A RECONSTRUCTION OF ABSTRACT ARGUMENTATION ADMISSIBLE SEMANTICS INTO DEFAULTS AND ANSWER SETS PROGRAMMING

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Keywords: Abstract argumentation frameworks, Answer-sets, Defaults.

Abstract: Given a default theory, we first show that the justified extensions of this theory characterize the maximal conflict-free sets of the corresponding abstract argumentation framework such as defined by Dung. We then show how to specialize justified extensions in order to represent admissible (and hence preferred and stable) extensions inside default theories. Relying on the correspondance of justified extensions with ι-answer sets on one hand, on the semi-monotonic character of justified extensions on the other hand, we then show that any admissible (or preferred) set of arguments of the initial argumentation framework can be directly computed from the ι-answer sets of the equivalent logic program. Eventually, this allows us to consider the addition of integrity constraints with whom the admissible sets are filtered from each ι-answer set.

1 INTRODUCTION

Since abstract argumentation frameworks have been introduced by Phan Minh Dung in his seminal paper (Dung, 1995), several authors have considered the links with default theories and answer set programming (Dung, 1995), (Bondarenko et al., 1997), (Nieves et al., 2008), (Egly et al., 2010). The whole of these works proceeds from a common approach which has successfully stressed, both in defaults and logic programs, the major role played by the idea of two conflicting informations. In this respect, abstract argumentation sheds a clear light on how nonmonotonicity is at work inside these formalisms. Without denying this fact, we propose however to return to the opposite and, in our opinion, barely explored question, that is: what default theories and logic programs can tell us about abstract argumentation frameworks? Because abstract argumentation frameworks appear formally to be a fragment of defaults, we especially would like to investigate how one of the most basic concepts of argumentation frameworks – admissible sets – is related to the same idea in default theories. Our motivations are manifold. We expect to clear up in a more precise way the links among these various formalisms: especially, while the question whether the definition of arguments should generally rely on logical criterions is controversial (e.g. in (Amgoud and Besnard, 2009)), we propose the basis for a reassessment of abstract argumentation under logic. Although out of the scope of this paper, but from the same point of view, our work intends to base the ability for a better understanding on how known results on preferences, cumulativity, or other logical properties could be applied to argumentation frameworks. Eventually, we expect to catch interesting and powerful methods of computation of arguments from a direct translation of argumentation frameworks into logic programming. The last section of the current paper is a step into this direction. Our paper is organized as follows: we first show that maximal conflict-free sets of arguments correspond strictly to justified extensions of a default theory (Łukaszewicz, 1988), and hence to the ι-answer sets (iota-answer sets) of a logic program (Gebser et al., 2009). We propose then a characterization of the admissible sets of arguments of any abstract argumentation framework obtained from a default theory *via* an additional constraint on justified extensions of this theory. Relying on the bijection of justified extensions with ι-answer sets, we then show that any admissible set of arguments of the initial argumentation framework can be characterized *via* the ι-answer sets of the equivalent logic program. It becomes then possible to add to any such program a set of integrity constraints that filters the admissible sets of arguments from its ι-answer sets. Since ι-

Nouioua F. and Risch V..
A RECONSTRUCTION OF ABSTRACT ARGUMENTATION ADMISSIBLE SEMANTICS INTO DEFAULTS AND ANSWER SETS PROGRAMMING.
DOI: 10.5220/0003717002370242
In *Proceedings of the 4th International Conference on Agents and Artificial Intelligence* (ICAART-2012), pages 237-242
ISBN: 978-989-8425-95-9

answer sets inherit semi-monotonicity (from justified extensions), this clearly emphasizes their central role in a possible incremental treatment of arguments with logic programs. In the second section below, we recall some basic notions about abstract argumentation frameworks (Dung, 1995) and default theories (Reiter, 1980). After reminding (after (Dung, 1995)) how to extract an argumentation framework from an equivalent initial default theory, we propose in the third section the converse translation, that is how to express an argumentation framework as a default theory. In the fourth section we establish a mapping between maximal conflict-free sets of arguments and justified extensions, which allows to further characterize admissible sets of arguments (and hence preferred extensions) as a special kind of justified extensions. The fifth section extends this characterization to ι-answer sets and describes the computation of admissible sets with help of integrity constraints.

