

HYBRIDIZING ANT COLONY SYSTEMS AND TABU SEARCH FOR A VEHICLE ROUTING PROBLEM WITH TIME WINDOWS

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Abstract: This paper describes a new approach for solving the vehicle routing problem that considers time windows (VRPTW). The proposal presents a hybrid approach that takes into account an ant colony system ACS and the meta-heuristic Tabu Search. Hybridizing meta-heuristics is one of the alternatives used for solving VRPTWs. Authors believe that a hybrid approach, with ACS providing good initial solutions for the Tabu Search heuristic can help to get acceptable final solutions. Tabu Search plays the role of keeping diversity in the population considered while searching a solution. The proposal was implemented and tested, and results obtained are discussed in the final part of this presentation.

1 INTRODUCTION

The key idea concerning VRP (Toth and Vigo, 2001) involves a routing problem with vehicles that have a specified capacity (CVRP); this is known as a problem having multiple variables to optimize, and therefore, is a classical combinatorial problem. The most popular version associated to this problem is the vehicle routing problem with time windows (VRPTW), due to his wide applicability to real problems.

All problems labeled as VRP may be described by using a graph G defined as $G = (V, A)$, where $V = \{0 \dots n\}$ is the set of nodes belonging to the graph, and A is the set of edges that connect a specific pair of nodes. Meta-heuristics are used to generate a set of circuits that include the V nodes exactly once, except one specific node that represents the starting and final node, known as node 0 (the depot in this work).

There have been proposed different mechanisms to deal with this problem; genetic algorithms (GA) as in (Zhu, 2000), where authors test different crossover operators hybridizing with hill climbing, or in (Ombuki et al., 2006) for optimizing distances and the number of vehicles. In (Jaszkiewicz and Kominek, 2003) the CVRP problem is presented under a genetic algorithm approach. Finally (Berger and Barkaoui, 2004) develops a parallel GA, where a population evolves focusing in minimizing distances.

Ant Colony System (ACO) is also used to solve this problem. In (Tao et al., 2009) the VRPTW is

solved by combining ACO with local search algorithms. Vehicles after returning to the depot can be re-used to visit different customers if time windows allow it. (Yu et al., 2011) deal with VRPTW combining ACO and Tabu Search. We consider, when selecting the next node in the ACS algorithm, not only distance and pheromone, we consider the unused capacity in the vehicles and the waiting intervals in time windows. Besides that, we use 2-opt search trying to find improvements between two paths, this is one of the differences we introduce. The objective of this proposal is to verify if hybridizing ACS with Tabu Search allows to improve results obtained by Solomon (Solomon, 2006).

This article is structured as follows; the first section is made up of the present introduction; the second section describes the problem; the third section is devoted to introduce the proposal, while in section four we present some results obtained. In section five we end with the conclusions.

2 PROBLEM DESCRIPTION

The practical problem we are interested in, that takes as a starting point the routing vehicle problem, deals with routing garbage trucks, a problem that presents various objectives to optimize, such as the number of trucks to be used, which in turn is associated to the number of paths and the distance involved in each

path. Not less important, it is necessary to consider the time involved in each path and constraints such as specific (and limited) time period in which the garbage collection can be done. The last constraint shows that we are solving a problem that can be classified as VRPTW.

VRPTW needs to satisfy a time window defined for a customer i as $[a_i, b_i]$, which expect to accomplish the job in a specific time interval. The starting time cannot be previous to a_i and the arrival time w_i plus the time devoted to a particular customer i cannot exceed b_i . The central depot, represented by 0 is also associated to a time window, which is defined as $[E, L]$, where E represents the time in which the trucks begin to traverse their routes, and L the time in which all trucks should be in the depot. If truck k arrives to a customer i location in a time w_i and satisfies $w_i < a_i$, the truck k has to wait until $w_i \geq a_i$.

A good solution optimizes (minimizes) the sum of costs involved in the K path considered for the vehicles.

The cost associated to a path for a vehicle k is determined by the sum of costs c_{ij} corresponding to the edges (i, j) that belong to the path. This solution must satisfy the following constraints:

i) A customer is visited by, exactly, one vehicle, ii) every path has only one edge from the depot, iii) every node in the path has only one edge arriving to it, and only one edge starting from it, iv) every path has only one edge arriving to the depot, v) the sum involving the customer's attention time for customer i and the time involved in going from i to j needs to be less or equal to the time in which attention for a new customer j begins, vi) the arrival time to a customer i location, for one vehicle, needs to be associated to the time interval defined for customer i , vii) starting time from the depot, and arrival time to the depot need to be considered in the interval associated to the depot and viii) the sum of customer's demands, for a particular path cannot exceed the vehicle capacity.

