A SIMPLE DERIVATION TO IMPLEMENT TRACKING DISTRIBUTIONS

Wei Yu, Jifeng Ning, Nan Geng and Jiong Zhang
College of Information Engineering, Northwest A&F University, No.22 Xinong Road, Yangling, China

Keywords: Tracking, Euler-Lagrange, Active Contours, Level Set.

Abstract: We present a simple and straightforward derivation to implement active contours for tracking distributions (Freedman and Zhang, 2004) and its improvement, i.e., distribution tracking through background mismatch (Zhang and Freedman, 2005). In the original work, two steps are performed in order to derive the tracking evolution equations. In the first step, curve flows are derived using Green's Theorem, and in the second step level set method is used to implement the curve flows, which seems to be somewhat complex. In our implementation, tracking evolution equations are derived directly by using variational theory. This is useful to understand the tracking method better. The final tracking evolution equations are identical to the previous work (Freedman and Zhang, 2004; Zhang and Freedman, 2005).

1 INTRODUCTION

Determining the location of an object contour is an important research topic in the field of object tracking. Tracking methods based on level set theory have received extensive attention (Freedman and Zhang, 2004; Zhang and Freedman, 2005; Bibby and Reid, 2008; Zhou et al., 2007; Prisacariu and Reid, 2011; Cremers, 2006; Fussenegger et al., 2006; Allili and Ziou, 2007) because level set method can implicitly represent almost any kind of contour and evolve an active contour naturally.

In the paper (Freedman and Zhang, 2004), authors devise trackers using distribution distances with three criteria, Kullback-Leibler distance, Bhattacharyya measure, and self-developed "simple criterion". To extremize distribution distances, first, they propose a proposition by converting a distribution distance from an area integral to a parameterized curve integral using Green's Theorem and calculus of variations which is very sophisticated. The representation of a parameterized curve is in a two dimensional space. So knowledge of differential geometry is required in this deduction. After the first step, a curve evolution equation is attained using the gradient method. Finally, implementation of the curve evolution function uses the level set method to attain the tracking evolution function. So the curve evolution is just an intermediate product and a tool for deduction. But this makes the deduction hard to be understood by readers. In the paper (Zhang and Freedman, 2005), a background density mismatching term is added to the original equation (Freedman and Zhang, 2004). Although the principle of background density mismatching term is similar to that of foreground density matching term, it allows the algorithm utilise more information so that it increases the robustness further.

In this paper, we present a new way to deduce a level set evolution equation without introduction of the curve flow. This is a really simple method since it only uses the knowledge of variational theory. Furthermore, this method can calculate a tracking evolution equation directly which reduces the difficulty of understanding the algorithm of tracking distributions. Although the derivation methods of Daniel Freedman's and ours are different, the final tracking evolution equations are the same.

2 BRIEF INTRODUCTION TO TRACKING DISTRIBUTIONS

2.1 Density Matching Criteria

In (Freedman and Zhang, 2004), Daniel Freedman et al. propose a new level set based tracking algorithm which finds the object of interest using foreground match. The tracker aims at finding the region ω in a given image such that the sample distribution

Yu W., Ning J., Geng N. and Zhang J..
A SIMPLE DERIVATION TO IMPLEMENT TRACKING DISTRIBUTIONS.
DOI: 10.5220/0003816603510354
In *Proceedings of the International Conference on Computer Vision Theory and Applications* (VISAPP-2012), pages 351-354
ISBN: 978-989-8565-04-4

(sample probability density) P_ω most closely matches the corresponding model distribution (model probability density) P_{std}. Daniel Freedman (Freedman and Zhang, 2004) verifies three criteria. In the following text, we introduce the algorithm (Freedman and Zhang, 2004) with Kullback-Leibler distance since this criterion is the simplest one to implement tracking distributions and this criterion is also used in the paper (Zhang and Freedman, 2005).

Consider a given image I to be a mapping from $\Omega \subseteq R^2$ in a plane coordinates system to y which could be an intensity, color vector and so on. The current sample probability density (e.g., RGB histogram) of a region of interest is denoted by $P_\omega(y)$ where $\omega \subseteq \Omega$ denotes the region of interest in the image.