In the following, we will denote atomic elements by lowercase letters and sets by shift case letters. Following a widespread tradition, greek letters are also used in definitions and theorems related to defaults and answer sets. We will use some of the standard operations of set theory ($\cup$ for union, $\setminus$ for set difference, $\times$ for cartesian product, 2^S for the power set of S). The symbols $\top$ and $\bot$ denote the usual truth values, and $\neg$, $\vee$, $\wedge$ the usual connectors of propositional logic.

2 PRELIMINARIES

We briefly recall some basic definitions, first on argumentation frameworks, then on default theories. Logic programs will be considered in a further section.

An *argumentation framework* is a pair $\langle AR, attacks \rangle$ where AR is a set and *attacks* is a relation over AR, i.e. $attacks \subseteq AR \times AR$. Each element of AR is called an *argument* and *a attacks b* means that there is an attack from a to b. Accordingly a is said to be an *attacker* of b (thus a is a *counterargument* for b). By extension, a set $S \subseteq AR$ *attacks* an argument $a \in AR$ iff some argument in S attacks a. On the contrary, *S defends a* iff for each $b \in AR$, if *b attacks a* then *S attacks b*. In this case, a is also said to be *acceptable with respect to S*. The *attacks* relation induces a kind of coherence with different degrees among arguments. First, $S \subseteq AR$ is *conflict free* iff there are no a and b in S such that *a attacks b*. Further, S is said *admissible* iff S is conflict free and defends all its elements. S is called a *complete extension* iff S is an admissible set such that each argument that S defends is in S. A *preferred extension* is then a $\subseteq$-maximal admissible subset of AR. Eventually, S is a *stable extension* iff S is conflict free and attacks each argument that is not in S.

Example 1. Consider the following argument framework *AF1*, in which the arrows represent the attack relation over the arguments a, b, c, d, e, f, g:

$$\begin{tikzcd}
& & & & & f \arrow[d] \\
a \arrow[r] & b & c \arrow[l] \arrow[r, bend right] & d \arrow[l, bend right] \arrow[r] & e \arrow[ur] & \\
& & & & & g \arrow[ul]
\end{tikzcd}$$

The admissible sets are: $\emptyset$, $\{a\}$, $\{c\}$, $\{d\}$, $\{a,c\}$, $\{a,d\}$, $\{d,f\}$, $\{a,d,f\}$. The preferred extensions are $\{a,c\}$, $\{a,d,f\}$. The unique stable extension is $\{a,d,f\}$. Remind that, whatever the kind of extension being under consideration (admissible, preferred, or stable), it is a subset of a maximal conflict-free set, being here one among $\{a,c,e\}$, $\{a,c,f\}$, $\{a,c,g\}$, $\{a,d,f\}$, $\{a,d,g\}$, $\{b,d,f\}$, $\{b,d,g\}$, $\{b,e\}$.