3 THE PROPOSAL

We propose to combine ACS with *Tabu Search* as follows: For each iteration in ACS, once the best route is found by an ant colony, this route is improved through 2-opt. The route obtained in this way is used as input data for applying the meta-heuristic *Tabu Search*. If *Tabu Search* effectively improves the solution, it is reintroduced into the colony, just before the process of global pheromone updating.

Ant Colony System (ACS) was proposed by Dorigo in (Dorigo and Stuetzle, 2004). As in other

ACO algorithms, ACS is a meta-heuristics inspired in ants' behavior when trying to find food. In the collaborative behavior, communication among ants is accomplished through a pheromone trail, that is secreted by the ants when moving from one point to another, and that is used to influence the ants behavior that contact this chemical factor. In this particular work, we use ACS, that is characterized for having a local and a global mechanism for updating the secreted artificial pheromone, allowing a search process in which random aspects are more relevant.

Tabu Search is a meta-heuristic algorithm that can be used for solving combinatorial optimization problems. Tabu search uses a local or neighborhood search procedure to iteratively move from a solution x to a solution x' in the neighborhood of x , until some stopping criterion has been satisfied. To explore regions of the search space that would be left unexplored by the global search procedure, ACS in this case, Tabu Search modifies the neighborhood structure of each solution as the search progresses (Glover and Laguna, 1997).

When building a solution to the VRPTW, each ant begins their path from the depot and visits different customers, until the capacity of the vehicle is completed. Then the ant goes back to the depot forming a path. This process is repeated until all customers have been visited. In this way, the solution for one ant builds as many path as times has started a search from the depot.

ACS features are: i) Each time an ant choose a customer, they have the possibility of doing it in a probabilistic or deterministic way; this possibility is given by a parameter called q_0 . ii) Pheromone updating is accomplished through a global process that considers the best solution that has been reached until this moment. iii) Each ant diminishes a certain quantity of pheromone each time it goes through an edge, allowing this way that future ants can explore different paths.

In ACS, when an ant that is in node i has to select a new node, not previously visited, from a set $N(i)$, it considers the quantity of pheromone and the cost of traversing an edge (i, j) , for each $j \in N(i)$. Besides that, in this proposal two additional variables as proposed in (Bullnheimer et al., 1997) were considered; the first one is the time to arrive to the customer j location, with respect to their time window (Δv) described in equation (1), and the second is the wasted capacity of a vehicle (Δc) described in equation (2). In this way equation (3) describes the mechanism an ant uses for selecting a node j .

$$\Delta v = \begin{cases} 1, & \text{if } w_{ik} = a_i; \\ 0, & \text{if } w_{ik} > b_i; \\ \frac{1}{|a_i - w_{ik}|}, & \text{otherwise;} \end{cases} \quad (1)$$

$$\Delta c = \begin{cases} 1, & \text{if } C = (\sum_{i \in N} d_i \sum_{j \in \Delta^+(i)} x_{ijk}); \\ 0, & \text{if } C < (\sum_{i \in N} d_i \sum_{j \in \Delta^+(i)} x_{ijk}); \\ \frac{1}{|C - (\sum_{i \in N} d_i \sum_{j \in \Delta^+(i)} x_{ijk})|}, & \text{otherwise;} \end{cases} \quad (2)$$

$$j = \begin{cases} \operatorname{argmax}_{l \in \mathcal{N}_i^h} \left\{ \tau_{il} [\eta_{il}]^\beta \right\}, & \text{if } q \leq q_0; \\ \frac{[\tau_{ij}]^\alpha [\eta_{ij}]^\beta [\Delta v]^\zeta [\Delta c]^\theta}{\sum_{l \in \mathcal{N}_i^h} [\tau_{il}]^\alpha [\eta_{il}]^\beta [\Delta v]^\zeta [\Delta c]^\theta}, & \text{if } j \in \mathcal{N}_i^h, \text{ otherwise;} \end{cases} \quad (3)$$

Where q is a random number, q_0 is the parameter mentioned above; τ_{ij} and η_{ij} are the level of pheromone and the cost of edge (i, j) , respectively. α , β , ζ and θ are the parameters that represent the influence of each variable.

For Ant Colony System (Dorigo and Gambardella, 1997) it was established the need to get a global pheromone updating to characterize the distance involved in the solution and the number of vehicles involved in this solution. This is described through the formula 4.

$$\tau_{ij} \leftarrow (1 - \rho) \tau_{ij} + \rho * (\Delta \tau_{ij}^{bs} + \Delta v^{bs}), \quad (4)$$

Where ρ is the parameter that indicates the level of pheromone to be updated; $\Delta \tau_{ij}^{bs} = \frac{1}{\text{cost} - \text{best} - \text{solution}}$ and $\Delta v^{bs} = \frac{1}{\text{vehicles} - \text{best} - \text{solution}}$.

