

A HIERARCHICAL APPROACH BASED ON LINEAR AND STOCHASTIC PROGRAMMING FOR THE EMPTY CONTAINER ALLOCATION PROBLEM

Jorge Luis Morales B. and Ciro Alberto Amaya G.

Department of Industrial Engineering, Universidad de los Andes, Bogotá, Colombia

Keywords: Empty container repositioning, Empty container allocation, Stochastic programming, Sample average approximation.

Abstract: This paper proposes a solution method to the problem of allocating an empty container fleet to a set of stocking yards in order to minimize empty container stock and repositioning costs under uncertainties in demand, supply, container damages and repairing times. We propose an approximate solution for the problem based on a hierarchical approach. We used random data from different probability functions to generate problem instances and evaluate robustness and performance. We find that the proposed model solves the single location inventory problem in a very short time while obtaining high robustness and each one can be solved independently. This approach allows liners to reduce the complexity of an aggregate stochastic problem by solving multiple independent stochastic inventory problems. Additionally to other similar works, the presented models consider random container damages and repairing times.

1 INTRODUCTION

In the new scenarios of increasingly globalized commerce, repositioning and container allocation problems are recurrent; however, commercial and operational contingency decisions dominate tactical policies and planning, leading to operational inefficiencies.

Imbalances are frequently observed between empty containers' demand and supply. Consequently, stock of empty containers may be not enough if replenishment is not made. Liners need to face the costs of these imbalances by allocating their container fleets to different yards, and repositioning empty containers in order to replenish yards with stock-outs while unnecessary inventory from other yards is evacuated. This problem has been denominated as the *Dynamic Empty Container Allocation Problem*.

This paper proposes a solution method to this problem, and is organized as follows. In Section 2 we describe the problem and state the randomness issues of the problem. In Section 3 we present and describe the previous works that have tackled this problem. In Section 4 we propose an approximate solution for the *Dynamic Empty Container*

Allocation Problem based on a hierarchical approach applying linear and stochastic programming, presenting and describing in detail our proposed models and the hierarchical structure of the solution. In Section 5 we describe the performed experiments for testing robustness and performance, and the data structure used. Finally, Section 6 presents the conclusions and perspectives.

2 PROBLEM STATEMENT

2.1 General Problem Statements

The dynamic empty container allocation problem addressed in this paper considers a liner—decision maker—that operates in a network of container storage yards located in both ports and inland points. The liner also offers shipping services on fixed and periodic routes and owns a finite container fleet. Each yard has an associated exogenous demand that is met with empty container inventory available at each location.

Shipping routes are weekly cyclic. The fixed and periodic routes allow each yard to be reached from any other.

Replenishment of empty containers at each yard is done from imports, which must consider the time elapsed after a container is discharged at a port and returned empty by the client. The empty returned containers can also present damages, and they are not always immediately available to be assigned to demand. Alternatively for replenishment, empty containers can be repositioned from a storage point to another when necessary, or the liner can sublease a finite number of empty containers to meet inventory requirements.

2.2 Present Uncertainties

The described process has a high level of uncertainty over time. The major sources of randomness are given by the demand, supply, damages and repair times of empty containers. On the other hand, the supply of containers in a yard is uncertain, when containerized cargo is transported from one port or yard to another, it loses time in waiting, from the moment the client receives the cargo in the ending port to the moment it is returned empty to storage yards and equipment is available.

Additionally, there is the possibility of container damages, in which case it must be repaired. Finally, the repair of empty containers takes a random time, after which the equipment will be available in storage to be assigned to new demand.

3 RELATED LITERATURE

The literature related to this subject is relatively short, and not until after the work of Crainic, Gendreau and Dejax (1993) was the dynamic and stochastic problem first really tackled. They proposed both deterministic and stochastic dynamic models for both single-product and multi-product cases; for the deterministic model, a decomposition strategy was proposed in subsequent works (Abrache, Crainic and Gendreau, 1999).

Shen and Khoong (1995) proposed a hierarchical decision support system for the multi-period planning of the distribution of empty containers. Cheung and Chen (1998) formulated the problem through a two-stage stochastic network. Leung and Wu (2004) presented a robust optimization model for the stochastic problem. Li et al. (2007) considered implementation of a (U,D) inventory interval policy for the replenishment. Crainic, Di Francesco and Zuddas (2007) presented a mixed-integer programming model associated with a stochastic program. Chang et al. (2008) use a

decomposition technique and branch and bound. Belmecheri et al. (2009) used integer programming. Yuanhui et al. (2009) used a hybrid integer code hybrid genetic algorithm. Feng and Chang (2010) used a revenue management model. Song and Zhang (2010) modeled container transportation as a continuous fluid flow. Wang et al. (2010) included decision maker risk preference in a robust optimization model. Yang (2011) considered an integer programming allocation model. Finally, Shi and Xu (2011) presented a Markov decision process model for empty container repositioning.