Let us now briefly remind some of the principal notions about default reasoning (Reiter, 1980). A *default* is an expression of the form $\frac{\alpha:\beta_1\ldots\beta_n}{\gamma}$ where α, β_i, $1 \leq i \leq n$, and γ are closed first-order sentences with α being called the prerequisite, β_i the justifications, and γ the conclusion. Considering a set of defaults D, the functions $PREREQ(D)$, $JUST(D)$, and $CONS(D)$ refer respectively to all prerequisites, justifications, and consequences of the defaults of D. A *default theory* Δ is a pair (W,D) where W is a set of closed first-order sentences, and D is a set of defaults. Intuitively, the consequence of a default holds if its prerequisite holds and nothing can prevent the justification to hold (i.e. the negation of the justification does not hold). The main consequence of this idea is captured by the notion of *extension*. The following characterizations of R- and J-extensions (respectively due to (Reiter, 1980) and (Łukaszewicz, 1988)) are given here after (Risch, 1996). Consider $\Delta = (W,D)$. A subset D' of D is *grounded in W* iff for all $d \in D'$, there is a finite sequence $d_0, \ldots d_k$ of elements of D' such that (1) $PREREQ(\{d_0\}) \in Th(W)$, (2) for $1 \leq i \leq k-1, PREREQ(\{d_{i+1}\}) \in Th(W) \cup CONS(\{d_0, \ldots d_i\})$, and $d_k = d$. Then, let D' be any subset of D; $E = Th(W \cup CONS(D'))$ is: (1) a *J-extension* of Δ iff D' is a maximal grounded subset of D such that for all $\beta \in JUST(D')$, $\neg\beta \notin E$; (2) a *R-extension* of Δ iff it is a J-extension, and for each default $d \in D \setminus D', d = \frac{\alpha:\beta_1\ldots\beta_n}{\gamma}$, either $\alpha \notin E$ or

$\neg\beta_i \in E$ for some β_i. When $E = Th(W \cup CONS(D'))$ is an extension (either J- or R-), the set D' is called the set of *generating defaults of* E, and is denoted by $GD(E,\Delta)$. Note that J-extensions have interesting properties: they always exist, they denote consistant sets (in the standard case), and R-extensions are a special case easily characterized among J-extensions. Moreover, they are semi-monotonic i.e. adding new defaults to a default theory does not remove the previous J-extensions of this theory. Note that in the sequel, since there is no need for the full expressive power of first-order logic, we restrict ourselves to propositional default theories.

3 TRANSLATING ARGUMENTS INTO DEFAULTS AND CONVERSELY

Following (Dung, 1995), let us remind how a default theory can be expressed as an abstract argumentation framework. Consider $\Delta = (W,D)$ a default theory and $\Lambda = \{\beta_1,\ldots,\beta_n\} \subseteq JUST(D)$. A sentence λ is said to be a *defeasible consequence of* Δ *and* Λ (Dung, 1995) if there is a sequence $(e_0,\ldots e_n)$ with $e_n = \lambda$ such that, for each $e_i, 0 \leq i \leq n$, either (1) $e_i \in W$ or (2) e_i is a logical consequence of the preceding members in the sequence, or (3) e_i is the conclusion γ of a default $\frac{\alpha:\beta_1\ldots\beta_n}{\gamma}$ whose prerequisite α is a preceding member in the sequence and whose justifications β_i belongs to Λ. Λ is said to be a support for λ with respect to Δ. The theory Δ is then interpreted as an argumentation framework $\langle AR_\Delta, attacks_\Delta\rangle$ as follows: (1) $AR_\Delta = \{(\Lambda,\lambda) \mid \Lambda \subseteq JUST(D), \Lambda \text{ is a support for } \lambda \text{ with respect to } \Delta\}$;
(2) (Λ,λ) $attacks_\Delta$ (Λ',λ') iff $\neg\lambda \in \Lambda'$. Conversely, let us introduce now a simple translation from any abstract argumentation framework into the langage of default theories. We first define a so-called *Attackers* function from AR to 2^{AR} such that for every a,b in AR, $Attackers(a) = \{\top\} \cup \{\neg b \mid b \text{ } attacks \text{ } a\}$ In other word, when an argument in $\langle AR, attacks\rangle$ is attacked by no argument, the function associates it with the "empty" attacker $\top$, otherwise it associates it with the set of its standard attackers logicaly negated. Any argumentation framework $AF = \langle AR, attacks\rangle$ is then interpreted modularly as a default theory $\Delta_{AF} = (\emptyset, D_{AF})$ with $D_{AF} = \{\frac{\top:\, Attackers(a)}{a} \mid a \in AR\}$ Note that, while a default with a set of justifications restricted to $\top$ will participate in the generation of a *consistant* extension, the same default but with an empty set of justifications may lead to an *inconsistant* set. This represents a degenerated case never considered in standard default reasoning. Hence, in order to ensure a standard behaviour, the empty attacker $\top$ is indeed the least element needed in the set of justifications of the defaults resulting from our translation.

