A TWO-PLAYER MODEL FOR THE SIMULTANEOUS LOCATION OF FRANCHISING SERVICES WITH PREFERENTIAL RIGHTS

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Abstract: We consider the discrete location problems faced by two decision-makers, franchisees, that will have to simultaneously decide where to locate their own services (unsure about the decisions of one another). All services compete among themselves. At most one service can be located at each potential location. We consider that one of the decision-makers has preferential rights meaning that if both decision-makers are interested in the same location, only to this decision-maker will be given the permission to open the service. We present a mathematical formulation and some conclusions based on computational results.

1 INTRODUCTION

Competitive location problems consider the situation where it is not sufficient for a given decision maker to consider only his own facilities when faced with a location decision (throughout the paper we will refer to facility and service interchangeably). Most of the times, these facilities will compete with similar facilities in the market, so that the customers' share that will be assigned to the decision-maker's facilities depends on his own choices as much as on the competitors' decisions. In this paper we work with a competitive discrete location problem where two decision makers (players) will have to decide simultaneously where to locate their own facilities, unsure about the decisions of one another.

Several authors have studied competitive location problems (for a review see, for instance, Plastria, 2001). Dobson and Karmarkar, 1987, study a discrete competitive location problem in which price and demand are fixed, and considering an existing firm, a competitor, and clients that want to minimize the distance traveled. Labbé and Hakimi, 1991, study the problem in which two firms have to make decisions regarding the location of facilities and also the quantities of a given commodity they will make available. Vetta, 2002, also proposes a location game where multiple decision makers (service providers) start by deciding where to locate their facilities and then define how much they charge their customers. Hande et al, 2011, study a sequential competitive location problem where the follower can react to the decisions made by the leader, adjusting the attractiveness of their own services.

Among the competitive location linear programming problems, most approaches either consider that the firms already present in the market will not be able to react to the decision-maker's new chosen locations or consider a Stackelberg problem, where there is a follower that will react to a leader, knowing what the leader has decided. These types of problems differ significantly from the problem tackled here. We consider a situation where a franchiser intends to open new facilities in a given area. There are two potential investors, and the facilities to be open will compete among themselves. They provide the same type of commodities to consumers, at the same prices, and it is assumed that customers patronize the closest available facility. The franchiser defines the finite set of potential locations for facilities, but he is not familiar with the demand patterns of the area. So, he will define more potential locations than he expects the investors to choose, and leave the choices among them to the investors, who are better acquainted with the area. The franchiser payoff will be a percentage calculated

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over the total demand assigned to the new facilities. Each investor is interested in maximizing the total demand that is assigned to his own facilities. Each investor is aware of the fixed costs incurred by opening each and every facility, which can be different for both investors. Each investor has a budget constraint. They are also aware of the demand associated with each customer. This demand will not increase with distance, meaning that the closer the assigned facility is of a given customer, the greater the demand from the customer. At each location at most one facility can be opened. If the decision-makers were to decide sequentially, this problem would be a sequential problem that could be formulated as a bilevel linear programming problem. But we consider that both decision-makers will have to decide simultaneously. In this situation, it will be necessary to clarify what happens if both investors apply for the same location. As at most one service can be opened, the franchiser will have to decide what to do in this case. We consider that the franchiser patronizes one investor, in detriment of the other. For the sake of simplicity, consider that the franchiser always chooses investor 1. This means that if both apply for the same location, then the franchiser will allow investor 1 to open the facility, and investor 2 will not be able to do so. We can say that investor 1 has preferential rights, which is known by both decision-makers. This problem can also be interpreted as a full information game (because each player knows the payoffs and strategies of the other), with a finite number of players (the two decision-makers), and a finite number of pure strategies (for each player, a pure strategy can be defined as a particular combination of locations, out of the set of potential new locations, where the player chooses to open facilities). That is why we will not distinguish between investor, decision-maker and player, and will use these terms interchangeably as having the same meaning.

We approach this simultaneous decision problem from a mathematical programming point of view (a preliminary mathematical formulation appeared in the research report Dias and Godinho, 2011) and from a game theory point of view. The game will have at least one Nash equilibrium, possibly with mixed strategies, that can be calculated algorithmically. Some computational results are presented and conclusions drawn.

2 PROBLEM FORMULATION

In this problem we are considering that both decision-makers will decide simultaneously, without knowing the decision made by the other. We will also accommodate the existence of already opened services. Let us consider that these services belong to investor 1.

