VISUALISING SMALL WORLD GRAPHS
Agglomerative Clustering of Small World Graphs around Nodes of Interest

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Abstract: Many graphs which model real-world systems are characterised by a high edge density and the small world properties of a low diameter and a high clustering coefficient. In the ”small world” class of graphs, the connectivity of nodes follows a power-law distribution with some nodes of high degree acting as hubs. While current layout algorithms are capable of displaying two dimensional node-link visualisations of large data sets, the results for dense small world graphs can be aesthetically unpleasant and difficult to read, due to the high level of clutter caused by graph edges. We propose an agglomerative clustering which allows the user to select nodes of interest to form the basis of clusters, using a heuristic to determine which cluster each node belongs to. We have tested three heuristics, based on existing graph metrics, on small world graphs of varying size and density. Our results indicate that maximising the average cluster clustering coefficient produces clusters that score well on modularity while consisting of a set of strongly related nodes. We also provide a comparison between our clustering coefficient heuristic agglomerative approach and Newman and Girvan’s top-down Edge Betweenness Centrality clustering algorithm.

1 INTRODUCTION

Many real-world networks across different fields have similar characteristics and can be classified as small world graphs (Watts and Strogatz, 1998). Small world networks are characterised by two properties. The first is the average of the shortest path between each pair of vertices for the entire graph. The second property is the average local clustering coefficient of the graph, which is defined as the average of the clustering coefficients for each vertex. Given an undirected graph $G = (V,E)$, where V is a set of vertices $\{v_1, v_2...v_n\}$ and E is a set of edges $e \in E$ connecting vertices $x \in V$ and $y \in V$with $e(x,y) = e(y,x)$, the *neighbourhood* of a vertex v, denoted Γ_v is defined as the set of all vertices adjacent to v, not including v itself. The *clustering coefficient* for a vertex , denoted by γ_v is most commonly defined as the ratio of edges connecting the neighbours of a vertex to the maximum number of edges that could possibly connect the neighbours of the vertex (Watts, 2003). The clustering coefficient c for a vertex v in an undirected graph is given by

$$\gamma_v = \frac{|E(\Gamma_v)|}{\binom{k_v}{2}} \tag{1}$$

where $|E(\Gamma_v)|$ is the magnitude of the set of edges connecting neighbours of the vertex, k is the neighbourhood size of the vertex, i.e.$|\Gamma_v|$and $\binom{k_v}{2}$ is the maximum possible number of edges in Γ_v. From the above it can be seen that a vertex needs at least two neighbours to have a valid clustering coefficient value. To determine if a graph can be considered a small world graph, it is compared to a randomly generated graph with the same number of vertices and edges. A small world graph has approximately the same average path length, but a considerably higher (by orders of magnitude) clustering coefficient.

1.1 Motivation

Our motivation is to make graphs more comprehensible. We are focusing on small world graphs specifically due to the presence of groups of highly connected nodes and the strong likelihood of clusters within the graph as well as the common occurrence of small world properties in real world networks. If a user has nodes of specific interest to them, reorganising the layout of the graph based on the nodes of interest may aid in their analysis. For example a user may want to view a graph describing a large program focusing on specific classes. The purpose of our clus-

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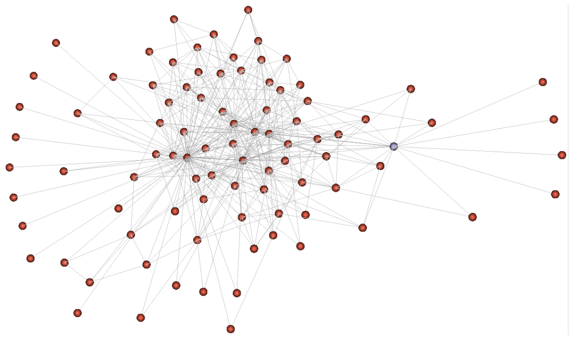


Figure 1: A small world graph based on the connection between a small set of Wikipedia articles ($|V| = 91|E| = 418$) laid out using a simple force directed algorithm.

tering approach is to aid in the layout by clustering nodes around the user's nodes of interest. The clustering assigns nodes in such a way that they are clustered around nodes that they are more conceptually related to. If grouping a node with one node set over another results in a higher heuristic score for that clusters, we can infer that the node conceptually belongs more to that set. In less dense graphs a clustering may be obvious as there will be few links between clusters. However for more dense graphs useful clusterings may not be so obvious and the density of edges can make the graph more difficult to read. The purpose of this paper is to determine what heuristic is best suited for our approach to clustering.

1.1.1 Edge Density

The links in a node link visualisation convey important information. However if they become too dense the graph becomes less comprehensible, resulting in nodes and other links becoming obscured. In terms of graph theory the density of a graph is usually considered the ratio of edges to the maximum possible number of edges in the graph (Coleman and Mor, 1983). For an undirected graph this can be described as

$$d = \frac{|E|}{|(V|(|V|-1)/2)} \tag{2}$$

A graph is then considered dense in theoretical terms if this ratio approaches 1.0. However in practical real world examples of information visualisation such dense graphs are rarely seen. If we consider the density of a graph to be the ratio of edges to nodes $d_l = |E|/|V|$, often referred to as the linear density, most real graphs have a value of $d_l <= 10$ (Melancon, 2006), however this is still enough to cause a large amount of clutter. Melacon et al. give an example of real world graphs which have even higher densities, such as webcrawl graphs with $d_l = 25.57$. The

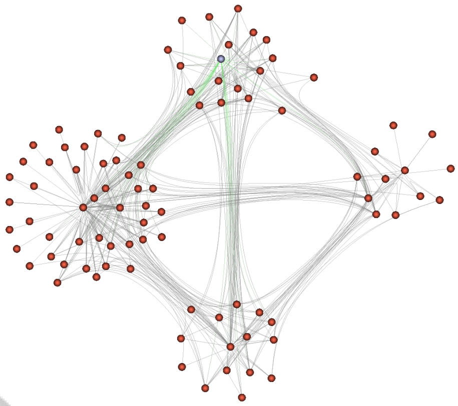


Figure 2: The graph from figure 1 clustered using our approach around the four most well connected nodes, using clustering coefficient as a clustering heuristic, and bundling(Holten, 2006) of inter-cluster edges.

graph in figure 1 has a density value $d_l = 4.59$, which, while not the most dense example, still appears difficult to read due to the number of edges. It is clear that graph theoretic density scales the number of edges more dramatically for a change in vertex count, so for comparison of densities between graphs with different node counts linear density provides a clearer result. However for understanding the impact of density on graphs with a constant node count, the graph theoretic density is clearer as it does scale evenly between the maximum and minimum edge count. Purchase (Purchase, 1997) has demonstrated how the crossing of edges is the graph aesthetic which affects most human understanding of the graph. Unfortunately in large dense graphs edge crossings are unavoidable. We hope that by clustering the graph intelligently, strongly related nodes will appear closer to each other within the same cluster. This will reduce long edges and the likelihood of edge crossings.