Example 2. (continued) Consider again the argument framework *AF1* given above. The corresponding default theory is given by $\Delta_{AF} = (\emptyset, D_{AF})$ with $D_{AF} = \{\frac{:\top}{a}, \frac{:\neg a,\neg c}{b}, \frac{:\neg d}{c}, \frac{:\neg c}{d}, \frac{:\neg d,\neg g}{e}, \frac{:\neg e}{f}, \frac{:\neg f}{g}\}$

Note that, where translating an *AF* to a default theory generates a number of defaults equal the number of arguments in *AF*, the translation in the other direction, that is from a default theory to an *AF*, generates many more (generaly infinitely many) arguments. This complexification is mainly due to the fact that we move from full propositional logics (on the side of defaults) to the simple flat fragment of propositional atoms, negated or not, with no operation of deductive closure (on the side of *AF*). Let us still point out that, as in the logical approaches of argumentation (cf. (Besnard and Hunter, 2008)), the standard translations defined here leads to define the arguments with a structure under the form $(support, conclusion)$. Eventually, note that the notion of *defeasible consequence* defined above allows precisely to express the arguments under this form by sort of removing the prerequisites from the initial default theory. This stresses indeed the fact that going from default theories to argument frameworks is an *abstraction* process (which, as noticed, is intractable in its full generality) while going from argument frameworks to default theories is a *modular translation* toward a fragment of defaults (linear, since there are as many defaults as arguments and, for each default, as many justifications as attackers, plus one for the empty attacker). Now, what remains to do regarding this translation is to ensure that any admissible set of arguments get an equivalent representation as some sort of default extension.

As shown in (Dung, 1995), there is an exact correspondence between the R-extensions of a default theory and the stable extensions of an abstract argumentation theory. Consider A, a first-order theory, and $A' \subseteq AR_\Delta$ a set of arguments obtained from a default theory Δ. Define (1) $arg(A) = \{(\Lambda,\lambda) \in AR_\Delta \mid \forall\beta \in \Lambda, \beta \cup A \nvdash\}$; (2) $flat(A') = \{\lambda \mid \exists(\Lambda,\lambda) \in A'\}$. Let Δ be a default theory. Then (lemma 42 and theorem 43 of (Dung, 1995)): (1) Given E any R-extension of Δ, $arg(E)$ is a stable extension of $\langle AR_\Delta, attacks_\Delta\rangle$; (2) Given E' any stable extension of $\langle AR_\Delta, attacks_\Delta\rangle$, $flat(E')$ is an R-extension of Δ. In order to consider into defaults the case of other types of extensions used in abstract argumentation frameworks, we establish the following corollary which characterizes the existence of arguments in AR_Δ regarding a default theory Δ.

Corollary 1. *Let* $\Delta = (W,D)$ *be a default theory and* $\langle AR_\Delta, attacks_\Delta \rangle$ *the corresponding argumentation framework. Then: (1)* $(\Lambda,\lambda) \in AR_\Delta$ *iff there exists* $D' \subseteq D$, D' *grounded in* W *such that* $\Lambda = JUST(D')$ *and* $\lambda \in Th(W \cup CONS(D'))$*; (2)* $flat(AR_\Delta) = \bigcup_{\substack{D' \in 2^D \\ D' grounded}} Th(W \cup CONS(D'))$.

