: Probability characteristics of a Hough detector with one dimensional signal processors - cell averaging CFAR (CA CFAR), excision CFAR processor (EXC CFAR) and with fixed threshold, for RAII probability - e_0.

It was shown that application of a binary CFAR processor significantly increases the detection quality (about 30dB) compared to the fixed threshold algorithm (Doukovska, 2007). Analyzed is a Hough detector with a more efficient structure of the two dimensional CFAR processor with excision censoring procedure in the reference window (EXC CFAR BI). The hypothesis that censoring techniques increase the detection efficiency with about 5dB was confirmed (Doukovska, 2008).

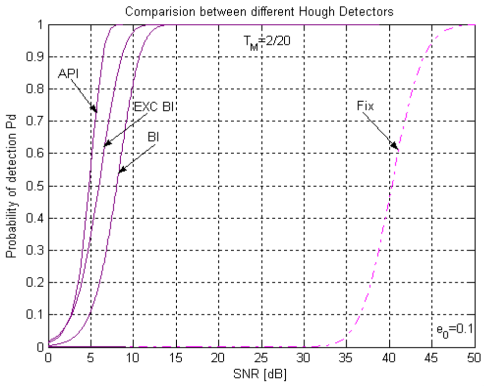


Figure 7: Probability characteristics of a Hough detector with two dimensional signal processors - adaptive CFAR processor (API CFAR), excision binary CFAR processor (EXC CFAR BI), binary CFAR processor (BI CFAR) and with fixed threshold, for RAII probability - e_0.

The most effective for noisy environment with high probability for randomly arriving impulse interference is the Hough detector with adaptive non coherent CFAR signal processor (API CFAR). This structure is by 37dB more effective than the one with fixed threshold Hough detector (Doukovska, 2007).

5 CONCLUSIONS

In conventional signal detection approach the process of target detection is separate from its trajectory detection. Unlike this wide spread technique Hough transform application allows for simultaneous target and trajectory detection. To detect a trajectory data from several consecutive radar scans is processed.

The presented paper considers the results obtained by the proposed adaptive threshold determination procedure and analysis of different Hough detector structures in intensive RAII environment. The need of an adequate threshold analysis procedure allowing better detection results for low values of the SNR is considered.

The obtained results are applicable for wide range of tasks like synthesis of radiolocation detectors, communication systems, medicine and other systems making use of infrared, ultrasonic and other sensor types.

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