SYNAPTIC TRANSMISSION AND FOKKER-PLANCK EQUATION

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Abstract: An important neurologic process consists in a time dependent transmission of the electric signal between neurons. The description of such signal is the objective of this work. In this way, the Fokker-Planck equation with a term of control which depends on time is used. The applied force is characterized by the existence of a barrier that increases with time and reduces the diffusion of particles. The solution of the equation is obtained by an ansatz that satisfies the initial conditions of the problem. Numerical examples of the time evolution of the found solutions are analyzed by calculating the escape rate and the time necessary to across the barrier for different values of diffusion constant.

1 INTRODUCTION

The process related to the flux of charge in the region between two neurons (synapse) is fundamental to the function of the nervous system. Thus, the understanding and description of ion transport in the synapses have a prominent place in modern neuroscience. This process occurs through fast picks of electrical currents between neurons (Hille, 1992, Ramakrishnan, 2011). Studies involving the synapse are very important considering that few neurological diseases have an effective treatment (Fassio, 2011). This fact has encouraged studies in this area, aiming to develop techniques that can bring greater understanding of the biological processes involved (Joshi, 2011 England, 2010, Guo, 2010, Fallon, 2011) and thus, develop more effective treatment methods for different types of neurological diseases.

A quantitative description of processes involving stochastic components can be made by using the Fokker-Planck equation (FPE). This equation has wide application in many branches of physics, chemistry and biology (Coffey, 2004), such as protein folding (Curtis, 1997) or ion transport across membranes (Lee, 2002). In some cases, the Fokker-Planck equation can be solved analytically as, for instance, for the linear and stationary problems involving one variable (Risken, 1989).

The usual form of EFP is

$$\frac{\partial}{\partial t}P(x,t) = -\frac{\partial}{\partial x}\left[f(x)P(x,t)\right] + Q\frac{\partial^2}{\partial x^2}P(x,t). \quad (1)$$

where t represents the time variable and the variable x can be identified, for example, with the velocity (Reichl, 1988). The function f(x) is known as the external force acting on the system. However, this terminology is only appropriate when x represents the velocity. Q this is related to the diffusion constant, and P(x,t) is the probability distribution.

Different methods have been proposed to determine and analyze the solution of the Fokker-Planck equation (1), as the use of the numerical method based on finite differences (Ames, 1992), Adomian polynomial method (Tatari, 2007) and the mapping of FPE in equation type Schroedinger (Risken, 1989).

In terms of synapse transmission, the potential to be used should reflect the transitory nature of the signal. Thus, it is suggested a potential which is initially metastable and converges to a harmonic potential for large value of time (Figure 1). Then, initially the particles in the region of minimum of potential can escape out this region. However, the system becomes confined for the stationary regime. The solution of the Fokker-Planck equation in this case is obtained through an ansatz.

Studies involving kinetics of reaction, studied for more than 70 years (Hänggir, 1990), are still based

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on the description of the system in quasi-stationary regime. Thus, the information related to the system outside that regime is lost. On the other hand, the solution of the Fokker-Planck equation (Risken, 1989) makes possible the analysis of physical and chemical processes from the beginning, when the system is in the no equilibrium condition.

In the section 2, the FPE and the probability distribution are presented. In section 3, the dynamic properties, the escape rates and a characteristic time, of the system are determined. In section 4, numerical examples are discussed. Finally, the conclusions are presented in Section 5.

2 THE FOKKER-PLANCK EQUATION

The potential studied in this work is:

$$V(x) = k_1 x^2 - k_2(t) x^3, \quad (2)$$

where k_1 is a constant and $k_2(t)$ is a time function. In order to describe the flux of ions through the synapse, the function $k_2(t)$ must go to a constant value for a large value of time ($k_2(t\rightarrow\infty)\rightarrow$constant) and the anharmonic term become small comparing the harmonic part. Thus, for large values of time, the barrier of potential to be overcome for occurs diffusion of particles from the region of minimum becomes very large. In this condition the system is governed by a quasi harmonic potential. The explicit form adopted for $k_2(t)$ in this work is:

$$k_2(t) = k_{2,0}\left[\lambda + (2k_1 - \lambda)e^{-4k_1 t}\right]^{-3/2}. \quad (3)$$

where $k_{2,0}$ is a constant. This expression attends the properties described above and allows an analytical solution for EFP (1) with the function $f(x)$ given by derivative of potential (2),

$$f(x) = -2k_1 x + 3k_2 x^2 \quad (4)$$

The solution of the equation (1), for $f(x)$ given by (4), can be found assuming that $P(x,t)$ is written as

$$P(x,t) = \frac{1}{\phi(t)}\exp\left\{-\frac{\lambda x^2}{2Q\phi(t)^2} + \frac{k_2(t)x^3}{Q}\right\} \quad (5)$$

where λ is a constant defined from the normalization of probability and the function $\phi(t)$ is given by

$$\phi(t)^2 = \frac{\lambda}{2k_1} + \left(1 - \frac{\lambda}{2k_1}\right)e^{-4k_1 t} \quad (6)$$

The initial condition adopted is that all particles are initially in the region of minimum in $x = 0$. Then, in the initial time ($t = 0$) the distribution of probability has value equal 1 in the position $x = 0$.