4 A J-EXTENSION BASED APPROACH OF ADMISSIBLE EXTENSIONS

Let us consider in deeper way how to get arguments of AR_Δ from a default theory Δ. Our idea is to map any subset of applied defaults to a subset of arguments as accurately as possible in the most possible general way, and hence to relate eventually extensions of AR_Δ with extensions of Δ. Note that Corollary 1 stresses the crucial role played by the subsets of D in the constitution of any argument (Λ,λ) of AR_Δ, since for any such argument there exists $D' \subseteq D$ such that $\Lambda = JUST(D')$ and $\lambda \in Th(W \cup CONS(D'))$. The last set is precisely of the form taken by the different kinds of extensions of Δ (with possibly different constraints on it). In other words, we expect to relate different types of subsets of AR_Δ with some type of extension of Δ *via* $JUST(D')$ (for the supports of the arguments) and $Th(W \cup CONS(D'))$ (for the consequences of the *same* arguments). To achieve this goal however, the operator *arg* defined earlier is too sloppy. Hence we introduce a more accurate operator, directly defined on a subset of defaults:

Definition 1. *Given a default theory* $\Delta = (W,D)$*, and* $D' \subseteq D$*, let* $AR_\Delta(D') = \{(\Lambda,\lambda) \in AR_\Delta \mid \lambda \in Th(W \cup CONS(D'))\}$

Obviously, $AR_\Delta(D) = AR_\Delta$. We can now come to the characterization of conflict-free sets of arguments *via* a subset D' of defaults:

Theorem 1. *Given a default theory* $\Delta = (W,D)$*, let* $D' \subseteq D$*,* D' *grounded in* W *and* $E = Th(W \cup CONS(D'))$*. Then* $AR_\Delta(D')$ *is conflict-free iff* $\forall \beta \in JUST(D'), \neg\beta \notin E$.

The two following corollaries show that J-extensions correspond to conflict-free maximal subsets of arguments. More precisely, from the definition of J-extensions and theorem 1, we get immediately:

Corollary 2. *Let* $\Delta = (W,D)$ *be a default theory and* $\langle AR_\Delta, attacks_\Delta \rangle$ *the corresponding argumentation framework. Let* E_Δ *be any conflict-free* $\subseteq$*-maximal subset of* AR_Δ*. Then* $flat(E_\Delta)$ *is a J-extension of* Δ.

Corollary 3. *Let* $\Delta = (W,D)$ *be a default theory and* $\langle AR_\Delta, attacks_\Delta \rangle$ *the corresponding argumentation framework. Let* E *be any J-extension of* Δ*. Then* $AR_\Delta(GD(E,\Delta))$ *is a conflict-free* $\subseteq$*-maximal subset of* AR_Δ.

The question is now to filter J-extensions in order to represent admissible extensions in default logic. We do it thank to the following characterization theorem:

Theorem 2. *Let* $\Delta = (W,D)$ *be a default theory and* $\langle AR_\Delta, attacks_\Delta \rangle$ *the corresponding argumentation framework. Let* $\{D_{i, i\in\mathbb{N}}\}$ *be any enumeration of the grounded subsets of* D *and* $E(D_i) = Th(W \cup CONS(D_i))$ *for any* $i \in \mathbb{N}$*. For any* $D' \subseteq D$*,* $AR_\Delta(D')$ *is an admissible set of* $\langle AR_\Delta, attacks_\Delta \rangle$ *iff*

(i) there is $i \in \mathbb{N}$ *such that* $D' = D_i$

(ii) there is $j \in \mathbb{N}$ *such that* $D' \subseteq D_j$ *and* $E(D_j)$ *is a J-extension*

(iii) for any $k \in \mathbb{N}$*,* $(\exists\beta \in JUST(D'), \neg\beta \in E(D_k)) \Rightarrow (\exists\gamma \in E(D'), \neg\gamma \in JUST(D_k))$

What is shown here is that in order for a subset of a J-extension to correspond to an admissible set, one has to check that if any negation of a justification used to derive this subset can be found in one of the grounded subsets of D (i.e. some argument is attacked) then some formula of this grounded subset will be found negated among the initial justifications (i.e. the argument is defended). In other words, in order to compute any admissible set inside a default theory (and hence any preferred extension when considering $\subseteq$-maximal subsets), it is sufficient to filter inside the J-extensions. The most interesting consequence of this result comes from the one-to-one correspondence between J-extensions and ι-answer sets, which is the matter of the following section.