2 RELATED WORK

2.1 Small World Graphs

Milgram (Milgram, 1967) first described small world graphs in his work focused on social networks. The concept was more recently revived by (Watts and Strogatz, 1998) and has been shown to hold true for a variety of networks, such as the relationships between actors and films (Auber et al., 2003) as well as computer systems (Cai-Feng, 2009), models of biological networks(Watts and Strogatz, 1998) and citation networks (van Ham, 2004).

2.2 Clustering

(Eades and Feng, 1997) describe clustered graphs as graphs with recursive clustering structure over the vertices. In their example the clustering structure is an attribute of the graphs and vertices. However in many cases, if a graph is to be clustered, there may be no intrinsic attribute or parameter which describes the clustering hierarchy. There are many different approaches to generating an optimum clustering as it is a difficult problem that is NP-complete (Newman and Girvan, 2004). Approaches to graph clustering can be considered geometric or non-geometric. The aim of *geometric clustering* is to have vertices that are geometrically close to each other share a cluster and vertices that are distant from each other appear in separate clusters. An example of such a clustering is given by Quigley and Eades' FADE algorithm (Quigley and Eades, 2001) in which a quad-tree is used alongside a modified force directed algorithm. There are many different methods of non-geometric clustering. Some methods such as Markov Clustering (MCL) (Van Dongen, 2000) and spectral portioning (Frishman and Tal, 2007) use an algebraic approach, working on a mathematical representation of the graph. Other methods such as Edge Betweenness Centrality Clustering (Newman and Girvan, 2004) use a graph based approach, calculating properties of vertices or edges that are then used to partition the graph into clusters. Quigley and Eades' geometric approach can be considered a bottom up (agglomerative) approach while Newman and Girvan's is considered top-down (divisive) approach. An agglomerative clustering algorithm merges set of nodes together to form clusters, a divisive approach divides the full set of nodes into clusters. Schaeffer (Schaeffer, 2007) provides an in depth review of clustering methods and methods for evaluating cluster quality.

2.2.1 Clustering Evaluation

Newman and Girvan (Newman and Girvan, 2004) define a measure of the quality of a division of a network graph, referred to as modularity. The measure is used to evaluate their community detection algorithm (which is essentially a top-down clustering algorithm). The measure has also been used in work by Newman (Newman, 2004) as a heuristic value for building clusters. This metric is based upon the number of edges that start and end in the same cluster (referred to as communities in Newman and Girvan's paper). Auber et al (Auber et al., 2003) and Chiricota et al. (Chiricota et al., 2003) use a quality measure developed by (Mancoridis et al., 1998) and utilised in their clustering tool "Bunch". This measure, denoted *MQ (Modularisation Quantity)* computes a value for any given partition of a graph. Chiricota et al. and Auber et al. use a slightly modified version of MQ that is defined only for undirected graphs as an evaluation measure. The MQ value is used by the Bunch tool as a function to be optimised to provide a good clustering, rather than as a metric to evaluate one. Boutin and Hascoet (Boutin and Hascoet, 2004) discuss many clustering evaluation approaches (referred to by them as clustering validation indices) and they note that these evaluations are often difficult to interpret and compare.

Difference between Modularity and Modularisation Quantity. The MQ metric differs to Newman and Girvan's modularity measure. The latter compares the fraction of all edges that are intra-cluster edges to the fraction of all edges that are inter-cluster edges. The former is a measure of the difference between the average ratio of actual intra-cluster edges to the maximum amount of intra-cluster edges possible and the average ratio of the amount of inter-cluster edges to the maximum amount of inter-cluster edges possible. This means that modularity depends purely on the number of edges, which is bounded to the number of nodes. MQ depends on the number of edges and the number of nodes directly, as the maximum possible number of edges between two clusters is a function of the number of vertices.

3 PROPOSED APPROACH

Our approach consists of an agglomerative clustering algorithm, focused on nodes of interest selected by the user. We grow the clusters around each of these nodes of interest by adding nodes based on a heuristic evaluation of the quality of the resulting clustering of the graph.

3.1 Chosen Heuristics

Modularity. Per Newman and Girvan (Newman and Girvan, 2004) the modularity, Q, is calculated as

$$Q = \sum_i (e_i i - a_i^2) \tag{3}$$

Where $e_i i$ is the fraction of all edges that start and end in cluster i and a_i is the fraction of all edges that terminate in cluster i. A modularity score of 1.0 indicates that all edges are intra-cluster edges, a score of 0.0 indicates the clustering is equivalent to a random one. Newman (Newman, 2004) uses modularity as a guiding heuristic for a greedy agglomerative clustering process.

Modularisation Quantity (MQ). We calculate MQ using Auber et al's approach (Auber et al., 2003) for undirected graphs. Let A and B be two sets of disjoint nodes in a graph $G = (V, E)$, let s equal the ratio of edges between the two sets to the maximum possible number of edges between the two sets.

$$s(A,B) = \frac{|E(A,B)|}{|A| \cdot |B|} \tag{4}$$

Note that this ratio can be calculated for a set with itself. For a cluster A in an undirected Graph without self linking edges

$$s(A,A) = \frac{2(|E(A,A)|)}{|A| \cdot (|A|-1)} \tag{5}$$

If cluster A is a *clique* $s(A,A) = 1$. A *clique* is a set of nodes where every node is connected to every other node in the set. If none of the nodes in A are connected $s(A,A) = 0$. If a cluster contains only a single node, we define the s-value for that cluster to be 0. Given a partition (also referred to as a clustering) $C = (C_1, C_2, ..., C_p)$ that divides the graph $G = (V, E)$ into p partitions the MQ score for that partition is given by:

$$MQ(C;G) = \frac{\sum_{i=1}^{p} s(C_i,C_i)}{p} - \frac{\sum_{i=1}^{p-1}\sum_{j=i+1}^{p} s(C_i,C_j)}{p(p-1)/2} \tag{6}$$

Essentially this is a measure of the difference between the s ratio of intra-cluster edges (denoted by $s(C_i,C_i)$ and the s ratio of inter-cluster edges (denoted by $s(C_i,C_j)$.