Consider the following definitions:

F – set of pre-existing facilities that belong to investor 1;

G – set of potential locations for new facilities;

J – set of customers;

d_{ij} – demand associated with customer j when he is assigned to a facility located at i;

c_{ij} – distance between customer j and location i;

f_{ip} – fixed cost associated with investor p opening a facility at location i (and such that $f_{ip} = 0, \forall i \in F$)

α_p – percentage over the demand captured to be paid to the franchiser by investor p;

O_p – maximum budget available to investor p.

We consider that demand will not increase with distance. We will additionally assume that potential locations at the same distance will capture the same demand. This means that:

$$c_{ij} \leq c_{kj} \Rightarrow d_{ij} \geq d_{kj}, \forall i,k \in F \cup G, \forall j \in J \tag{1}$$

$$c_{ij} = c_{kj} \Rightarrow d_{ij} = d_{kj}, \forall i,k \in F \cup G, \forall j \in J \tag{2}$$

Let us define the following decision variables:

$$y_i = \begin{cases} 1, \text{ if investor 1 either opens} \\ \quad \text{a facility at } i \text{ or has a} \\ \quad \text{pre-existing facility at } i \\ 0, \text{ otherwise} \end{cases}, \forall i \in F \cup G$$

$$w_i = \begin{cases} 1, \text{ if investor 2 bids for} \\ \quad \text{opening a facility at } i \\ 0, \text{ otherwise} \end{cases}, \forall i \in F \cup G$$

$$z_i = \begin{cases} 1, \text{ if investor 2 opens a facility at } i \\ 0, \text{ otherwise} \end{cases}, \forall i \in F \cup G$$

$$x_{ij} = \begin{cases} 1, \text{ if client } j \text{ is assigned to} \\ \quad \text{facility } i \text{ that belongs to} \\ \quad \text{investor 1} \\ 0, \text{ otherwise} \end{cases}, \forall i \in F \cup G, \forall j \in J$$

$$m_{ij} = \begin{cases} 1, \text{ if client } j \text{ is assigned to} \\ \quad \text{facility } i \text{ that belongs to} \\ \quad \text{investor 2} \\ 0, \text{ otherwise} \end{cases}, \forall i \in F \cup G, \forall j \in J$$

In most location problems, only binary variables similar to y_i and z_i are needed. For this problem, however, another set of variables, w_i, is essential to allow the distinction between two different situations: of *bidding for* and of *being able to open* a facility. This distinction is not needed for investor 1: he will open every facility that he bids for, because he has preferential rights. But investor 2 can bid for a location and still not be able to open a facility there if investor 1 has also shown interest for the same location.

In this problem, different sets of variables are controlled by different stakeholders: investor 1 controls variables $y_i, \forall i \in G$; investor 2 controls variables $w_i, \forall i \in G$; the franchiser controls variables $z_i, \forall i \in G$ (according to a predefined rule known by both decision makers); customers control variables $x_{ij}, m_{ij} \forall i \in F \cup G, j \in J$ (also according to known rules – in this case resorting to the minimum distance criteria). Decisions made by the franchiser and by customers are not controlled by the two decision-makers, despite the fact that these decisions play a crucial role in the determination of each players' payoff. An important point to make is that despite not being under their control, decision makers are both fully aware of how these decisions are made. As a matter of fact, once variables y_i and w_i are fixed it is possible to immediately compute the corresponding values for x_{ij}, m_{ij} and z_i.

Each investor will make his own decisions conditioned only by his own constraints. A set of *connection constraints* is then considered that will determine the values of the remaining variables according to the pre-established rules.

Let us now formulate the problem, following the representation introduced in Godinho and Dias, 2010:

Decision-Maker 1

$$Max \sum_{i \in F \cup G} \sum_{j \in J} (1-\alpha_1) d_{ij} x_{ij} \tag{3}$$

Subject to:

$$\sum_{i \in G} f_{i1} y_i \le O_1 \tag{4}$$

$$y_i = 1, \forall i \in F \tag{5}$$

Decision-Maker 2

$$Max \sum_{i \in F \cup G} \sum_{j \in J} (1-\alpha_2) d_{ij} m_{ij} \tag{6}$$

Subject to:

$$\sum_{i \in G} f_{i2} w_i \le O_2 \tag{7}$$

$$w_i = 0, \forall i \in F \tag{8}$$

Connection restrictions

$$y_i + z_i \le 1, \forall i \in F \cup G \tag{9}$$

$$m_{ij} \le z_i, \forall i \in F \cup G, j \in J \tag{10}$$

$$x_{ij} \le y_i, \forall i \in F \cup G, j \in J \tag{11}$$

$$\sum_{i \in F \cup G} (x_{ij} + m_{ij}) = 1, \forall j \in J \tag{12}$$

$$m_{ij} + x_{ij} \le 1 - z_k - y_k, \forall i \in G, j \in J, k \in T_{ij} \tag{13}$$

$$z_i \le w_i, \forall i \in F \cup G \tag{14}$$

$$\begin{gathered} z_i, w_i, y_i \in \{0,1\}, \forall i \in F \cup G \\ x_{ij} \in \{0,1\}, \forall i \in F \cup G, j \in J \\ m_{ij} \in \{0,1\}, \forall i \in F \cup G, j \in J \end{gathered} \tag{15}$$