Thus, a sample probability density $P_\omega(y)$ is given by

$$P_\omega(y) = \frac{\int_\omega K(y - I(x))\,dx}{\int_\omega dx} = \frac{N(y;\omega)}{A(\omega)} \qquad (1)$$

where $K(\cdot)$ is an n-dimensional function, e.g., for RGB histogram, $n = 3$ (RGB vector).

$$K(a) = \begin{cases} 1 & a = \overrightarrow{0} \\ \overrightarrow{0} & otherwise \end{cases} \qquad (2)$$

$A(\omega)$ is the area of the region ω and $N(y,\omega)$ denotes the number of pixels of y in the region ω.

For Kullback-Leibler distance:

$$D(\omega) = \int P_{std}(y)\, log\left(\frac{P_{std}(y)}{P_\omega(y)}\right) dy \qquad (3)$$

For Bhattacharyya measure:

$$D(w) = \int \sqrt{p_{std}(y)\, p_\omega(y)}\, dy \qquad (4)$$

It is clear to see that the more closely they match, the smaller the Kullback-Leibler distance is and the bigger the Bhattacharyya measure is. So the object of tracking turns to extremizing D.

2.2 Curve flow

In order to attain the extremal, how to evolve the active contours is taken into account. First, a proposition is proposed as follows:

Proposition: Let ω be an elementary region of R^2, let $c = \partial\omega$ be its boundary, and let $\Gamma(\omega) = \int_\omega \mu(x)\,dx$, where μ is C^1. Additionally, let $\frac{\delta\Gamma}{\delta c}$ be a 2-vector whose i^{th} component is the variational derivative $\frac{\delta\Gamma}{\delta c_i}$, assuming a particular parameterization for c. Then there exists a parameterization of c for which

$$\frac{\delta\Gamma}{\delta c} \propto \mu(c)\,\vec{n} \qquad (5)$$

where $\vec{n}$ is the normal to c.

This proposition is used to calculate curve flows. Taking the Kullback-Leibler Flow for example,use the above proposition to calculate $\frac{\delta D}{\delta c}$:

$$\frac{\delta D}{\delta c} = \frac{P_\omega(I(c)) - P_{std}(I(c))}{N(I(c))}\vec{n} \qquad (6)$$

Attain the gradient descent flow using the method of steepest descent:

$$\frac{\partial c}{\partial t} = -\frac{\delta D}{\delta c} = \frac{P_{std}(I(c)) - P_\omega(I(c))}{N(I(c))}\vec{n} \qquad (7)$$

For Bhattacharyya measure, tracking turns to maximising $D(\omega)$. In a similar way, attain Bhattacharyya gradient ascent flow:

$$\frac{\partial c}{\partial t} = \frac{\delta D}{\delta c} = \frac{1}{2A(\omega)}\left[\frac{\sqrt{P_{std}(I(c))}}{\sqrt{P_\omega(I(c))}} - D(\omega)\right]\vec{n} \qquad (8)$$

To increase robustness, background density mismatching is introduced (Zhang and Freedman, 2005). Background density mismatching is to maximise the disparity of probability density between the region of interest and the background $\Omega\backslash\omega$. Therefore, a curve flow becomes a combination flow based on both background-mismatching and foreground-matching.

2.3 Implementation with Level Set

Then these flows are implemented using the level set framework (Osher and Sethian, 1988) for its unparalleled advantages in topology.

3 OUR METHOD

In this part, we take Kullback-Leibler distance for example to present our deduction.

3.1 Implementation of Energy with Level Set

In level set theory, the boundary of a region of interest is a curve c which is represented in 3-dimension space. One more dimension ingeniously allows automatically topological changes, such as merging and breaking.

Let $\omega \subseteq R^2$ be a region of interest in a given image. Then the curve c (the boundary of ω, i.e., $c = \partial\omega$) is represented as the zero level set of a scalar Lipschitz continuous function $\phi : R^2 \to R$. The level set function ϕ is usually defined as a signed distance function for the sake of stability.

$$\phi(x) = \begin{cases} > 0 & in\, \omega \\ < 0 & in\, \Omega \backslash \omega \\ = 0 & on\, \partial\omega \end{cases} \tag{9}$$

where $x \in \Omega \subseteq R^2$ in a plane coordinates system of a given image.