Clustering Coefficient. The clustering coefficient of a graph is described in section 1. When we calculate the average clustering coefficient of a cluster, we only consider the nodes and their neighbours from within that cluster. The average clustering coefficient of a cluster describes how well inter-connected the cluster is. This implies that the higher the clustering coefficient of a cluster the more strongly related the nodes within the cluster are. Unlike the previous two metrics, this metric only applies to a single cluster and not to all of the clusters so far defined within the graph. Therefore a high average of the average clustering coefficient for each cluster does not imply that all clusters have a high average clustering coefficient. A large standard deviation between the average clustering coefficients of the clusters indicates that some clusters have been created with a low quality of clustering. In the cases where a clustering using a heuristic other than clustering coefficient produces clusters containing only one or two nodes, it is not possible to calculate the average clustering coefficient of the cluster so we assign the cluster a clustering coefficient of -1.0. This results in a suitably decreased score that reflects to poor quality of the clustering, when rating the graph using the average cluster clustering coefficient.

Random Assignment. In order to provide a comparison clustering in which no heuristic is used, we have also implemented a random assignment of nodes to clusters. A node is chosen at random from the combined neighbourhood of the clusters and then randomly assigned to one of the clusters that it is connected to. The process is then repeated until all nodes are assigned. Nodes can only be assigned to clusters in which they have a neighbour, to allow a reasonable comparison with the preceding heuristics.

3.2 Initial Cluster Set Up

The initial set of nodes that are to form the basis of the clustering is selected by the user. We term these nodes supernodes. If our heuristic is either modularity or MQ, or we are using the random approach, we can begin to add further nodes to the clusters once the initial cluster nodes have been specified. This is because it is possible to calculate modularity and MQ heuristic value, or randomly choose a node, if a cluster contains only 1 node. However this is not the case for the clustering coefficient heuristic, as we need to have at least two nodes existing already in the cluster before we can calculate a valid heuristic value for a new node being added. Therefore, before we start adding candidate nodes to the cluster using clustering coefficient as a heuristic we need to add a second node to the cluster of each of the supernodes. The nodes that are considered to be added are nodes within the neighbourhood of the supernode. We would like to add a node that is similar as possible to each of the supernodes, so we use the *Jaccard index* of the supernode and the candidate second node's respective neighbourhoods. The Jaccard index of two sets of nodes A and B, ρ is defined as:

$$\rho(A,B) = \frac{|A| \cap |B|}{|A| \cup |B|} \tag{7}$$

If the super-node is denoted by v and the candidate node is denoted by u we can write the Jaccard index as

$$\rho(\Gamma(v),\Gamma(u)) = \frac{\Gamma(v) \cap \Gamma(u)}{\Gamma(v) \cup \Gamma(u)} \tag{8}$$

The node which is used as the secondary node of the cluster based around v is the node u for which $\rho(\Gamma(v),\Gamma(u))$ is the largest. Ideally we aim to select a node where the neighbourhood Jaccard index is 1. If the node chosen has already been assigned as a neighbour of one of the other supernodes, we assign the node to the supernode which results in the

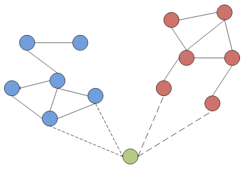


Figure 3: An example considering whether the green node should be clustered with either the red or blue clusters using the clustering coefficient heuristic. The green node is added to the blue cluster increasing the cluster's local average clustering coefficient to 0.48. Adding it to the red cluster would reduce its average local clustering coefficient to 0.33.

best Jaccard neighbourhood index. The supernode with the lower resulting Jaccard index is assigned the node with the next highest resulting Jaccard index. If this replacement node has also been assigned, we repeat the revaluation until all supernodes have been assigned distinct neighbours.

3.3 Assignment of Nodes to Clusters

Once the initial clusters are created, we store the neighbourhood of each cluster and use this as input set of nodes which can be potentially added to a cluster. Given a clustering of p clusters $C = (C_1, C_2, ..., C_p)$ where each element of C contains a disjoint subset of the graph $G = (V, E)$ such that $C_i = v_1, v_2, ...v_n$, $n = |C_i|$ and $C_i \subset V$, we define the neighbourhood of a cluster i as

$$\Gamma(C_i) = (\Gamma_G(v_1) \cup \Gamma_G(v_2) ... \cup \Gamma_G(v_n)) \qquad (9)$$

Each of the candidate nodes is added temporarily to a cluster and a score based on that addition is calculated. The node which maximises the heuristic score of the graph (or of each cluster) is permanently added to the cluster. Once a node is added the process is repeated until all nodes have been assigned to clusters. The modularity and MQ heuristics are scored for the graph as a whole, so once a node is added all scores will have to be recalculated in the next round of assignments. If the clustering coefficient heuristic is being used, only the cluster average clustering coefficient of the cluster which has had a node added will have changed. The average clustering coefficients calculated for the other clusters and their candidate nodes will be unchanged from previous rounds. This allows caching of the results for later reuse, therefore it decreases computation time.