Regarding decision-maker 1, he will maximize his payoff subject to the restriction that he has to afford to open all the facilities he bids for (constraint (4)). Constraint (5) guarantees that the existing facilities will stay open. A similar objective function is considered by decision-maker 2, and similar constraints: a budget constraint (7) and a constraint that does not allow him to bid for already opened services (8). Constraint (9) guarantees that at most one service is opened at each location. Customers can only be assigned to opened facilities (constraints (10) and(11)), and have to be assigned to exactly one facility (12). Each customer is assigned to the closest opened facility (13). We are not considering the situation such that a customer is equally distant from two or more opened facilities. This possibility can easily be considered, assuming that the demand of a customer is equally split by two or more opened facilities (see Godinho and Dias, 2010).

Constraints (14) state that investor 2 can only open facilities he has bid for.

Each solution to this problem is composed of a set of y_i variable values, which we will denote as vector $\mathbf{y}$, and a set of w_i variables' values, which we will denote as vector $\mathbf{w}$. Interpreting this problem as a game, $\mathbf{y}$ is a strategy for player 1, and $\mathbf{w}$ is a strategy for player 2. An admissible solution is a Nash equilibrium solution. In the case of a Nash equilibrium with pure strategies, this means that $(\mathbf{y}, \mathbf{w})$ is admissible if $\mathbf{y}$ is a best response to $\mathbf{w}$ and vice-versa.

3 COMPUTATIONAL RESULTS

There is no obvious procedure for solving the two-player simultaneous decision problem presented in the previous section. Therefore, in order to calculate the game equilibria, we resorted to an algorithm based on the best responses of each player to the other one's strategy, proposed by Godinho and Dias (2010).

The algorithm was implemented in C programming language, using LP Solve routines for solving the linear programming problems (source: http://lpsolve.sourceforge.net). For each instance, we applied the algorithm twice for the game in which player 1 has preferential rights. The first time we chose a null strategy for player 1 (opening no locations) as the starting point; the second time, we chose a null strategy for player 2 as the starting point. In fact, in a model without preferential rights, the algorithm will often find solutions that are more favorable to the player whose best response is considered first (the algorithm will only find one equilibrium, and the game may have several equilibria, so the results may be somewhat biased by the choice of the starting point, as shown in Godinho and Dias, 2010).

However, in the problem here addressed, the equilibrium solution that is found is usually independent of the starting point of the algorithm; moreover, when different starting points lead to different equilibria, the differences in the players payoffs in the two equilibria are small.

Test set 1 was used as a reference, the parameters of the remaining test sets being defined as changes over the parameters of this test set. For test set 1, we defined a network with 100 nodes (that is, 100 possible locations for the customers), with both players being able to open facilities at 48 of these locations. The budget for each player was set to 1000, and the average cost of opening a facility was set to 350.

Test sets 2-4 were designed to allow us to analyze the impact of simultaneously changing the number of potential locations for both players' facilities. The number of potential locations for the players' facilities was set to 36, 24 and 12 in test sets 2, 3 and 4, respectively, and the other parameters' values were identical to the ones used in test set 1.

The results obtained with test sets 1-4 are summarized in Table 1. As expected, the average payoffs of both players increase as the number of potential facility locations increase, but this increase takes place at a decreasing rate. This behavior occurs both when there are preferential rights and when they do not exist, and it is consistent with the results of Godinho and Dias (2010). Both the benefit that player 1 gets from having preferential rights and the loss player 2 incurs when player 1 has such rights, seem fairly stable in absolute terms. Since payoffs increase with the number of potential locations, this means that the relative gain of player 1 and the relative loss of player 2 become less significant as the number of potential locations increase.

This makes sense because an increase in the number of potential locations leaves player 2 with more places in which he can avoid player 1, and provides player 1 with more interesting locations, so he has a relatively smaller incentive to try to choose the same locations as player 2.