The figure 1 shows a numerical representation of the potential (2) in function of position for different values of time, with $k_2(t)$ given by (3) and the parameters $k_1 = 0.2$, $\lambda = 1$ and $k_{2,0} = 0.05$.

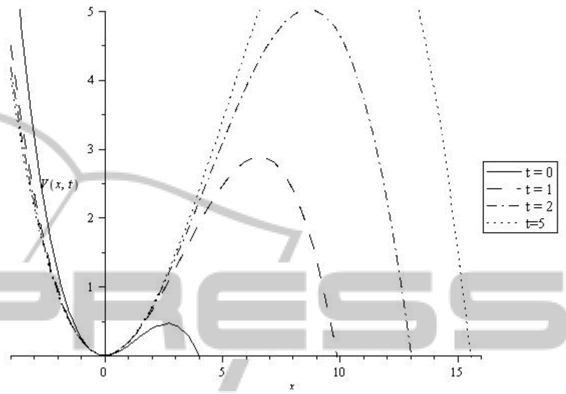
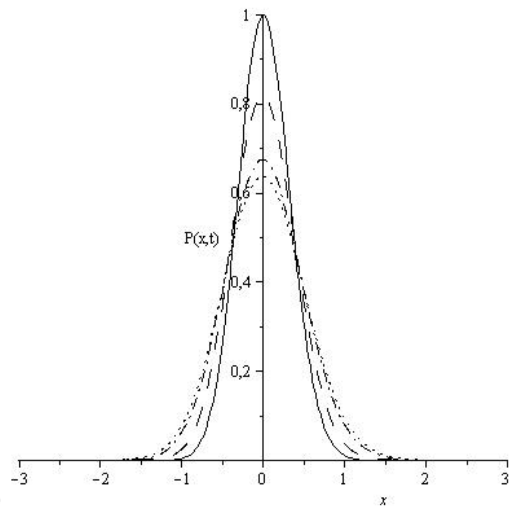


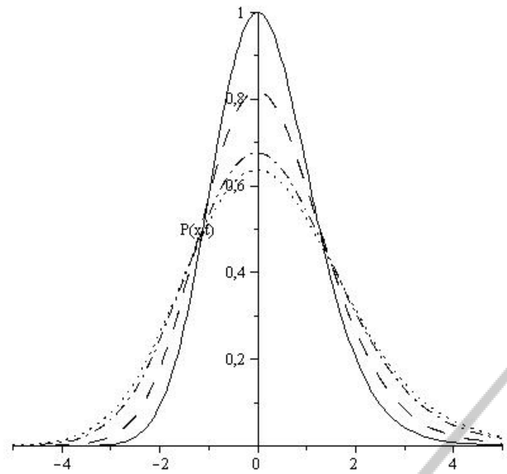
Figure 1: Curves of the potential V(x,t) defined in equation (2) for different values of time, $k_1 = 0.2$, $\lambda = 1$ and $k_{2,0} = 0.05$.

The distribution of probabilities, equation (5), is showed in the figure 2 for different values of time and diffusion constant. The values of k_1, λ and $k_{2,0}$ are the same used in the figure 1 ($k_1 = 0.2$, $\lambda = 1$ e $k_{2,0} = 0.05$).



a)

Figure 2: Distribution of probability for different times with a) Q = 0.1 and b) Q = 1.



b)

Time: —— t = 0.0; – – t = 0.5; – · t = 2.0; ·····t = 5.0

Figure 2: Distribution of probability for different times with a) Q = 0.1 and b) Q = 1. *(Cont.)*

3 DYNAMIC PROPERTIES OF THE SYSTEM

Important results for the transmission at synapses are the diffusion rate (r) and or time of passage through the barrier (τ). In order to compute these quantities, one consider the initial potential metastable (2) (t = 0), as showed in figure 3.

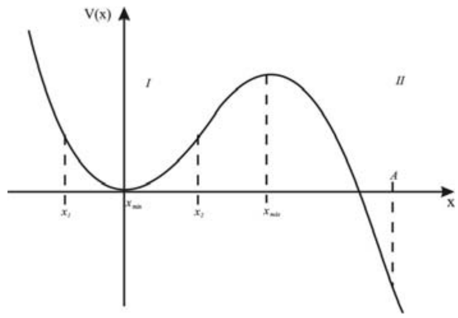


Figure 3: Representation of a metastable potential.