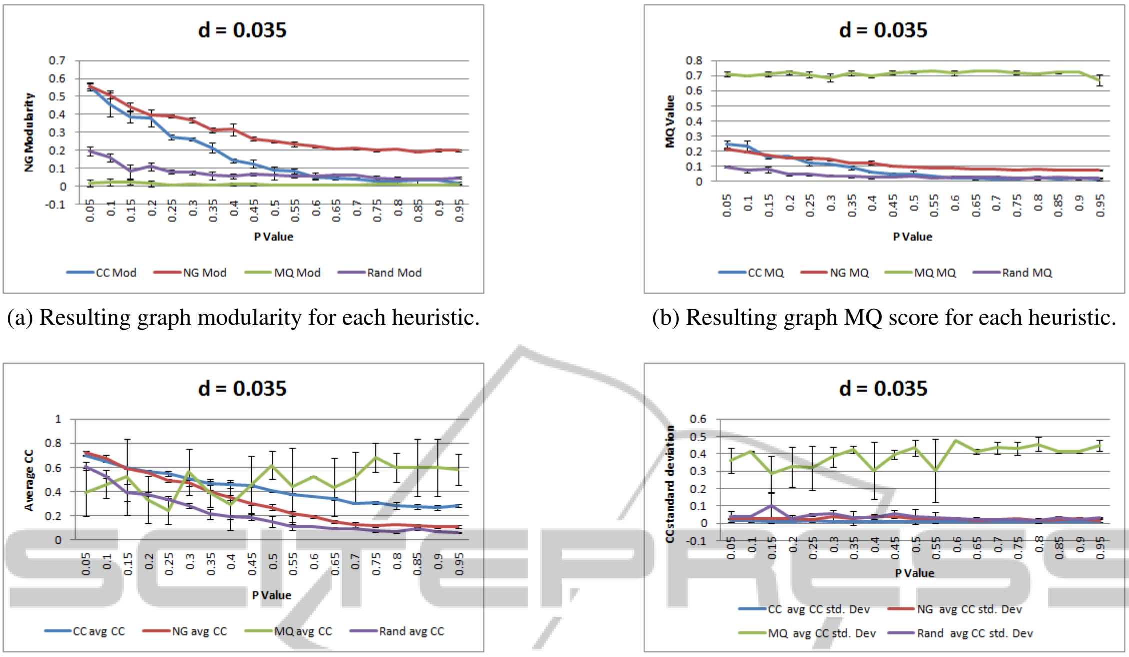
4 EVALUATION

4.1 Evaluation Graphs

We use Watts and Strogatz's beta approach for creating small world graphs (Watts and Strogatz, 1998) for evaluating the effectiveness of the heuristics. This approach allows us to create a large set of graphs of various densities and various levels of structure, from regular lattices, to small worlds graphs, to completely random graphs. The approach begins by creating a lattice like structure with edges uniformly distributed across vertices, k edges per-vertex resulting in $|E| = k|V|$. Each edge is rewired to a random target vertex with a probability P. For a low value of P the resulting graph is well structured and exhibits small world properties. As P approaches 1 the graph becomes more like a randomly connected graph. Each graph in our test set consists of 200 vertices. We have generated graphs varying the input probability to the beta model from 0.5 to 0.95. We have also varied the Edge density of the graph from a graph theoretic value of value of $d = 0.015$ to $d = 0.3$, resulting in the most dense graph having 11,940 edges. This is equivalent to a range of linear edge densities from $d_l = 1.49$ to $d_l = 29.85$. We have clustered each graph using our described heuristics. For evaluating the graphs we also rate them using the MQ score and modularity score of the resulting clustered graph, as well as examining the average of per cluster average clustering coefficients and the standard deviation of the per cluster average clustering coefficient. The four nodes with the largest neighbourhoods have been selected as the nodes of interest, resulting in four clusters. Due to the random nature of the graph generation we have averaged each result across 3 graphs generated with the same input parameters. Our full test set of data consist of 285 graphs for each of the three generation runs. We display the results of the clustering for a sample of the low density graphs in figure 4, a sample of the high density graphs in figure 5, a sample of the more structured graphs in figure 6 and a sample of the more random graphs in figure 7. The standard deviation of the heuristics across the 3 graphs is displayed as the error bounds.

4.2 Results and Analysis

The effectiveness of each heuristic differs depending on the density of the graph, how random the graph is as a result of the input probability p of the generation algorithm, and the metric used for evaluation. There are some constants however. Throughout the graphs displayed in figures 4 through 7 it can be seen



(a) Resulting graph modularity for each heuristic.

(b) Resulting graph MQ score for each heuristic.

(c) Resulting average cluster clustering coefficient for each heuristic.

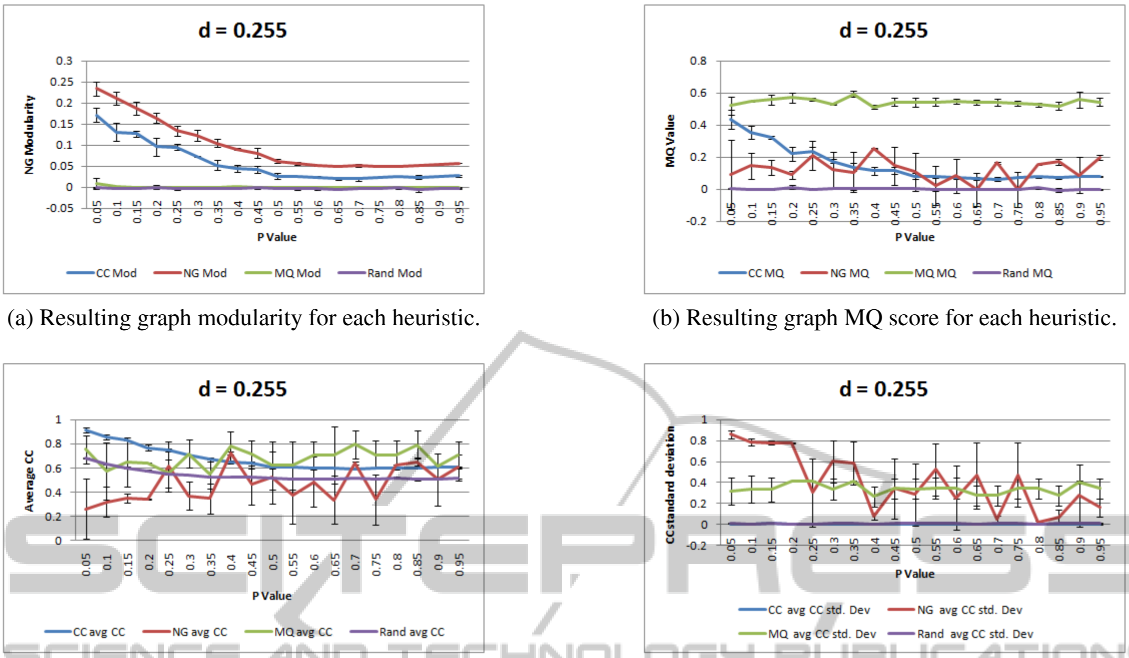
(d) The standard deviation of the average cluster clustering coefficient of each of the four clusters for each heuristic.