4 MODEL DESCRIPTION

Parameters include random container demands, transit times, expected return times for containers, storing and transportation costs, leasing costs and capacities, random supplies, damage probabilities, and probability for repair times.

The proposed models is based on the following assumptions: a) The whole observed demand is met along the time horizon; b) Uncertainty is given by: demand, supply, damages, repair times; c) The random variables are independent and not self-correlated along the time horizon; d) There is no capacity limit for replenishment containers; e) Storage capacity in yards is unlimited; f) A returning yard or time period for subleased containers is not specified. Based on these assumptions and the topology shown above, two optimization programs will be presented.

4.1 Empty Container Aggregated Replenishment (Agre) Model

The objective of the model will be to minimize the costs of aggregate replenishment for empty containers.

The decisions that must be made along the planning horizon include the container stock to having available the in each yard, transportation of empty containers between yards, and containers subleasing.

Following this, a cost function can be defined considering storage, transportation and subcontracting costs. Contingency supplies must be also defined to guarantee the problem feasibility.

Constraints of the Agre model include the dynamic balance for the available stock of empty containers, arrivals during a given time period, subleasing capacities, contingency supplies, and safety stock.

The Agre model does not take into account the randomness of the parameters under uncertainty. The impact of this randomness must be captured by the safety stock that must be guaranteed at each storage point along the planning horizon. This minimum stock should be sufficient to deal with uncertainty. The model which estimates the minimum stock level is described in the next section.

4.2 Minimum Stock (Mist) Model

The empty container minimum stock (Mist) model should estimate a minimum inventory at each storage point, to meet the randomness of the problem.

Based on certain demand for initial stock at the first time period, we must make a decision on the inventory to stock. In each period, when making a decision, the history of previous decisions and realizations of random variables is known, ignoring the future demand and supply.

The cost function of the Mist model considers only the stock costs, specifically, the mean cost along the planning horizon. Note that, since the solution of depends on random parameters of demand and supply, then it also is random.

Contrary to the aggregated replenishment model, the demand and the supply are random variables with known probability functions. Additionally, the damage rate and the devolution times are considered probabilistically.

Solution of the Mist model provides the minimum container stock, which will be used as a parameter by the Agre model in order to capture the randomness of the system and face the uncertainty.

4.3 Solution to the Mist Model based on the Sample Average Approximation

In order to solve the Mist stochastic non-linear model described previously, the sample average approximation, or SAA, method is used. This method generates an approximation of the probability distribution of the random variables by Montecarlo simulation. On each period of the time horizon, realizations of the random variables are generated, about which decisions should be made.

The motivation for using this method lies in the following reasons: a) Multiple randomness sources exist, with heterogeneous probability distributions; b) The planning horizon considers multi-periods; c) The probability distributions of the random variables can be expressed through discrete functions; e) Based on the previously established assumption of

independence and no self-correlation for the random variables, random scenarios can be easily generated; f) Since the Mist model does not include binary or integer variables, the problem can be solved by a polynomial size linear program.

5 IMPLEMENTATION AND RESULTS

We code the models in GMPL to test the performance robustness of the model. It is important to note that the computational complexity of the solution is determined by the complexity of the Mist problem, given the large number of scenarios that have to be generated in order to simulate the randomness of the system. For an instance of one yard, one container type, a fourweek planning horizon, and a tree with 1,000 random scenarios, the matrix of the associated program has a size of 62,000 columns and 5,000 constraints before pre-processing.

We construct four data sets with four different demand probability distributions, empirical, normal, lognormal and uniform, for forty experiments corresponding to forty probability trees. To test the robustness of the model, the variation of the optimal solution for expected cost and average minimum stock over the four periods was recorded, using the sample variability coefficient as the estimator. To measure the performance of the model, solution times and memory usage were recorded; note that it is expected that the running frequency of the application is not greater than one run per week.

5.1 Conclusions and Perspectives

Inspired by the work of Shen and Khoong (1995), the model presented in this paper proposes a hierarchical solution approach to the Dynamic Empty Container Allocation Problem that divides the problem into two models: Mist, the estimation of the minimum stock necessary to deal with uncertainty at every storage point over a planning horizon, and Agre, the estimation of the flow of container replenishment to ensure compliance with those minimum stocks levels and business demand budgets. These models are also inspired by several of the works mentioned before and operating conditions of the specific problem described. However, unlike the works mentioned, the proposed models also consider the possibilities of observing damage to containers and repairs' random times.