In order to represent above equations by the level set function, we need to introduce the Heaviside function H:

$$H\{z\} = \begin{cases} 1 & z \geq 0 \\ 0 & z < 0 \end{cases} \tag{10}$$

Thus the level set formulation of equation (1) is:

$$P_\omega(y) = \frac{\int_\Omega K(y - I(x)) H(\phi)\, dx}{\int_\Omega H(\phi)\, dx} \tag{11}$$

In our method, equation (3) is referred to as an energy functional. Minimising this energy is equivalent to finding a region which represents the closest distribution as the model distribution (i.e., prior distribution). Then equation (3) can be implemented by level set framework:

$$E(\phi) = \sum_y P_{std}(y) \log \left(\frac{P_{std}(y)}{\frac{\int_\Omega K(y-I(x))H(\phi)dx}{\int_\Omega H(\phi)dx}} \right) \tag{12}$$

Equation (12) is a fraction-type energy functional for level set which is different from classic equations (Chan and Vese, 2001; Vese and Chan, 2002; Li et al., 2007; Zhang et al., 2010).

3.2 Euler-Lagrange Differential Equation

Calculus of variations (Wei-chang, 1980) is a common tool to search for a function that minimizes a certain functional. The Euler-Lagrange differential equation is the fundamental equation of calculus of variations.

$E(\phi)$ is a functional and we want to find the ϕ minimizing $E(\phi)$. When $E(\phi)$ reaches its extreme, the segmentation gets the ideal result (The probability density of the region $\phi \geq 0$ is the same as the model probability density). A necessary condition for ϕ to yield the minimum of E is : $\delta E = 0$, where δ is the first variation of E. In terms of the algorithms of calculus of variations, we have:

$$\delta E = -\sum_y \frac{P_{std}(y)}{P_\omega} \delta P_\omega(y) \tag{13}$$

$$\delta P_\omega(y) = \frac{A(\omega)\,\delta N(y;\omega) - N(y;\omega)\,\delta A(\omega)}{A(\omega)^2} \tag{14}$$

So, we attain:

$$\delta E = -\sum_y \frac{P_{std}(y)}{N(y;\omega)} \delta N(y;\omega) - \frac{P_{std}(y)}{A(\omega)} \delta A(\omega) \tag{15}$$

where

$$\delta N(y;\omega) = \int_\Omega K(y - I(x))\, \delta(\phi)\, \delta\phi dx \tag{16}$$

$$\delta A(\omega) = \int_\Omega \delta(\phi)\, \delta\phi dx \tag{17}$$

and $\delta(\phi)$ is the Dirac delta function $\delta(z) = \frac{d}{dz} H(z)$, $\delta\phi$ is the first variation of ϕ.

Combine these equations (13)-(17) above and take account of the arbitrariness of $\delta\phi$, then we get the Euler-Lagrange differential equation:

$$\frac{P_\omega(I(x)) - P_{std}(I(x))}{N(I(x), \omega)} \delta(\phi(x)) = 0 \tag{18}$$

So gradient descent with respect to the Euler-Lagrange differential yields the following evolution:

$$\frac{\partial\phi(x)}{\partial t} = \frac{P_{std}(I(x)) - P_\omega(I(x))}{N(I(x), \omega)} \delta(\phi(x)) \tag{19}$$

We also calculate the evolution equation of the energy model using Bhattacharyya measure which is the same as the results of the work (Freedman and Zhang, 2004; Zhang and Freedman, 2005).

In Bhattacharyya criterion-based model, we attain the Euler-Lagrange differential equation as follows:

$$\frac{1}{2A(\omega)} \left[\sqrt{\frac{p_{std}(I(x))}{p_\omega(I(x))}} - D(\omega) \right] \delta(\phi(x)) = 0 \tag{20}$$

and its tracking evolution equation is:

$$\frac{\partial\phi}{\partial t} = \frac{1}{2A(\omega)} \left[\sqrt{\frac{p_{std}(I(x))}{p_\omega(I(x))}} - D(\omega) \right] \delta(\phi(x)) \tag{21}$$

So comparing the equations above, equations (18) and (20) correspond to equations (7) and (8). After equations (7) and (8) is implemented by level set method, two deductions get the same results as equation (19) and equation (21).