The figure 3 shows a maximum peak in x_{max} and a point of local minimum in the region I (x_{min}). The points x_1 and x_2 refer to a region I around the minimum x_{min} and A referes to a point in the second region, region II, after the barrier. The escape rate of particles through the barrier can be calculated from the relation,

$$r = \frac{J}{w}, \tag{7}$$

where J is the current probability or particle flow through a particular region and w represents the population within the region of minimum. They are defined, respectively, as

$$w(x,t) = \int_{x_1}^{x_2} P(x,t)dx \quad \text{e} \quad J(x,t) = \int_{x_{\min}}^{A} j(x,t)dx. \tag{8}$$

The function j(x,t) is obtained from the Fokker-Planck equation, such that,

$$j(x,t) = f(x)P(x,t) - Q\frac{\partial}{\partial x}P(x,t). \tag{9}$$

Then, the rate r can be rewritten as

$$r(t) = \int_{x_{\min}}^{A} j(x,t)dx \Bigg/ \int_{x_1}^{x_2} P(x,t)dx. \tag{10}$$

This literal definition allows obtaining the escape rate in the various situations of potential.

Another important quantity to characterize the diffusion through the barrier is the first passage time or particle escape time (τ) (Lenzi, 2001). This time corresponds to the inverse of the escape rate, i.e.:

$$r(t) = \frac{1}{\tau} \quad \Rightarrow \quad \tau = \int_{x_1}^{x_2} P(x,t)dx \Bigg/ \int_{x_{\min}}^{A} j(x,t)dx \tag{11}$$

4 NUMERICAL RESULTS

Using the solution for the Fokker-Planck equation (5), the curves obtained for the escape rate of particles (equation (10)) for different values of diffusion (Q) are presented in figure 4. The constants values are adopted as $k_1 = 0.2$, $\lambda = 1$ and $k_{2,0} = 0.05$.

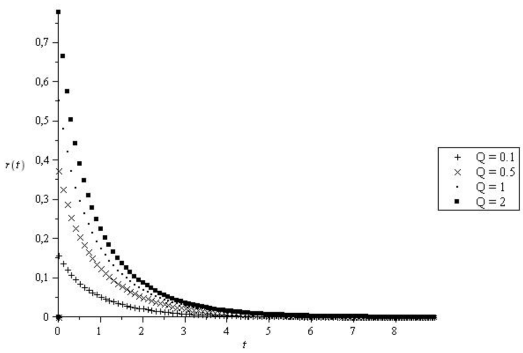


Figure 4: Curves of the rate escape (r) versus t for different values of Q with $k_1 = 0.2$, $\lambda = 1$ and $k_{2,0} = 0.05$.

In the figure 4 it is showed that the rate of diffusion of particles decays with time until the value zero. This behavior is associated with the potential that is not constant on time. For large value of time, the potential presents a high and large barrier. The result is the confinement of the particles.

This phenomenon is also observed by the first passage time (τ) of the particles through the barrier, equation (11). In the figure 5 the value of τ as a function of the time is plotted. Initially, the time is short featuring a fast passage of particles through the barrier, but this value increase due to the increased and diverges ($\tau \rightarrow \infty$) for large values of time.

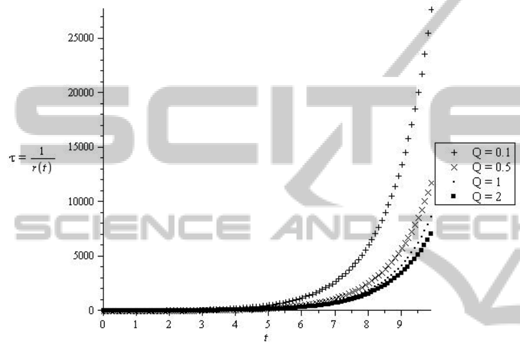


Figure 5: Escape time (τ) versus t for different values of Q, with $k_1 = 0.2$, $\lambda = 1$ and $k_{2,0} = 0.05$.

From the curves in figure 5, it is possible to observe that for large values of the Q the diffusion through the barrier become more fast and effective. In figure 4 we note that for a time t = 4 there is a small diffusion of particles when Q equal to 2, but for a low value of the diffusion coefficient (for instance, Q = 0.1) the diffusion is null for this time. Following the same reasoning, it is observed that for a low value of the diffusion coefficient, the time of first passage (figure 5) becomes long for large Q values.

5 CONCLUSIONS

The results obtained from the solution of the Fokker-Planck equation for the potential suggested (2) exhibit a behavior consistent with that expected from the electrical transmission at synapses. The flow of charge is most intense at the beginning of the signal and tends to vanish for large values of time. This result combined with the construction of the potential allows concluding that this approach can be effective to describe the dynamics of the physical process that occurs at synapses.

The models proposed in literature usually consider the diffusion through a fixed barrier of energy on time and the escape rate is calculated using only the results of the system at the stationary state (rate of Kramer (Risken, 1989, Hänggir, 1990)), which limits quite the applicability of these models. Then, the potential proposed here can be useful to build more realistic models.

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