Figure 4: Evaluation of graphs with 200 Nodes and a density of 0.035 ($d_l = 3.483$), and an increasing level of randomness, denoted by p value.

that the random approach of assignment of nodes to clusters generally scores close to zero when evaluated using modularity or MQ. This is to be expected as both of these metrics lie in the range $[-1,1]$ with zero being equivalent to a random clustering. In the less dense graphs sometimes the random approach does score slightly above zero for a low input probability p, when the graphs are less random. This is because of the fact that our random approach does rely on nodes to be connected to the clusters they are added to, reducing the number of options for less well connected nodes. For higher density graphs this is not evident as a node will have a larger set of clusters it can be assigned to. It is clear from each of these figures that a graph scores well when it is rated with a metric that is also used as the heuristic to build the clusters. It also seems surprising that using MQ as a heuristic results in a high average per cluster clustering coefficient, however looking at the standard deviation of the per cluster clustering coefficients shows that the individual clusters vary wildly in quality. This is a result of a very imbalanced clustering, which will not be of benefit to a user if the majority of nodes are placed in a cluster with a low average clustering coefficient. This means that the nodes within the cluster will be less strongly related to each other.

4.2.1 Low Density Graphs

Figure 4 shows the resulting modularity, MQ and clustering coefficient values when the algorithm is run on graphs of increasing randomness with a relatively low density, $d = 0.035, d_l = 3.483$. Due to the relatively low density, there are fewer nodes to be chosen from when adding new nodes to the clusters, so nodes being added to a cluster are more likely to closely relate to several of the other nodes within the cluster. This is reflected by the higher scores for the random layout approach for each heuristic for a low levels of rewiring probability, see figure 4. Rating the graph based on modularity (figure 4a) results in the best results for the modularity heuristic with the clustering coefficient approach not far behind. The MQ approach noticeably scores similarly to the random approach. Rating the graph based on MQ (figure 4b) results in a consistently high score regardless of the level of randomness when using MQ as a heuristic. Using the average cluster clustering coefficient as a heuristic results in a low score of 0.2 for the more structured graphs, but this diminishes toward 0.0 as the graph becomes more random, making it no more effective than the random approach. The modularity heuristic scores similarly to the clustering coefficient



(a) Resulting graph modularity for each heuristic.

(b) Resulting graph MQ score for each heuristic.

(c) Resulting Average Cluster clustering coefficient for each heuristic.

(d) The standard deviation of the average cluster clustering coefficient of each of the four clusters for each heuristic.

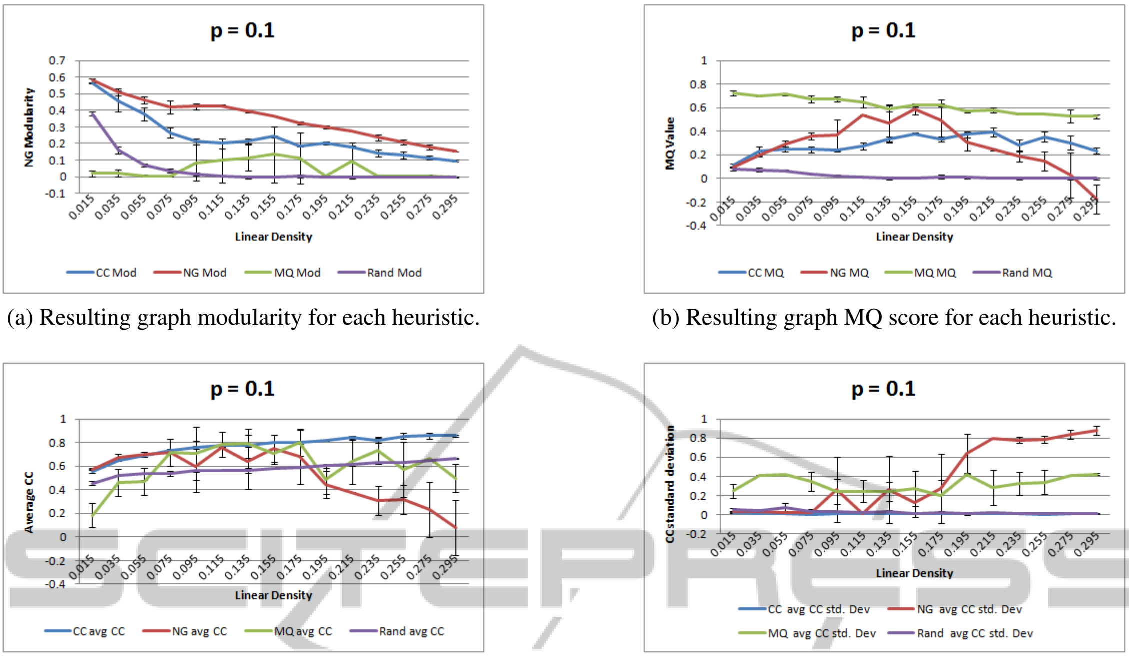
Figure 5: Evaluation of a graphs with 200 Nodes and a density of 0.255 ($d_l = 25.3725$), and an increasing level of randomness, denoted by p value.

heuristic, for structured graphs and also diminishes, but to a lesser degree than the clustering coefficient approach. Rating the graph using average cluster clustering coefficient (figure 4c), we can see that clustering coefficient and modularity perform similarly for structured graphs and diverge as the graphs become more random. We can also see from the error bounds that the MQ heuristic produces varying results for each of the 3 input graphs, where the other approaches are consistent in their results. Even with the large error bounds, once the graph becomes sufficiently random, MQ appears to provide the highest average clustering coefficient of the resulting clusters, but if we look at the standard deviation of the average clustering coefficient of each of the clusters created by the MQ heuristic (figure 4d), we can see that the quality of the clustering is quite poor. A large standard deviation of the average of the clustering coefficients of the 4 clusters indicates that while some clusters have a high clustering coefficient others will have a very poor one. This means the MQ does not in fact provide a good consistent average cluster clustering coefficient, therefore clusters will be created where adjacent nodes do not have many mutual neighbours, and will be less conceptually alike.