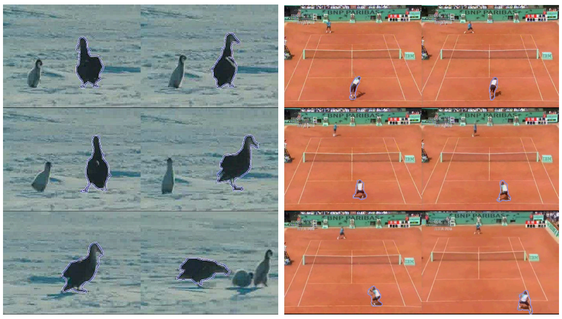

Figure 1: Left: Tracking results of bird sequence with Kullback-Leibler criterion. Frame 20, 40, 60, 80, 100 and 120 are displayed.(resolution of 320x240, 25 FPS) Rright: Tracking results of player sequence with Kullback-Leibler criterion. Frame 2, 13, 24, 35, 46 and 79 are displayed.(resolution of 512x380, 25 FPS)

4 EXPERIMENTAL RESULTS

We used an Intel Core2 E7300 (2.66GHz) machine to run all our experiments using the algorithm of distribution tracking through background mismatch (Zhang and Freedman, 2005). The model distributions are built as 8-bin RGB histograms out of the bird and out of the player taken from the first frame respectively. We show 2 experimental results in Fig.1. The results show that this algorithm can evolve the boundary of the tracking object correctly although there exist large non-rigid deformations.

5 CONCLUSIONS

In this paper, we present a simple way of deduction to implement two important tracking algorithms (Freedman and Zhang, 2004; Zhang and Freedman, 2005) based on level set theory. Our deduction results are identical to the previous work (Freedman and Zhang, 2004; Zhang and Freedman, 2005). Further, our evolution equations for level set are deduced in a straightforward and direct way. This way of deriving an evolution equation can provide readers with an intuitive explanation of the foreground density matching algorithm and the background density mismatching algorithm, which helps understand and uses these two algorithms better.

ACKNOWLEDGEMENTS

This work is partially supported by the National Science Foundation of China (NSFC) under Grant No.61003151, the Fundamental Research Funds for the Central Universities under Grant No.QN2009091, Northwest A&F University Research Foundation under Grant No.Z111020902 and the International Cooperation Foundation of Northwest A&F University.

REFERENCES

Allili, M. S. and Ziou, D. (2007). Active contours for video object tracking using region, boundary and shape information. *Signal, Image and Video Processing*, 1(2):101–117.

Bibby, C. and Reid, I. (2008). Robust real-time visual tracking using pixel-wise posteriors. *Proceedings of the European Conference on Computer Vision (ECCV) 2008*, page 831–844.

Chan, T. F. and Vese, L. A. (2001). Active contours without edges. *IEEE Transactions on Image Processing*, 10(2):266–277.

Cremers, D. (2006). Dynamical statistical shape priors for level set-based tracking. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, page 1262–1273.

Freedman, D. and Zhang, T. (2004). Active contours for tracking distributions. *IEEE Transactions on Image Processing*, 13(4):518–526.

Fussenegger, M., Deriche, R., and Pinz, A. (2006). Multiregion level set tracking with transformation invariant shape priors. *Proceedings of Asian Conference on Computer Vision 2006*, page 674–683.

Li, C., Kao, C., Gore, J., and Ding, Z. (2007). Implicit active contours driven by local binary fitting energy. In *2007 IEEE Conference on Computer Vision and Pattern Recognition*, page 1–7.

Osher, S. and Sethian, J. A. (1988). Fronts propagating with curvature-dependent speed: algorithms based on Hamilton-Jacobi formulations. *Journal of computational physics*, 79(1):12–49.

Prisacariu, V. and Reid, I. (2011). Nonlinear shape manifolds as shape priors in level set segmentation and tracking.

Vese, L. A. and Chan, T. F. (2002). A multiphase level set framework for image segmentation using the mumford and shah model. *International Journal of Computer Vision*, 50(3):271–293.

Wei-chang, Q. (1980). *Calculus of variations and finite element [M]*. Beijing: Science Press.

Zhang, K., Song, H., and Zhang, L. (2010). Active contours driven by local image fitting energy. *Pattern Recognition*, 43(4):1199–1206.

Zhang, T. and Freedman, D. (2005). Improving performance of distribution tracking through background mismatch. *IEEE transactions on pattern analysis and machine intelligence*, page 282–287.

Zhou, X., Hu, W., and Li, X. (2007). An adaptive shape subspace model for level set-based object tracking. In *Subspace 2007. Workshop on Asian Conference on Computer Vision*, page 9–16.