The computational complexity of the application is given by the stochastic complexity of the Mist model. For this program, we introduced the sample average approximation method approach, which offers advantages regarding uncertainty capture by generating a large number of random scenarios given by realizations of the random variables considered. Through this method, the stochastic program became a stochastic linear program and the expected cost function was expressed as the sample average cost observed over the generated scenarios.

The computational performance and robustness of the models was tested, finding that the Mist model, which governs the complexity of the application, solved each of four runs and four data set instances in a negligible time, considering that the running frequency of the model should not be greater than one run per week. Also, the solution, of minimum stock values and optimal cost estimates, was found to show good robustness. Only the lognormal demand data set showed a high variation coefficient for the solution, due the dispersion of the lognormal distribution.

Further work may eliminate some assumptions, such as the infinite capacity of empty container shipping; include uncertainty in the capacity; and include constraints on the container leasing process, limiting the devolution time and location of leased equipment to a time window and a specific geographic location. Finally, is possible to perform a statistical analysis to generate a measurement of efficiency for the number of random scenarios that should be generated in the tree, and to estimate confidence intervals for the calculated solution.

REFERENCES

- Abrache J, Crainic T. G, Gendreau M, A New Decomposition Algorithm for the Deterministic Dynamic Allocation of Empty Containers (1999) *Université de Montréal Publication CRT-99-49*.
- Belmecheri F, Cagniard T, Amodeo L, Yalaoui F, Prins C (2009) Modelling and optimization of empty container reuse: a real case study. *Proc Int Conf Comput Ind Eng*, 1106-1109.
- Chang H, Jula H, Chassiakos A, Ioannou P (2008) A heuristic solution for the empty container substitution problem. *Tran Res Part E*, 44: 203-216.
- Cheung R. K, Chen C-Y (1998) A Two-Stage Stochastic Network Model and Solution Methods for the Dynamic Empty Container Allocation Problem. *Transp. Sci.* doi: 10.1287/trsc.32.2.142.
- Crainic G, Gendreau M, Dejax P (1993) Dynamic and Stochastic Models for the Allocation of Empty Containers. *Oper. Res.* doi: 10.1287/opre.41.1.102
- Crainic T. G, Di Francesco M, Zuddas P (2007) An Optimization Model for Empty Container Reposition under Uncertainty. *The Sixth Triennial Symposium on Transportation Analysis*, Phuket Island, Thailand.
- Di Francesco M, Crainic T. G., Zuddas P (2009) The effect of multi-scenario policies on empty container repositioning. *Transp Res Part E*, 45: 758-770.
- Feng C-M, Chang C-H (2010) Optimal slot allocation with empty container reposition problem for Asia ocean carriers. *Int J Shipp Transp Logist* Vol 2, 1:22-43.
- Leung S. C, Wu Y (2004) A Robust Optimization Model for Dynamic Empty Container Allocation Problems in an Uncertain Environment. *Intern J Oper Quant Manag.* 10(4): 315-334.
- Li J-A, Leung S C, Wu Y, Liu K (2007) Allocation of Empty Containers Between Multi-ports. *Eur J Oper Res.* doi: 10.1016/j.ejor.2006.09.003.
- Shen W, Khoong C (1995) A DSS for empty container distribution panning. *Decis Support Syst* 15: 75-82.
- Shi N, Xu D (2011) A Markov Decision Process Model for an Online Empty Container Repositioning Problem in a Two-port Fixed Route. *Int J Oper Res* Vol. 8, 2:8-17.
- Song D-P, Zhang Q (2010) A Fluid Flow Model for Empty Container Repositioning Policy with a Single Port and Stochastic Demand. *SIAM J. Control Optim* 48: 3623-3642.
- Wang et al (2010) Robust optimization for empty container allocation problems under uncertainties. *29th Chin Control Conf* 1791-1796.
- Yang Y (2011) Integer Programming Allocation Model of Empty Container by Sea Transportation for Liner Operators' Cooperation. *J Shanghai Jiaotong Univ* Vol 45, 1:120-124.
- Yuanhui Li, Jingjing Cao, Jun Li, Jian Sun (2009) Optimizing empty container allocation based on hybrid genetic algorithm. *Int Conf Transp Eng*, 1548- 1553. doi : 10.1061/41039(345)256.