Conclusions: For graphs of low density clustering coefficient and modularity are the preferred heuristics when the graph contains structure. As the graphs become more random modularity becomes the sole preferred heuristic.

4.2.2 High Density Graphs

Changing the density of the graph has an impact on the performance of each of the heuristics. Figure 5 show results for graphs with a high density, $d = 0.255, d_l = 25.3725199$. Rating the graphs based on modularity (figure 5a), we see that overall the scores are lower when compared to the less dense graphs, however the same trends hold true as for the lower density graphs. Rating the graphs based on MQ score (figure 5b), we again see the MQ heuristic perform well. A noticeable difference is the improved performance of the clustering coefficient heuristic for the more structured graphs. The modularity metric performs better relative to clustering coefficient as graphs become more random, but the scores are less consistent, with larger standard deviations. Rating the graphs based on average cluster clustering coefficient results in the clustering coefficient heuristic performing the best for the more structure graphs. As the graphs become more random, the MQ heuristic per-



(a) Resulting graph modularity for each heuristic.

(b) Resulting graph MQ score for each heuristic.

(c) Resulting average cluster clustering coefficient for each heuristic.

(d) The standard deviation of the average cluster clustering coefficient of each of the four clusters for each heuristic.

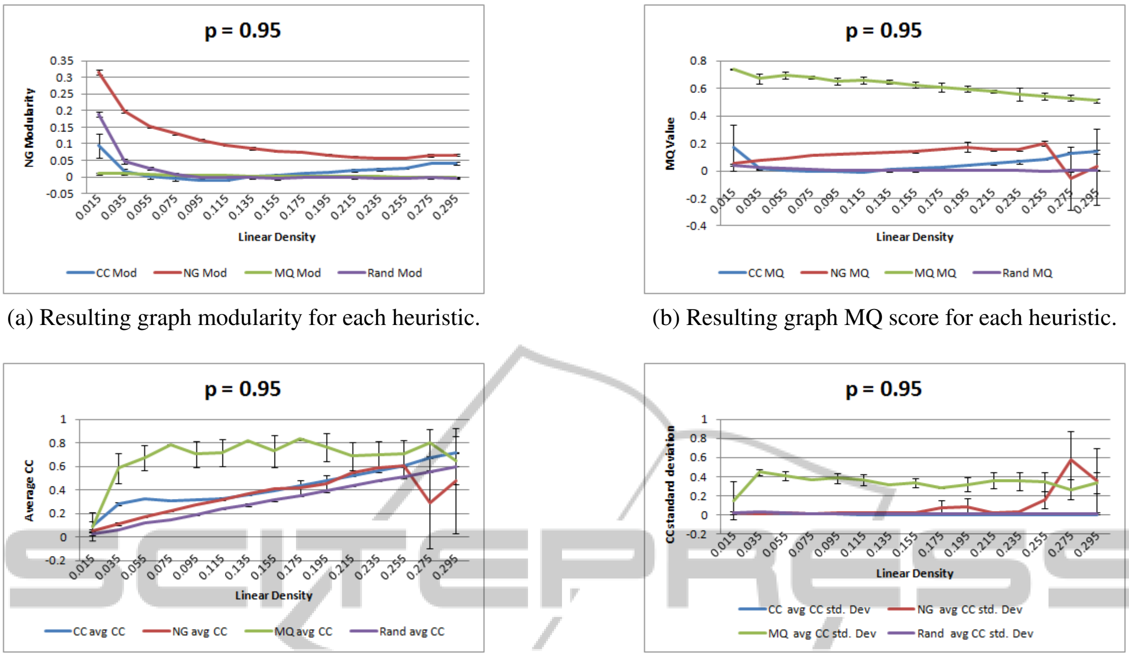
Figure 6: Evaluation of a graph with 200 Nodes and a constant input rewiring probability p = 0.1, and an increasing density.

formance does appear to perform slightly better, but the larger standard deviation in results across the input graphs reveals it does not do so. Also, as for the less dense graphs, the standard deviation of the average clustering coefficient (figure 5d) of each cluster is much higher than the clustering coefficient approach. It is noticeable that for most of the graphs the modularity heuristic performs even worse than using the random approach and that the clusters generated vary largely in average clustering coefficient. *Conclusions:* Clustering Coefficient is the most consistently high performing heuristic across all metrics. Modularity results in a large deviation in the average clustering coefficient of individual clusters within a graph as long as there is some structure in it. As the graph approaches random, modularity performs similarly to clustering coefficient, but there is not enough difference to recommend it above clustering coefficient.

4.2.3 Low Randomness Graphs

These are the graphs which exhibit small world properties. Rating the graphs based on modularity (figure 6a), we see the modularity heuristic score best as expected, however the score decreases as the graphs become more dense. The average clustering coefficient scores best out of the other heuristics and also decreases similarly to the modularity approach as the graphs become more dense. Using MQ as a heuristic, modularity behaves erratically, with large error bars and scores worse than random for the less dense graphs, and similar to random for the more dense graphs, with a large standard deviation. Rating the graphs based on MQ (figure 6b), we see the expected high score for MQ. Interestingly we see low scores for modularity for both the less dense and most dense graphs, however for graphs in the mid range of densities it does improve considerably, just about outperforming the clustering coefficient approach. Looking at the average cluster clustering coefficient (figure 6c), we see clustering coefficient performs the best at all densities, with close competition from modularity at lower graph densities. The average cluster clustering coefficient rating for the MQ heuristic still exhibits a large standard deviation between individual clusters (6b) for all densities. For more dense graphs, the use of modularity as a heuristic performs quite poorly.

Conclusions: The clustering coefficient heuristic performs relatively well across all levels of density for all metrics. The closest rival is modularity, which is similarly effective until a density of approximately 0.19 ($d_l = 18.905$) is reached.



(a) Resulting graph modularity for each heuristic.

(b) Resulting graph MQ score for each heuristic.

(c) Resulting average cluster clustering coefficient for each heuristic.

(d) The standard deviation of the average cluster clustering coefficient of each of the four clusters for each heuristic.