Note that in the case where the consequence $(\exists\gamma \in E(D'), \neg\gamma \in JUST(D_k))$ of the implication of *(iii)* is always true, we characterize stable extensions, which indeed correspond directly to the R-extensions *via* the characterization considered earlier.

5 LINK WITH ι-ANSWER SETS

Put into the context of answer set programming, J-extensions have been shown by (Delgrande et al., 2003) to correspond to some way of relaxing answer sets. This idea was further fully developed in (Gebser et al., 2009) who defines the ι-answer sets of a logical program as the exact counterpart of the J-extensions of the corresponding default theory. Following (Gebser et al., 2009), remind that a normal logic program is a finite set of rules of the form

$$p_0 \leftarrow p_1, \ldots, p_m, not\ p_{m+1}, \ldots, not\ p_n$$

where each p_i is an *atom*. For a rule r, $head(r)$ and $body(r)$ denote the usual corresponding parts of r, while $body^+(r)$ and $body^-(r)$ denote respectively the positive part and the negative part of $body(r)$. This definition are extended from a rule to a program Π, e.g. $head(\Pi) = \{head(r) \mid r \in \Pi\}$. Eventually, note that an empty head is similar to $\bot$, while an empty body is similar to $\top$. A program Π is called *basic* if $body^-(\Pi) = \emptyset$. Each basic program Π has a unique $\subseteq$-minimal model, denoted by $Cn(\Pi)$, that is the smallest set of atoms closed under the rules of Π.

Let $Cn^+(\Pi) = Cn(\Pi^\emptyset) = Cn(head(r) \leftarrow body^+(r) \mid r \in \Pi)$. Considering Π a logic program and X a set of atoms, X is an ι-answer set of Π if $X = Cn^+(\Pi')$ for some maximal $\Pi' \subseteq \Pi$ such that (1) $body^+(\Pi') \subseteq Cn^+(\Pi')$ and (2) $body^-(\Pi') \cap Cn^+(\Pi') = \emptyset$. The ι-answer sets of a program Π correspond to the justified extensions of the default theory given by the following known modular translation: each rule r of Π yields a default $\frac{\bigwedge body^+(r) : \neg|body^-(r)|}{head(r)}$ (where $| S | = \{a \mid not\ a \in S\}$), and $W = \emptyset$. Given $\Pi' \subseteq \Pi$, let (1) $AR_\Pi(\Pi') = \{(body^-(r), head(r)) \mid r \in \Pi'\}$, (2) $flat_\Pi(\Pi') = \{head(r) \mid r \in \Pi'\}$ Given any argumentation framework $AF = \langle AR, attacks \rangle$, from the translation defined earlier we get a default theory $\Delta_{AF} = (\emptyset, D_{AF})$. In turn, from the modular translation defined just above, this default theory yields a logic program Π_{AF} with an empty positive body (i.e. $body^+(\Pi)_{AF} = \emptyset$). Clearly, $AR_\Pi(\Pi_{AF}) = AR$, and for every $D' \subseteq D_{AF}$ there exists $\Pi' \subseteq \Pi_{AF}$ such that $AR_\Delta(D') = AR_\Pi(\Pi')$ and $flat(AR_\Delta(D')) = flat_\Pi(\Pi')$. As a consequence of corollaries 2 and 3 we then get immediately:

Corollary 4. *Let Π be a program with an empty positive body and $AR_\Pi(\Pi)$ the corresponding argumentation framework. For any $\Pi' \subseteq \Pi$, $AR_\Pi(\Pi')$ is a $\subseteq$-maximal conflict-free subset of $AR_\Pi(\Pi)$ iff $head(\Pi')$ is a ι-answer set of Π.*

From theorem 2 we get directly:

Corollary 5. *Let Π be a program with an empty positive body and $AR_\Pi(\Pi)$ the corresponding argumentation framework. Let $X_1, \ldots, X_k$ be a collection of all the ι-answer sets of Π and $AR_\Pi(\Pi_1), \ldots, AR_\Pi(\Pi_k)$ the corresponding conflict-free maximal subsets of $AR_\Pi(\Pi)$. For any $\Pi' \subseteq \Pi$, $AR_\Pi(\Pi')$ is an admissible set of $AR_\Pi(\Pi)$ iff there is i, $1 \leq i \leq k$, such that $\Pi' \subseteq \Pi_i$ and $(\forall r' \in \Pi')(\exists r \in \Pi \setminus \Pi')(body^-(r') \cap head(r) \neq \emptyset \Rightarrow head(\Pi') \cap body^-(r) \neq \emptyset)$.*

Example 3. (continued) Back to *AF1*, we get the following logic program Π_{AF1}:

$$\begin{array}{llll}
r^1: a \leftarrow & & r^5: e \leftarrow not\ d, not\ g \\
r^2: b \leftarrow not\ a, not\ c & & r^6: f \leftarrow not\ e \\
r^3: c \leftarrow not\ d & & r^7: g \leftarrow not\ f \\
r^4: d \leftarrow not\ c & &
\end{array}$$

Eight ι-answer sets are generated, namely $X_1 = \{a,c,e\}$, $X_2 = \{a,c,f\}$, $X_3 = \{a,c,g\}$, $X_4 = \{a,d,f\}$, $X_5 = \{a,d,g\}$, $X_6 = \{b,d,f\}$, $X_7 = \{b,d,g\}$, $X_8 = \{b,e\}$. For instance, consider especially X_1 and X_5 that respectively correspond to the following two conflict-free $\subseteq$-maximal subsets of $AR_\Pi(\Pi_{AF1})$: $AR_\Pi(\Pi_1) = \{(\{\}, a), (\{not\ d\}, c), (\{not\ d, not\ g\}, e)\}$, $AR_\Pi(\Pi_5) = \{(\{\}, a), (\{not\ c\}, d), (\{not\ f\}, g)\}$, with $\Pi_1 = \{r^1, r^3, r^5\}$, $\Pi_5 = \{r^1, r^4, r^7\}$. Applying corollary 5, it is easy to check that while $(\{not\ f\}, g)$ from $AR_\Pi(\Pi_5)$ *attacks* $(\{not\ d, not\ g\}, e)$ from $AR_\Pi(\Pi_1)$ (that is $body^-(r^5) \cap head(r^7) \neq \emptyset$), $AR_\Pi(\Pi_1)$ does not defends itself from this attack (that is $head(\Pi_1) \cap body^-(r^7) = \emptyset$). This means that $AR_\Pi(\Pi_1)$ is not admissible and that the argument $(\{not\ d, not\ g\}, e)$ has to be removed in order to get $\{(\{\}, a), (\{not\ d\}, c)\}$ as an admissible subset of $AR_\Pi(\Pi_{AF1})$. Similar checks apply to all the ι-answer sets found here.