Test sets 5-7 allow us to analyze the consequences of changing the potential locations available to just one of the players. Player 1 has 48 potential locations, and the number of potential locations for player 2's facilities is 48, 36, 24 and 12 in test sets 1, 5, 6 and 7, respectively. This is done by randomly choosing a subset of G and considering $f_{i2} = +\infty$, for all facilities i in this subset. The other parameters' values were identical to the ones used in test set 1. The results are summarized in Table 2. As the number of locations available to player 2 increases, player 2's payoff increases and player 1's payoff tends to decrease. The relative loss of player 2 from the preferential rights of player 1 is fairly stable. In the case of player 1, both the absolute and the relative gain increase with the number of potential locations for player 2. This means that, as player 2 gets more

Table 1: Summary of the results obtained with test sets 1-4.

Test set	Potential locations	Average return (with preferential rights)		Average return (without preferential rights)		Player 1 benefit from preferential rights		Player 2 loss from player 1 rights	
						Absolute	Relative	Absolute	Relative
		$\overline{\pi_1^{with}}$	$\overline{\pi_2^{with}}$	$\overline{\pi_1^{w/out}}$	$\overline{\pi_2^{w/out}}$	$\overline{\pi_1^{with}} - \overline{\pi_1^{w/out}}$	$\overline{\pi_1^{with}} / \overline{\pi_1^{w/out}} - 1$	$\overline{\pi_2^{w/out}} - \overline{\pi_2^{with}}$	$1 - \overline{\pi_2^{with}} / \overline{\pi_2^{w/out}}$
1	48	1427.8	952.8	1197.6	1196.9	230.2	19.2%	244.1	20.4%
2	36	1416.9	932.7	1179.2	1180.6	237.7	20.2%	247.9	21.0%
3	24	1310.8	813.4	1084.2	1060.7	226.6	20.9%	247.4	23.3%
4	12	1089.9	633.1	840.3	866.9	249.6	29.7%	233.7	27.0%

$\overline{\pi_1^{with}}$, $\overline{\pi_2^{with}}$: average payoffs for player 1 and player 2, respectively, when player 1 has preferential rights;

$\overline{\pi_1^{w/out}}$, $\overline{\pi_2^{w/out}}$: average payoffs for player 1 and player 2, respectively, when there are no preferential rights.

Table 2: Summary of the results obtained with test sets 1 (repeated for easier reference) and 5-7.

Test set	Potential locations for player 2	Average return (with preferential rights)		Average return (without preferential rights)		Player 1 benefit from preferential rights		Player 2 loss from player 1 rights	
						Absolute	Relative	Absolute	Relative
		$\overline{\pi_1^{with}}$	$\overline{\pi_2^{with}}$	$\overline{\pi_1^{w/out}}$	$\overline{\pi_2^{w/out}}$	$\overline{\pi_1^{with}} - \overline{\pi_1^{w/out}}$	$\overline{\pi_1^{with}} / \overline{\pi_1^{w/out}} - 1$	$\overline{\pi_2^{w/out}} - \overline{\pi_2^{with}}$	$1 - \overline{\pi_2^{with}} / \overline{\pi_2^{w/out}}$
1	48	1427.8	952.8	1197.6	1196.9	230.2	19.2%	244.1	20.4%
5	36	1419.1	916.8	1243.7	1152.0	175.4	14.1%	235.2	20.4%
6	24	1519.5	882.5	1365.7	1092.5	153.7	11.3%	210.0	19.2%
7	12	1543.6	742.3	1421.8	932.4	121.8	8.6%	190.1	20.4%

$\overline{\pi_1^{with}}$, $\overline{\pi_2^{with}}$: average payoffs for player 1 and player 2, respectively, when player 1 has preferential rights;

$\overline{\pi_1^{w/out}}$, $\overline{\pi_2^{w/out}}$: average payoffs for player 1 and player 2, respectively, when there are no preferential rights.

Table 3: Summary of the results obtained with test sets 1 (repeated for easier reference) and 8-10.

Test set	Player 2's budget	Average return (with preferential rights)		Average return (without preferential rights)		Player 1 benefit from preferential rights		Player 2 loss from player 1 rights	
						Absolute	Relative	Absolute	Relative
		$\overline{\pi_1^{with}}$	$\overline{\pi_2^{with}}$	$\overline{\pi_1^{w/out}}$	$\overline{\pi_2^{w/out}}$	$\overline{\pi_1^{with}} - \overline{\pi_1^{w/out}}$	$\overline{\pi_1^{with}} / \overline{\pi_1^{w/out}} - 1$	$\overline{\pi_2^{w/out}} - \overline{\pi_2^{with}}$	$1 - \overline{\pi_2^{with}} / \overline{\pi_2^{w/out}}$
1	1000	1427.8	952.8	1197.6	1196.9	230.2	19.2%	244.1	20.4%
8	750	1567.4	763.8	1340.8	1000.9	226.6	16.9%	237.1	23.7%
9	500	1576.7	505.6	1425.2	675.4	151.4	10.6%	169.8	25.1%
10	250	1678.4	244.4	1576.4	348.0	102.0	6.5%	103.7	29.8%

$\overline{\pi_1^{with}}$, $\overline{\pi_2^{with}}$: average payoffs for player 1 and player 2, respectively, when player 1 has preferential rights;

$\overline{\pi_1^{w/out}}$, $\overline{\pi_2^{w/out}}$: average payoffs for player 1 and player 2, respectively, when there are no preferential rights.