Figure 7: Evaluation of a graph with 200 Nodes and a constant input rewiring probability p = 0.95, and an increasing density.

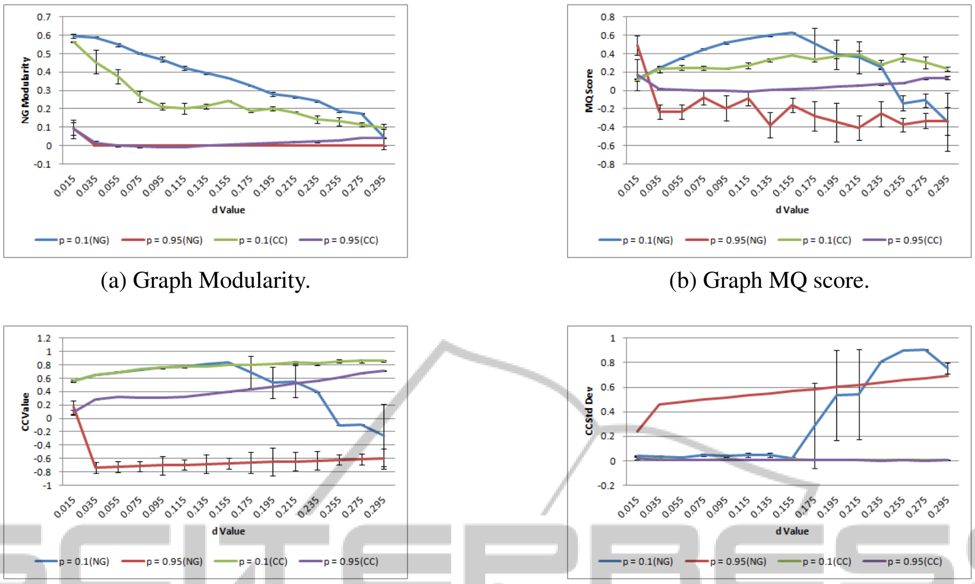
4.2.4 High Randomness Graphs

These are the graphs which exhibit a high level of randomness, and thus exhibit no small world properties. These graphs can give us insight into what approaches are affected most by the absence of a high clustering coefficient. All heuristics other than modularity perform poorly when rated using graph modularity (figure 7a). Rating the graph using MQ (figure 7b), we see, as expected, MQ performs very well, with modularity performing poorly but better than random or the clustering coefficient heuristic. When we rate the graphs using average cluster clustering coefficient (figure 9c) we see that there are some small improvements over random using modularity and clustering coefficients as heuristics, and that as graph density increases the scores for these approaches increases in a manner similar to the random approach. This is to be expected given the random nature of the graph. From this figure and figure 5b, we can see that when using MQ as a heuristic and rating the final clustered graph using MQ, the results are not reliable as they appear to be independent of graph randomness and only slightly affected by graph density.

Conclusions: Overall the best heuristic for graphs which are more random and less structured appears to be modularity, until the graphs become very dense $d = 0.26, (d_l = 25.87)$, when average clustering coefficient becomes marginally better.

4.2.5 Comparison with Edge Betweenness Centrality Clustering

To provide a comparison with a state of the art clustering approach we performed a similar analysis on our test data set having applied Edge Betweenness Centrality clustering (Newman and Girvan, 2004). This is a top town clustering approach which tries to find naturally occurring clusters within the data. Unlike our approach, the number of clusters is not usually specified and there is no equivalent of a user specifying nodes of interest. However it is an effective algorithm which can distinguish the clusters which naturally occur within a small world graph. The algorithm generates a hierarchy of partitions. The partitioning with the best modularity score is chosen from this hierarchy as the final clustering. This can result in a high number of clusters depending on the density and structure of the graph, as can be seen in figure 10. In many cases a very large number of clusters are created, thus for our comparison we are constraining the number of clusters formed by the Newman and Girvan approach to 4, the same number used for our agglomerative clustering analysis. Evaluation uses the same approach as that used for evaluating our clustering heuristics and the results can be seen in figures 8



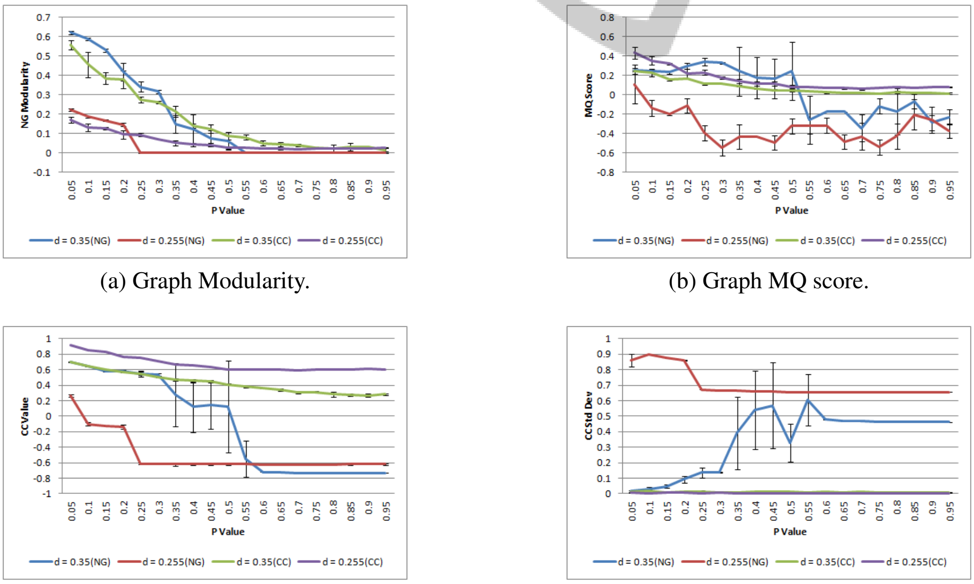
(a) Graph Modularity.

(b) Graph MQ score.

(c) Average cluster Clustering Coefficient of the 4 clusters.

(d) The standard deviation of the average cluster Clustering Coefficient of each of the clusters.

Figure 8: Evaluation of test graphs when clustered using Newman and Girvan's Edge Betweenness Centrality clustering (NG) and our clustering coefficient heuristic (CC) for comparison. The graphs display the metrics for well structured (p = 0.1) and unstructured graphs (p = 0.95) of increasing density, where the number of clusters is constrained to 4.