Let us now define the counterpart of an admissible set of arguments inside a logic program:

Definition 2. *Let Π be a logic program and X be a set of atoms. X is called an* admissible answer set of Π *iff there is $\Pi' \subseteq \Pi$ such that $X = flat_\Pi(\Pi')$ and $AR_\Pi(\Pi')$ is an admissible set of $AR_\Pi(\Pi)$.*

Following (Gebser et al., 2009), we can augment our framework with integrity constraints whose purpose here will be to filter inside the ι-answer sets the subsets that are admissible. Remind that an integrity constraint is a rule c with an empty head, that is

$$\leftarrow p_1, \ldots, p_m, not\ p_{m+1}, \ldots, not\ p_n$$

After (Gebser et al., 2009), we consider a constraint c *satisfied with respect* to a set X of atoms if for any rule r of Π, $body^+(c) \not\subseteq X$ or $body^-(c) \cap X \neq \emptyset$. In order to eliminate into ι-answer sets the subsets that would not correspond to admissible sets of $AR_\Pi(\Pi)$ for a given program Π obtained from an abstract argumentation framework, let

$$C_\Pi^{Ad} = \{\leftarrow head(r'), body^-(r) \mid r, r' \in \Pi, head(r) \in body^-(r')\}$$

Remark: Just as we label every rule of a program with a unique natural number, we find convenient to label every constraint from the two rules that yield it in a program, i.e. in the sequel c^{ij} will denote $\leftarrow head(r^i), body^-(r^j)$ when $head(r^i) \in body^-(r^j)$.

Definition 3. *Let Π be a logic program, $\mathcal{C}_\Pi^{Ad}$ be a set of integrity constraints and X be a set of atoms. X is* admissible with respect to $\mathcal{C}_\Pi^{Ad}$ *iff X is a subset of an ι-answer set of Π such that every $c \in \mathcal{C}_\Pi^{Ad}$ is satisfied with respect to X.*

Theorem 3. *Let Π be a logic program and X be a set of atoms. Then X is an admissible answer set of Π iff X is admissible with respect to $\mathcal{C}_\Pi^{Ad}$.*

Example 5. (continued) Back to *AF1*, we add to the logic program Π_{AF1} the set of constraints $\mathcal{C}_{\Pi_{AF1}}^{Ad}$:

$$\begin{array}{lll@{\qquad}lll} c^{12}: & \leftarrow & b & c^{56}: & \leftarrow & f, not\ d, not\ g \\ c^{32}: & \leftarrow & b, not\ d & c^{67}: & \leftarrow & g, not\ e \\ c^{45}: & \leftarrow & e, not\ c & c^{75}: & \leftarrow & e, not\ f \end{array}$$

$X_1 = \{a, c, e\}$ is eliminated by c^{75} since $body^+(c^{75}) \subseteq X_1$ while $body^-(c^{75}) \cap X_1 = \emptyset$, and hence c^{75} is not admissibly satisfied with respect to X_1. On the contrary, $X_1 \setminus \{e\}$ is an admissible set of Π_{AF1} with respect to $\mathcal{C}_{\Pi_{AF1}}^{Ad}$. Note that any set containing b is immediately eliminated by c^{12}. Note that in order to automatize the enumeration of the possible candidates among the subsets of the X_i, it is necessary to relax the definition of an ι-answer set by removing the condition on maximality.

6 CONCLUSIONS

In this paper, we have established a characterization of the admissible semantics defined by Dung inside defaults and answer-set programming. Although not surprising, to our opinion these results show a closer relation of abstract argumentation frameworks with defaults and answer sets than initially described by Dung. Notably, and contrary to the approaches used for instance by (Dung, 1995), (Nieves et al., 2008) or (Egly et al., 2010), a first-order encoding appears useless for processing abstract argumentation frameworks with logic programs. Especially, this means that no grounding is necessary for the logic programs obtained from our transformation of argumentation frameworks. Among further perspectives, we are concerned with the ability to extend the current characterization to other different semantics, e.g. obviously complete extensions, but also the CF2 (Baroni et al., 2005) or the semi-stable (Caminada, 2006) semantics. Of course, and regarding complexity issues, we are also concerned with a detailed comparison with the computation methods proposed in (Nieves et al., 2008) and (Egly et al., 2010). Finally, note that another direction under way concerns the possibility to extend the bipolar approach of abstract argumentation frameworks in order to provide them with the same expressive power as normal logic programs.

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