The local updating of pheromone is given by equation (5)

$$\tau_{ij} \leftarrow (1 - \xi) \tau_{ij} + \xi \tau_0, \quad (5)$$

Where $0 < \xi < 1$ is the quantity of pheromone to evaporate and τ_0 is the initial level of pheromone initially allocated to every edge.

Additionally, solutions obtained from ACS have been improved with *2-opt*, that is able to obtain results rapidly. It is due to his simple approach, intending to replace the edges under analysis by other edges having a lower cost, and that it can be done by restructuring the nodes associated to those edges.

4 RESULTS

The following table shows the values for parameter used in testing the hybrid algorithm, that were obtained through experimentation.

In the table, *NACS* represents the number of ant's colonies; *H* represents the number of ants per colony.

Table 1: Set of parameters for *ACS/2-opt*.

	<i>NACS</i>	<i>H</i>	α	β	ζ
	10	1000	3	4	2
θ	q_0	ρ	ξ	τ_0	
1	0,4	0,1	0,1	3.6724741E - 6	

In graphics shown in figures 1 and 2, we can see the hybrid algorithm performance that correspond to two of the three categories proposed by Solomon. When analyzing these graphics, it is possible to deduce that the proposed algorithm offers a good average behavior. The worst case corresponds to a difference of two additional vehicles for only one problem (*R112*). There is a difference of (at most) one additional vehicle in some problems. For all the other instances our proposal offers solutions that are similar to solutions in Solomon.

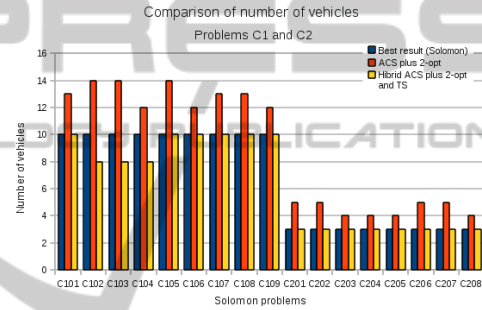


Figure 1: Comparison of *Best result*, *ACS/2-opt* and *Hybrid approach* in problems with prefix C (on the number of vehicles).

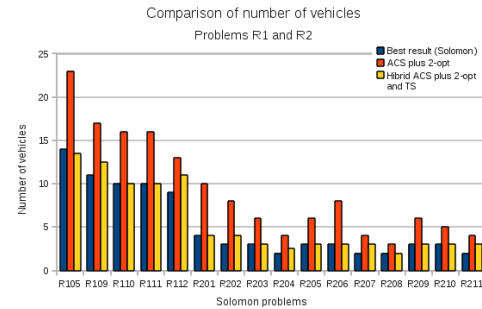


Figure 2: Comparison of *Best result*, *ACS/2-opt* and *Hybrid approach* in problems with prefix R (on the number of vehicles).

One of the problems this approach presents, is the distance involved in the vehicles routes, because in some cases the distance increases. It is important to notice that this behavior is likely to occur when customers are grouped into clusters, having time windows with large intervals.

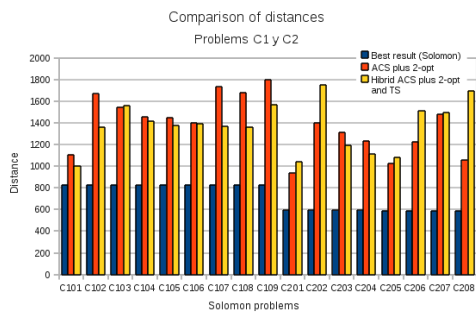


Figure 3: Comparison of Best result, ACS/2-opt and Hybrid approach in problems with prefix C (on the distance).

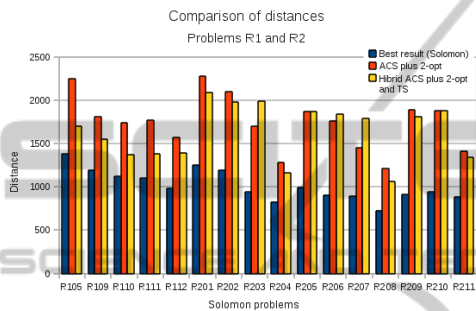


Figure 4: Comparison of Best result, ACS/2-opt and Hybrid approach in problems with prefix R (on the distance).

5 CONCLUSIONS

Hybridizing meta-heuristics appears as a promising strategy for solving combinatorial problems; but there is a lot of work still to be done. Results illustrate a wide variety of cases; but it is necessary to handle these cases carefully. On one hand, we got good results in the case of specific instances, but in most of other instances results were definitely poor.

Solomon, on the other hand, presents academic examples, that not necessarily reflect a real city customer distribution. The euclidean computation of the distances should be probably replaced by a Manhattan distance computation. In the same sense, the random distribution of customers in a city doesn't reflect, in most of cases, a real city.

However, the basic idea of combining different mechanism that provides, from their own features, promising pre-processed solutions is something that should be explored more deeply.

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