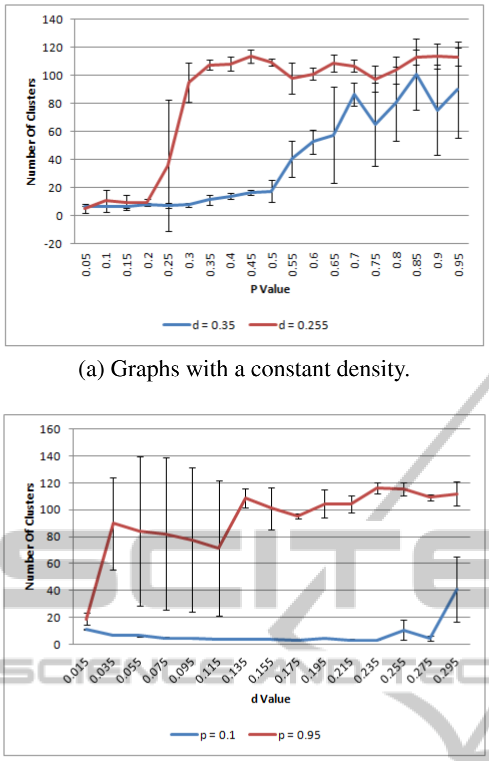
(a) Graph Modularity.

(b) Graph MQ score.

(c) Average cluster Clustering Coefficient of the 4 clusters.

(d) The standard deviation of the average cluster clustering coefficient of each of the clusters for high and low density graphs, both with and without a predetermined number of clusters.

Figure 9: Evaluation of test graphs when clustered using Newman and Girvan's Edge Betweenness Centrality Clustering (NG) and our clustering coefficient heuristic (CC) for comparison. The graphs display the metrics for low density (d = 0.035) and high density graphs (d = 0.255) with an decreasing level of structure (p increasing), where the number of clusters is constrained to 4.



(a) Graphs with a constant density.

(b) Graphs with a constant rewiring probability.

Figure 10: Number of clusters generated using Edge betweenness Centrality Clustering.

and 9.

Less Dense Graphs. When clustering lower density graphs, compared to cluster coefficient heuristic agglomerative approach, the cluster count limited version of Edge Betweenness Centrality lustering predictably scores higher on modularity (figure 9a). This is as the clustering is not constrained by the user specifying supernodes to form the basis of clusters and the algorithm is very effective at finding the small amount of clusters in the more structured graphs (see figure 10). Predictably as the graph becomes more random this difference diminishes until the cluster coefficient approach produces cluster with a higher level of modularity, as there are fewer naturally occurring communities for more random graphs. For the MQ score, (see figures 9b , 8b) Edge Betweenness Centrality clustering is superior, however the difference is not as large, and once the graphs become less structured the performance of the approach drops off significantly. In terms of clustering coefficient Edge Betweenness Centrality clustering performs similarly for the more structured graphs but drops off significantly as the graphs become more random.

More Dense Graphs. From figure 8 it can be seen that graphs with stronger small world graph characteristics modularity is slightly better, but the clustering coefficient approach performs better once the graphs become slightly more random (at approximately p = 0.2, so the underlying structure is still quite strong). However for the MQ score we find that, for the more dense graphs, the clustering coefficient consistently outperforms the Edge Betweenness Centrality clustering approach. Our approach also provides equivalent and better average clustering coefficient for clusters and far higher clustering coefficient values for the more random graphs (due to all of the singleton clusters). Our approach also maintains more consistently high clustering coefficient values for the more dense graphs $(d > 0.235, d_l > 23.38)$ than Newman and Girvan's approach. The low standard deviation between the clustering coefficients also indicates that the resulting average clustering coefficient is balanced across multiple clusters.

5 CONCLUSIONS AND FUTURE WORK

Based on the preceding analysis the most consistently effective heuristic for agglomerative clustering around nodes of interest is clustering coefficient, especially for small world graphs. It scores well on modularity and produces clusters with a high average clustering coefficient that is balanced across all clusters. The MQ scores for all heuristics other than MQ are generally quite low, but average clustering coefficient does perform well for dense graphs and with a high level of structure. The clustering coefficient heuristic was also was more stable when run over different graphs generated with the same input parameters, as evidenced by the smaller error bars on the preceding graphs. Modularity also works as an effective heuristic for agglomerative clustering, and is more effective than clustering coefficient when the graphs become more random. However for small world graphs clustering coefficient produces more consistent results in terms of the average clustering coefficient of resulting graphs. MQ performed the least successfully of the heuristics when used for agglomerative clustering. We also compared our agglomerative approach using clustering coefficient as a heuristic to Newman and Girvan's Edge Betweenness Centrality algorithm, constrained to produce four clusters. The comparison is not a direct one as the agglomerative algorithm focuses on building clusters around nodes of interest and the betweenness centrality algorithm defines clusters without any such constraints. As expected the Edge Betweenness Centrality clustering algorithm performs very well on structured graphs with low density. However as the graphs become more dense the agglomerative algorithm performs close to the level of the centrality algorithm and by some metrics (MQ and clustering coefficient) it outperforms the algorithm for graph with a density of $d = 0.255, d_l = 25.373$. Given that the agglomerative approach is designed to focus around nodes of interest to aid in visualisation rather than discover communities, we feel our algorithm compares favourably with the Edge Betweenness Centrality algorithm.

This paper has examined the effectiveness of the clustering heuristics purely using calculated metrics. Further evaluation is required using user experiments to determine fully the effect of the clustering on graph comprehensibility. Such an evaluation could also be extended to cover examples of real-world graphs, rather than large sets of procedurally created ones. Further work is required concerning the layout of these clusters and their visualisation. Currently when visualising the graphs we use a simple force directed layout of individual clusters, however a graph layout with consideration given to inter-cluster edges to reduce edge crossing could be very beneficial. Node hierarchies are frequently used to aid layout, so one potential application of the above clustering approach is to recursive apply it to generated cluster to generate a hierarchy to aid in layout and in the routing of edges within large graphs. The routing of edges between and within clusters also impacts graph comprehensibility, so an approach such as Holten's hierarchical edge bundling (Holten, 2006) may be useful here.

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