Table 4: Summary of the results obtained with test sets 1 (repeated for easier reference) and 11-13.

Test set	Average fixed facility cost	Average return (with preferential rights)		Average return (without preferential rights)		Player 1 benefit from preferential rights		Player 2 loss from player 1 rights	
						Absolute	Relative	Absolute	Relative
		$\overline{\pi_1^{with}}$	$\overline{\pi_2^{with}}$	$\overline{\pi_1^{w/out}}$	$\overline{\pi_2^{w/out}}$	$\overline{\pi_1^{with}} - \overline{\pi_1^{w/out}}$	$\overline{\pi_1^{with}} / \overline{\pi_1^{w/out}} - 1$	$\overline{\pi_2^{w/out}} - \overline{\pi_2^{with}}$	$1 - \overline{\pi_2^{with}} / \overline{\pi_2^{w/out}}$
11	175	1961.2	1150.1	1590.0	1580.0	371.2	23.3%	429.9	27.2%
12	262.5	1709.6	1089.9	1408.5	1429.0	301.1	21.4%	339.1	23.7%
1	350	1427.8	952.8	1197.6	1196.9	230.2	19.2%	244.1	20.4%
13	525	1109.6	832.4	965.0	989.0	144.6	15.0%	156.7	15.8%

$\overline{\pi_1^{with}}$, $\overline{\pi_2^{with}}$: average payoffs for player 1 and player 2, respectively, when player 1 has preferential rights;

$\overline{\pi_1^{w/out}}$, $\overline{\pi_2^{w/out}}$: average payoffs for player 1 and player 2, respectively, when there are no preferential rights.

potential locations, it becomes more important to player 1 to get preferential rights, in order to secure exclusive benefits from the most interesting locations.

Test sets 8-10, considered along with test set 1, allow us to analyze the consequences of changing the budget of a player, while keeping the other player's budget constant. We defined that player 1's budget is 1000, and set player 2's budget to 1000, 750, 500 and 250 in test sets 1, 8, 9 and 10, respectively, with all other parameters' values held constant. The results are summarized in Table 3.

As expected, player 2's payoff increases when his budget increases, and player 1's payoff decreases in that situation. The benefit from having preferential rights becomes more significant for player 1 as player 2's budget increases. This means that, as player 2 is able to build more facilities, it becomes more important for player 1 to secure exclusive benefits from the best locations. As for player 2, the absolute loss from player 1's rights increases with his budget, but the relative payoff reduction becomes less significant for higher budgets.

Test sets 11-13, considered along with test set 1, allow us to analyze what happens when the average fixed cost of each facility changes and the players' budgets are kept constant. We set the average cost of each facility to 175, 262.5, 350 and 525 in test sets 11, 12, 1 and 13, respectively. The other parameters' values were identical to the ones used in test set 1. The results are summarized in Table 4. The payoffs of both players decrease as the average cost of each facility increases. When the average cost increases, players are able to open less facilities, thus reducing their payoffs. At the same time, the increase in average facility cost reduces the absolute and relative benefit player 1 gets from preferential rights, and it also reduces the absolute and relative loss incurred by player 2. In fact, with the increase in average facility cost, and the consequent reduction in the number of facilities, the level of competition between players decreases, reducing the impact of preferential rights.

4 CONCLUSIONS

We have introduced a simultaneous discrete location problem with two decision-makers, in a franchising environment, where one of the players has preferential rights. This model has several distinguishing features, namely the fact of considering explicitly simultaneous decisions instead of sequential decisions. We have formulated the problem as a linear programming problem, and have defined as admissible solutions those that are Nash equilibrium solutions.

The computational results show us that if the level of competition increases, then the importance of having preferential rights also increases. The level of competition is higher when there are fewer potential locations for opening facilities, when fixed opening costs decrease keeping the budget constant, or when the budget sizes are similar.

The developed work raises other questions, namely what happens if it is given to the player without preferential rights the possibility of bidding for more facilities than the ones he can afford. This will be the subject of further research.

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